Zadanie 1.

Find approximate numerical solution to the Laplace'a equation in 2 dimensions

$$\Delta f(x,y) = 0,$$

in the square $[0,\pi] \times [0,\pi]$ with boundary condition resulting from known analytical exact solution

$$f(x,y) = \Re e(\sin z + z), \quad z = x + iy.$$

To achieve goal, discretize interval $[0,\pi]$ into $N-1$ cells and use derivative formulas derived previously. Resulting linear system cast into matrix form

$$A.x = b.$$

where x denotes discrete values $f_{ij} \equiv f(x_i, x_j)$ on the grid $\{x_i, y_j\}$.

Analyze structure of the $(N-2)^2 \times (N-2)^2$ matrix A

- non-zero elements
- structure of the matrix
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Linear system solve using solvers

- LU decomposition
- sparse matrix,
- multidiagonal,
- iterative,
- other, e.g. using GPU (see Project IV).

Analyze CPU and memory utilization, and compare results to known analytical formulas.

Zadanie 2*.

Similar to previous, but inside disk of radius $R = 1$, with Dirichlet condition defined by function $u(\phi)$. Compare with analytical formula

$$f(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2 - r^2}{R^2 - 2Rr\cos(\theta - \phi) + r^2}\, u(\phi)\, \mathrm{d}\phi.$$