Scalar fields with V-shaped self-interaction

The canonical example of the V-shaped self-interaction is $U(\phi) = \lambda|\phi|$, where ϕ is a real scalar field, λ is a positive constant. Because the modulus function is not differentiable at $\phi = 0$, we regularize it,

$$U(\phi) \to U_\epsilon(\phi) = \lambda\sqrt{\epsilon + \phi^2(x)},$$

where ϵ is a positive constant. The action reads

$$S[\phi] = \int d^4x\, (\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U_\epsilon(\phi)).$$

The corresponding Euler-Lagrange equation in the limit $\epsilon \to 0+$ has the form

$$\partial_\mu\partial^\mu\phi(x) + \lambda\,\text{sign}(\phi(x)) = 0, \tag{1}$$

where the signum function is defined as the limit

$$\text{sign}(\phi(x)) = \lim_{\epsilon\to 0+} \phi(x)/\sqrt{\epsilon + \phi^2(x)}.$$

Note that this prescription implies that $\text{sign}(0) = 0$. The real scalar field with equation (1) is called the signum-Gordon model, as suggested by Benny Lautrup from the Niels Bohr Institute.

The signum-Gordon model is interesting for at least two reasons. First, one can find exact analytic nontrivial solutions of Eq. (1), e.g., non radiating oscillons (breathers), or various self-similar solutions. Second, the vacuum field, $\phi = 0$, is reached exactly on a finite distance from localized external sources. Therefore, the force mediated by such a field has a strictly finite range. For this reason, the field can be called 'hyper-massive'. See the papers [1-5].

In the case of complex scalar field Φ we take the field potential in the form $U_\epsilon(\Phi, \Phi^*) = \lambda\sqrt{\epsilon + |\Phi(x)|^2}$. The field equation in the limit $\epsilon \to 0_+$ reads

$$\partial_\mu\partial^\mu\Phi(x) + \lambda\,\frac{\Phi(x)}{|\Phi(x)|} = 0,$$

where $\Phi/|\Phi| = 0$ if $\Phi = 0$. This model inherits the main features of the signum-Gordon model. Here we have found various nontopological solitons of the Q-ball type, [6, 7]. Their common feature is compactness in the sense that the scalar field exactly vanishes outside certain finite region in space.

The work is continued by Paweł Klimas and his collaborators at the University of Santa Catarina in Florianópolis, Brasil, see, e.g., the papers [8, 9].

I plan to consider applications in particle physics, and to construct a quantum model with the V-shaped self-interaction.

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