$$\overrightarrow{F} = \{2xz^2 - 2y, -2x - 6yz, 2x^2z - 3y^2\}$$
$$\overrightarrow{G} = \{x^2z, -xy, 0\}$$

Dywergencja: $\nabla \cdot \overrightarrow{F} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \cdot (F_x, F_y, F_z) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$$\nabla \cdot \overrightarrow{F} = 2z^2 - 6z + 2x^2$$
$$\nabla \cdot \overrightarrow{G} = 2xz - x$$

Rotacja: $\nabla \times \overrightarrow{F} = det \begin{bmatrix} \widehat{x} & \widehat{y} & \widehat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{bmatrix} = \widehat{x}(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}) + \widehat{y}(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}) + \widehat{z}(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})$

$$\nabla \times \overrightarrow{F} = [-6y + 6y, 4xz - 4xz, -2 + 2] = [0, 0, 0] = \overrightarrow{0}$$
$$\nabla \times \overrightarrow{G} = [0 - 0, x^2 - 0, -y - 0] = [0, x^2, -y]$$

Pole jest zachowawcze gdy rotacja = $\overrightarrow{0}$
Potencjal pola istnieje tylko gdy pole jest zachowawcze

Potencjal pola:

$$u = \int (2xz^2 - 2y)dx = z^2x^2 - 2yx + \phi(y, z)$$

$$\frac{\partial u}{\partial y} = -2x + \frac{\partial \phi}{\partial y}$$

$$-2x + \frac{\partial \phi}{\partial y} = F_y$$
$$-2x + \frac{\partial \phi}{\partial y} = -2x - 6yz$$

$$\frac{\partial \phi}{\partial y} = -6yz$$

$$\phi(y, z) = \int (-6yz)dy = -3zy^2 + \psi(z)$$

$$u = z^2x^2 - 2yx - 3zy^2 + \psi(z)$$

$$\frac{\partial u}{\partial z} = 2zx^2 - 3y^2 + \frac{\partial \psi}{\partial z}$$

$$2x^2z - 3y^2 + \frac{\partial \psi}{\partial z} = F_z$$
$$2x^2z - 3y^2 + \frac{\partial \psi}{\partial z} = 2x^2 - 3y^2$$

$$\frac{\partial \psi}{\partial z} = 0$$

$$\psi(z) = C$$

$$u(x, y, z) = z^2x^2 - 2yx - 3zy^2 + C$$