$$\overrightarrow{A} \times (\overrightarrow{A} \times \overrightarrow{B}) = (\overrightarrow{A} \cdot \overrightarrow{B}) \cdot \overrightarrow{A} - (\overrightarrow{A} \cdot \overrightarrow{A}) \cdot \overrightarrow{B}$$

$$\overrightarrow{A} \times (\overrightarrow{A} \times \overrightarrow{B}) = \overrightarrow{A} \times det \begin{bmatrix} \widehat{x} & \widehat{y} & \widehat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = det \begin{bmatrix} \widehat{i} & \widehat{j} & \widehat{k} \\ A_x & A_y & A_z \\ A_yB_z - A_zB_y & A_zB_x - A_xB_z & A_xB_y - A_yB_x \end{bmatrix}$$

Policzmy najpierw tylko $\widehat{i}$:

$$\widehat{i}(A_yA_xB_y - A_yA_yB_x + A_zA_xB_z - A_zA_zB_x)$$

Dodajmy i odejmijmy $A_xA_xB_x$

$$\widehat{i}(A_yA_xB_y - A_yA_yB_x + A_zA_xB_z - A_zA_zB_x + A_xA_xB_x - A_xA_xB_x)$$

Nastepnie wyciagnijmy A_x i B_x przed nawias

$$\widehat{i}(A_x(A_xB_x + A_yB_y + A_zB_z) - B_x(A_xA_x + A_yA_y + A_zA_z))$$

Jedyne co nam pozostao to wynik iloczynu skalarnego A z B w pierwszym nawiasie oraz A z A w drugim

$$\widehat{i}(A_x(\overrightarrow{A} \cdot \overrightarrow{B}) - B_x(\overrightarrow{A} \cdot \overrightarrow{A}))$$

Nastepnie powtarzamy powyzsze kroki dla $\widehat{j}$ oraz $\widehat{k}$. Otrzymujemy wiec:

$$\begin{aligned} \overrightarrow{A} \times (\overrightarrow{A} \times \overrightarrow{B}) = & \widehat{i}A_x(\overrightarrow{A} \cdot \overrightarrow{B}) - \widehat{i}B_x(\overrightarrow{A} \cdot \overrightarrow{A}) + \widehat{j}A_y(\overrightarrow{A} \cdot \overrightarrow{B}) - \widehat{j}B_y(\overrightarrow{A} \cdot \overrightarrow{A})) + \widehat{k}A_z(\overrightarrow{A} \cdot \overrightarrow{B}) - \widehat{k}B_z(\overrightarrow{A} \cdot \overrightarrow{A})) = \\ & = (\overrightarrow{A} \cdot \overrightarrow{B})(\widehat{i}A_x + \widehat{j}A_y + \widehat{k}A_z) - (\overrightarrow{A} \cdot \overrightarrow{A})(\widehat{i}B_x + \widehat{j}B_y + \widehat{k}B_z) = \\ & = (\overrightarrow{A} \cdot \overrightarrow{B}) \cdot \overrightarrow{A} - (\overrightarrow{A} \cdot \overrightarrow{A}) \cdot \overrightarrow{B} \end{aligned}$$