Commentary on
"Lectures on Classical and Quantum Theory of Fields"

1. In the second edition, the chapter "Relativistic Spinor Fields" has been modified, especially in the part devoted to the Majorana field. Pseudoclassical fermionic field, which has anticommuting components, is now discussed in the new Section 5.5.

2. We plan to add a chapter on QED in the Coulomb gauge. It will be posted on this web page when ready.

3. Writing a textbook is an opportunity to think about various fundamental concepts. Inevitably, one notices loopholes in existing explanations, but also interesting possibilities for research. In the case of theory of fields there are plenty of them. Two examples are given below.

4. Let us have a look at the notion of pseudoclassical fermionic fields. It is rather abstract concept. Such fields are neither quantum nor classical. We expect that total energy, total momentum, etc., of truly classical fields have numerical values (in a system of units). In the case of pseudoclassical fields, the Noether theorem gives expressions which are elements of the Grassmann algebra, not numbers. On the other hand, quantum fields are represented by operator-valued distributions, which involve an infinitely dimensional Hilbert space, while in the standard description of pseudoclassical fields it is assumed that their components at each point of the spacetime belong to a finite dimensional Grassmann algebra, and that algebras at different points are independent.

The question arises what is mathematical definition of integral of pseudoclassical field (or of a product of such fields) over spacetime. Does it suffice to regard it as a formal expression which has all algebraic properties of an integral, but which does not have any definite value? In such approach we do not care about convergence of the integral because convergence is not even defined.

One can have questions also about derivatives of the pseudoclassical fields. Is the r.h.s. of the expression

$$\partial\psi^\alpha(x)/\partial x^\mu = lim_{a\to 0}\frac{\psi^\alpha(x + a\, e_{(\mu)}) - \psi^\alpha(x)}{a}$$

defined at all? Remember that $\psi^\alpha(x + a\, e_{(\mu)})$ and $\psi^\alpha(x)$ belong to different Grassmann algebras if $a \neq 0$. Here $e_{(\mu)}$ is the unit four-vector with components $e^\nu_{(\mu)} = \delta^\nu_\mu$.

We regard the derivative as a linear transformation of the Grassmann elements $\psi^\alpha(x)$, namely

$$\frac{\partial\psi^\alpha(x)}{\partial x^\mu} = \int d^4y\ \frac{\partial\delta(x-y)}{\partial x^\mu}\psi^\alpha(y).$$

The kernel of this transformation is singular. Hence partial derivatives of pseudoclassical field might require a regularization!

Such approach to pseudoclassical fields is sufficient for calculations of Feynman's path integrals. Nevertheless, I think there is a need of a more complete mathematical treatment.

5. Quantum fields are operator-valued distributions. Therefore, expressions such as $\int d^4x\ \hat{\phi}^4(x)$, where $\hat{\phi}(x)$ is a free scalar field, are mathematically incorrect. As discussed in the textbook, we should use its regularized version, e.g.,

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4\ g(x_1, x_2, x_3, x_4)\ \hat{\phi}(x_1)\hat{\phi}(x_2)\hat{\phi}(x_3)\hat{\phi}(x_4), \quad (1)$$

where $g(x_1, x_2, x_3, x_4)$ is a fixed test function. Such test function is smooth and it vanishes at infinity. However, it turns out that the presence of it breaks relativistic invariance (the Poincaré symmetry). In order to preserve this symmetry, we use the Pauli-Villars regularization instead of the test function. This is equivalent to replacing the test function g with certain distribution. Expression (1) then becomes a convolution of distributions. Nevertheless, it works at least in the perturbative expansion. It is rather intriguing that in such a relativistically invariant regularization we use distributions, not test functions. This may be regarded as a hint that relativistic fields are distributions of a special kind. What exactly are these special features?