$$\overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C}) + \overrightarrow{C} \cdot (\overrightarrow{B} \times \overrightarrow{A}) = 0$$

Najpierw wyliczamy $\overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C})$

$$\overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C}) = det \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix} = A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Identycznie wyliczamy $\overrightarrow{C} \cdot (\overrightarrow{B} \times \overrightarrow{A})$ i dodajemy do $\overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C})$:

$$\begin{aligned} \overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C}) + \overrightarrow{C} \cdot (\overrightarrow{B} \times \overrightarrow{A}) &= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x + \\ &+ C_xB_yA_z - C_xB_zA_y - C_yB_xA_z + C_yB_zA_x + C_zB_xA_y - C_zB_yA_x = \\ &= A_xB_yC_z - C_zB_yA_x + C_zB_xA_y - A_yB_xC_z + C_yB_zA_x - A_xB_zC_y + \\ &+ A_yB_zC_x - C_xB_zA_y + A_zB_xC_y - C_yB_xA_z - A_zB_yC_x + C_xB_yA_z = 0 \end{aligned}$$