Quantum mechanics of the relativistic Majorana particle.
The Majorana field.

This project has emerged from the work on our textbook *"Lectures on Classical and Quantum Theory of Fields"*. Writing about the Majorana field, I could not find in literature any Lagrangian for it in the case it is represented by a real bispinor with commuting components (I call such field the classical Majorana field). On the other hand, in the case the components anti-commute, i.e., when they are Grassmann elements, the Lagrangian is well-known and simple. However, in the parallel case of the Dirac field Lagrangians are well-known and simple in both cases. This difference puzzled me. I regarded it, and still regard, as a hint about something important. Some time later I obtained a Lagrangian for the massless classical Majorana field. Actually, I rediscovered S. Tanaka's Lagrangian from 1960. But the massive case remained unsolved – here the Tanaka Lagrangian gives tachyons.

Equally puzzling is quantum mechanics of a single relativistic Majorana particle. Here the wave equation has the form of the Dirac equation in which the Dirac matrices γ^μ are purely imaginary (this is so called Majorana representation of these matrices). Quantum states of the particle are represented by the bispinor with real components mentioned above. It is a quantum mechanics based on real numbers, complex numbers are not allowed. Observables have to respect this fact. In consequence, the standard momentum operator $\hat{p} = -i\nabla$ can not be accepted because of the factor i. Nabla operator ∇ is real, but it is not Hermitean. Hermitean and real is the operator $\hat{p}_5 = -i\gamma_5\nabla$ (the matrix γ_5 is purely imaginary and Hermitean). We call it the axial momentum. Its properties are discussed in papers [1], [2]. In the Heisenberg picture, it oscillates with a frequency proportional to the rest mass of the particle. Pedagogical introduction to the relativistic quantum mechanics of the Majorana particle is given in [3].

Eigenfunctions of the axial momentum (called the axial plane waves) provide a convenient mode expansion of the classical Majorana field. Using it we have quantized the field, [4]. While the Fock space, particle spectrum, etc., essentially coincide with the ones described in literature, the procedure itself differs in interesting way.

Moreover, using that mode expansion we have found a Lagrangian for the classical Majorana field in the massive case. Unfortunately, it is nonlocal, it is not Lorentz invariant, and it looks really ugly. For these reasons we have not published it yet – we hope there exists a simpler Lagrangian.

Literatura

[1] H. A., *Axial momentum for the relativistic Majorana particle*, Phys. Lett. A 383, 1242 (2019) (arXiv:1805.03016).

[2] H. A., Z. Świerczyński, *On relativistic quantum mechanics of the Majorana particle: quaternions, paired plane waves, and orthogonal representations of the Poincaré group*, J. Phys. G: Nucl. & Part. Phys. 48, 065001 (2021) (arXiv: 1910.13920).

[3] H. A., *Relativistic quantum mechanics of the Majorana particle*, Acta Phys. Polon. B 50, 2165 (2019) (arXiv: 2002.07482).

[4] H. A., *Axial momentum and quantization of the Majorana field*, Acta Phys. Polon. B 53, 2-A4 (2022) (arXiv: 2202.05133).