Project description

Goal of the project is to compute integral of the form

$$I[f(x)] = \int_0^\pi f(x)\ \sin x\ \mathrm{d}x,$$

using as few as possible calls of the function $f(x)$, assumed to be very intense computationally, however „smooth". For example, to evaluate $f(\pi/2)$ one might require few hours of CPU time. To solve the problem Gaussian quadrature should work:

$$I[W_{2n+1}(x)] \equiv \sum_{i=0}^{n} w_i W_n(x_i),$$

where $W_n(x)$ is **arbitrary** polynomial of the order $2n+1$, and above equation becomes identity.

Integral is then evaluates using:

$$I[f(x)] \simeq \sum_{i=0}^{n} w_i f(x_i).$$

Analitycal solution for $n = 1$:

$$\begin{aligned} x_0 &= \tfrac{\pi}{2} - \tfrac{1}{2}\sqrt{\pi^2 - 8}, \\ x_1 &= \tfrac{\pi}{2} + \tfrac{1}{2}\sqrt{\pi^2 - 8}, \\ w_0 &= w_1 = 1 \end{aligned}$$

Numerical solution for $n = 2$:

$$\begin{aligned} x_0 &= 0.5581950867965893545098494001963, \\ x_1 &= 1.5707963267948966192313216916398, \\ x_2 &= 2.58339756667932038839527939830832, \\ w_0 &= 0.4558404080391222561098574237169, \\ w_1 &= 1.0883191839217554877802851525663, \\ w_2 &= 0.45584040803912225610985742371686. \end{aligned}$$

For project solution to be considered at all, $n > 6$ is required. Additionally, one can assume vanishing derivatives of $f(x)$ at interval endpoints: $f'(0) = f'(\pi) = 0$.