Solution of any of the problems below is equivalent to passing an examination.

1. MESA. Evolve $4M_\odot$ star until black dwarf.

MESA has a problem with default settings (http://mesa.sourceforge.net/) to evolve star up to cooling white dwarf stage (Q_limit). Using examples for $0.25, 0.5, 1, 2, 8M_\odot$ stars included in lecture notes, and test_suite of MESA, find appropriate settings (inlist). Problems are: slow stellar wind (infinitely long calculations) and or envelope instabilities (zero timestep).

2. Find orbital elements from numerical solution.

It is relatively easy to find N-body solution with slowly varying elliptical orbit. Propose and verify numerical method to extract elliptical orbit parameters $(a, e, i, T$ etc.) as a function of time using previously found numerical N-body evolution.

3. Verify linear mass approximation method.

In theory we could under some conditions replace orbiting body with linear mass density smeared out on the elliptical orbit. This is particularly easy for circular orbit, replaced with uniform density circle. Compare evolution in circular potential with true point mass in circular orbit.

Zadanie 4. Van der Waals planet.

Find mass-radius relation for izotermal Van der Waals gas ball.

Zadanie 5. Copernicus on self-gravity

Translate from Latin original De Revolutionibus part, related to spherical shape of the Sun, Earth and Moon. Did Copernicus predict self-gravity indeed?

6. Ultradense material.

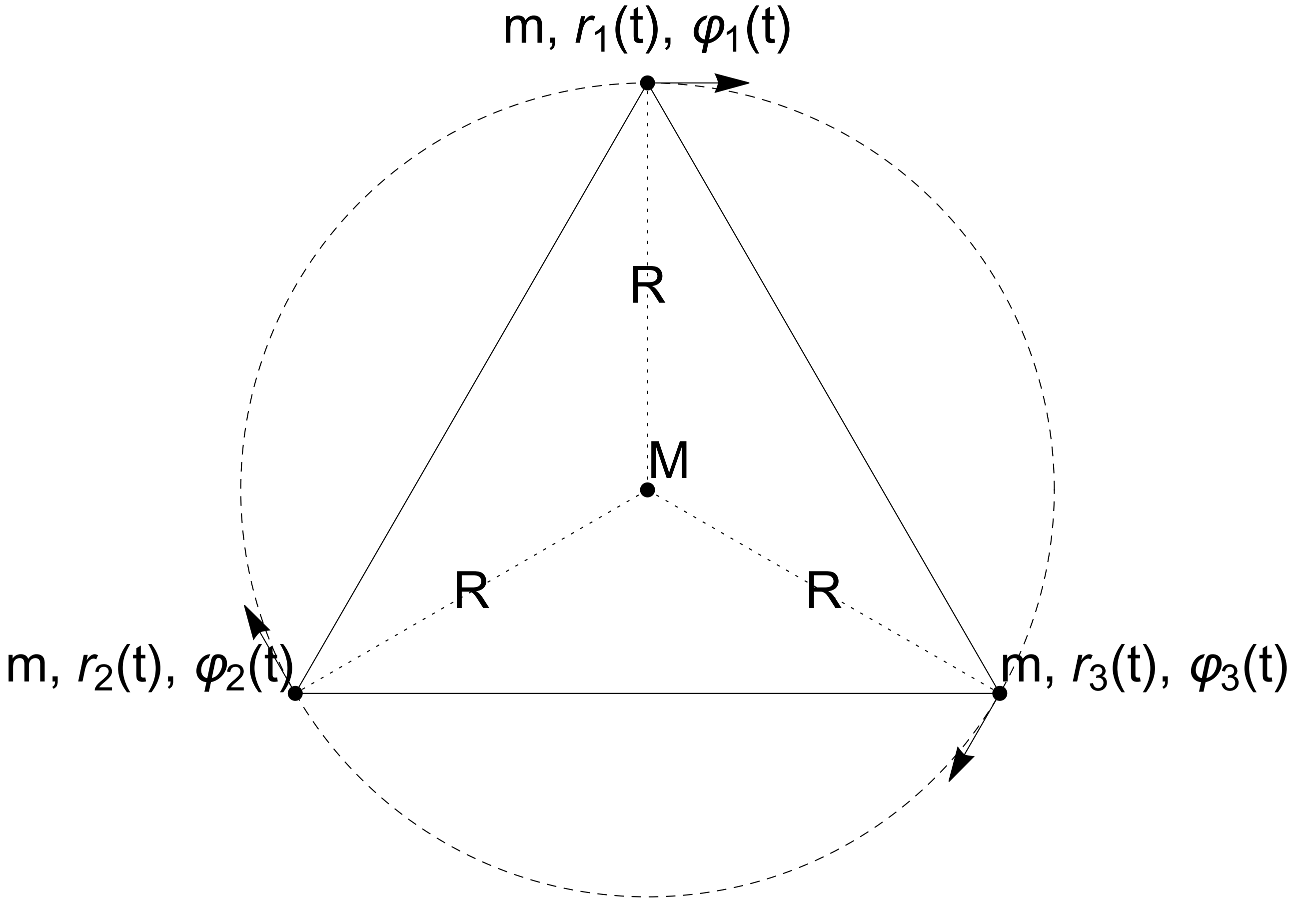
Propose material with AVERAGE (incl. aparatus) density far above those of heavy metals (gold, osmium, iridium), for long-time use in gravitational constant G measurements.

7. Elastic planetoid.

Spherically symmetric planetoid is composed of elastic material with initial density ρ_0 and Younga modulus E. Under self-gravity it will compress increasing average density. Find „cheapest" material to produce densest planetoid. Assume EOS $P = K \ln\left(\rho/\rho_0\right)$.

8. Stabilize triangular N-body solution.

Check stability of 4-body system, including one mass M initially in the center-of-mass, and three masses m initially in the corners of equilateral triangle:



orbiting in circular fashion with radius R and angular velocity:

$$\Omega^2 = \frac{GM}{R^3}\left(1 + \frac{1}{3}\frac{m}{M}\right).$$

In case of $M = 0$ circular solution exists, but is known to be unstable. In case of $M > 0$ and $m = 0$ solution is stable, because no self-gravity operates between masses $m = 0$ - system becomes uncoupled, with three separated circular stable two-body problems.

Main goal of the exercise is to check stability of the above system in case of $M > 0$ and $m > 0$. Both numerical (N-body) and analytical (linear stability) solutions are acceptable. At initial stage, problem could be reduced to planar case, with $M \gg m$, i.e., fixed position of mass M at origin of the coordinate system. Using above assumptions, initial solution is:

$$r_1(t) = r_2(t) = r_3(t) = R = const, \qquad \varphi_1(t) = \varphi_2(t) - 2\pi/3 = \varphi_3(t) + 2\pi/3 = \Omega\, t.$$

See also Lectures 14/2018 and 13/2019, eqns. (6,7) in particular for a hint.

9. Piotrowski Ring Paradoks

Compute sum resulting from orbital velocity of self-gravitating regular n-polygon:

$$\frac{1}{4n}\sum_{i=1}^{n-1}\frac{1}{\sin\left(i\pi/n\right)},$$

exactly, or approximately.

10. Negative mass in positive mass system?

Propose stable N-body system, with one mass being negative (gravitational mass).

11. Future/past solar neutrino spectra

Calculate neutrino spectrum for the Sun at different than present age, or for similar star with slightly different mass.

12. Self-gravitating pile of sand.

Assuming sand with density ρ and friction coefficient μ find non-spherical solution, or prove they cannot exist.

13. MESA binary.

Use MESA to evolve binary star.

14. Realistic gravitational collapse

Use GR1D http://www.stellarcollapse.org/codes.html to compute collapse with Hybrid EOS and $\Gamma = 2$ (aka soft EOS) and for $\Gamma = 4$ (hard EOS). Compare evolution of the bounce and shock propagation.

15.

Interstellar planetoid Omuamua is believed to be elongated as much as 1:10. Assuming it was rotating liquid (Jacobi ellipsoid) compute angular velocity, momentum and check stability timescale.

16. Realistic visualization of phenomena in rotating artificial gravity environment.

„The Expanse" TV series http://www.imdb.com/title/tt3230854/ at least twice we have seen naked eye visible Coriolis effects:

Are the effects real, exaggerated, of just fake?