

## ZESTAW ZADAŃ 2

### Zadanie 2.1

Obliczyć granice ciągów:

$$\lim_{n \rightarrow \infty} \sqrt{1+n} \quad (1a) \quad \lim_{n \rightarrow \infty} \sqrt{n^2+n} - \sqrt{n^2+1} \quad (1e) \quad \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{1-\frac{1}{2+n}}}{n^2+1} \quad (1i)$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n-3} \quad (1b) \quad \lim_{n \rightarrow \infty} \left( \frac{3n-1}{1+n^2} - n \right) \quad (1f)$$

$$\lim_{n \rightarrow \infty} (2^n + 5^n + 7^n)^{1/n} \quad (1c) \quad \lim_{n \rightarrow \infty} \frac{1+n(n-2)}{n^2-n} \quad (1g) \quad \lim_{n \rightarrow \infty} \frac{n[n(n-1)-1]}{n^2(n^2+1)+1} \quad (1j)$$

$$\lim_{n \rightarrow \infty} \sqrt{n+2} - \sqrt{n+1} \quad (1d) \quad \lim_{n \rightarrow \infty} \frac{2}{2 - \frac{2}{1+\frac{1}{n}}} \quad (1h)$$

### Zadanie 2.2

Rozłożyć podane wyrażenia wymierne na ułamki proste:

$$\frac{n^2 - 2n - 5}{n^3 + 2n^2 - n - 2} \quad (2a) \quad \frac{2n^3 + n^2 + 1}{n^2 + n^4} \quad (2c)$$

$$\frac{1}{k(k+1)} \quad (2b) \quad \frac{1}{(n^2 - n - 2)(n^2 - \frac{9}{4})} \quad (2d)$$

### Zadanie 2.3

Wyznaczyć ciąg sum częściowych dla szeregów:

$$1 + 2 + 3 + 4 + \dots + (n-1) + n \quad (3a)$$

$$\sum_{k=1}^n \frac{1}{k(k+1)} \quad (3b)$$

$$1 - 3 + 5 - 7 + 9 - 11 \dots \quad (3c)$$

**Zadanie 2.4**

Wyznacz  $n$ -ty wyraz  $a_n$  szeregu o sumie częściowej równej:

$$\sum_{k=0}^n a_k = \frac{1}{n} \quad (4a)$$

$$\sum_{k=0}^n a_k = n + n_0 \quad (4c)$$

$$\sum_{k=0}^n a_k = 1 - 2 \cos\left(\frac{2+n}{3}\pi\right) \quad (4b)$$

$$\sum_{k=0}^n a_k = 2^{n+1} \quad (4d)$$

**Zadanie 2.5**

Zbadaj zbieżność szeregów:

$$\sum_{k=1}^{\infty} \frac{\pi}{k^2} \quad (5a)$$

$$\sum_{n=1}^{\infty} \sqrt[n]{n} \quad (5f)$$

$$\sum_{k=1}^{\infty} \frac{3 \cdot 4^k}{4 k^2 3^k} \quad (5j)$$

$$\sum_{k=1}^{\infty} \operatorname{tg}(1/k) \quad (5b)$$

$$\sum_{k=0}^{\infty} \frac{3k}{3^k} \quad (5g)$$

$$\sum_{k=0}^{\infty} \frac{1}{(2k)!} \quad (5c)$$

$$\sum_{n=1}^{\infty} \frac{\cos(2n-1) - \cos(2n+1)}{n} \quad (5k)$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{k!}\right)^2 \quad (5d)$$

$$\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k} \quad (5h)$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n} \quad (5e)$$

$$\sum_{k=1}^{\infty} \frac{k!}{k^k} \quad (5i)$$

$$\sum_{n=1}^{\infty} \frac{(7n+5)(4n!)}{10^{4n}(n!)^4} \quad (5l)$$

**Zadanie 2.6**

Udowodnij, że:

$$\sum_{i=0}^n i^3 = \left(\sum_{i=0}^n i\right)^2. \quad (6)$$

**Zadanie 2.7**

Oblicz sumę:

$$\sum_{k=0}^{\infty} \left(\frac{1}{k} \frac{1}{k+1} \frac{1}{k+2} \frac{1}{k+3}\right). \quad (7)$$