

Task 1.

Find numerical approximation for definite integral

$$\int_0^1 f(x) \, dx.$$

Please use the following methods:

1. trapezoidal rule,
2. Simpson rule,
3. apply above using recursive interval subdivision until desired error measure has been reached,
4. gaussian quadrature,
5. Monte Carlo method,
6. other methods.

Test your implementation of the above algorithms for a variety of functions $f(x)$. Compare with analytical result, if applicable. Estimate convergence and speed of the algorithm.

Find numerical approximation for improper integral:

$$\int_0^\infty \frac{f(x)e^{-x}}{\sqrt{x}} \, dx.$$

Please use the following methods:

1. methods from Taks 1,
2. Gaussa-Laguerre quadrature with weight e^{-x} ,
3. Gaussa-Laguerre quadrature with weight e^{-x}/\sqrt{x} ,
4. tanh – sinh quadrature using transformation $x = e^{t-e^{-t}}$ and trapezoidal rule,
5. use $x = t^2$ transform to Gauss-Hermite quadrature.

Test your implementation of the above algorithms for a variety of functions $f(x)$. Compare with analytical result, if applicable. Estimate convergence and speed of the algorithm.

Zadanie 2.