Solution of any of the problems below is equivalent to passing an examination.

1. MESA. Evolve $4M_{\odot}$ star until black dwarf.

MESA has a problem with default settings (http://mesa.sourceforge.net/) to evolve star up to cooling white dwarf stage (Q_limit). Using examples for $0.25, 0.5, 1, 2, 8M_{\odot}$ stars included in lecture notes, and test_suite of MESA, find appropriate settings (inlist). Problems are: slow stellar wind (infinitely long calculations) and or envelope instabilities (zero timestep).

2. Find orbital elements from numerical solution.

It is relatively easy to find N-body solution with slowly varying elliptical orbit. Propose and verify numerical method to extract elliptical orbit parameters (a, e, i, T etc.) as a function of time using previously found numerical N-body evolution.

3. Verify linear mass approximation method.

In theory we could under some conditions replace orbiting body with linear mass density smeared out on the elliptical orbit. This is particularly easy for circular orbit, replaced with uniform density circle. Compare evolution in circular potential with true point mass in circular orbit.

Zadanie 4. Van der Waals planet.

Find mass-radius relation and structure of the self-gravitating body composed of isothermic Van der Waals gas.

Zadanie 5. Model of plate tectonics.

Recently it has been proposed, that Moon gravity, not internal mantle motion, is responsible for movement of tectonic plates. Analyse simplified model:

1. Moon is modeled as point mass m,
2. Earth is rigid sphere with mass $M_{\oplus}$ and radius $R_{\oplus}$,
3. continental plate is modelled by point mass μ sliding on the sphere representing Earth,
4. mass μ moves without friction (assumptions to be released).

Next, replace point mass μ with something more realistic, e.g. colliding disks or polygons.

6. Ultradense material.

Propose material with AVERAGE (incl. aparatus) density far above those of heavy metals (gold, osmium, iridium), for long-time use in gravitational constant G measurements.

7. Elastic planetoid. UPDATED 7 May 2023.

A spherically symmetric planetoid is composed of an elastic material with an initial density (ρ_0) and Young's modulus (E). Under the influence of its own gravity, the planetoid will compress, resulting in an increased average density ($\bar{\rho} > \rho_0$). The task is to identify the cheapest material that can produce the densest planetoid while considering the equation of state (EOS) $P = K \ln(\rho/\rho_0)$.

The motivation for this project comes from the idea of space measurement of the gravitational constant, $G = 6.67 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{s}^2\mathrm{kg}}$, using orbiting manufactured balls. The orbital period (T) on circular orbits depends only on the average density ($\bar{\rho}$) of the balls. Dense materials like tungsten (W), iridium (Ir), osmium (Os), and gold (Au) can achieve minimum orbital times of around 12 minutes. However, these materials are expensive or troublesome, like uranium. It might be more practical to use cheaper materials that can be compressed under their own gravity, but it is difficult to predict which properties are more advantageous: a larger initial density (ρ_0) or a smaller elastic modulus (K).

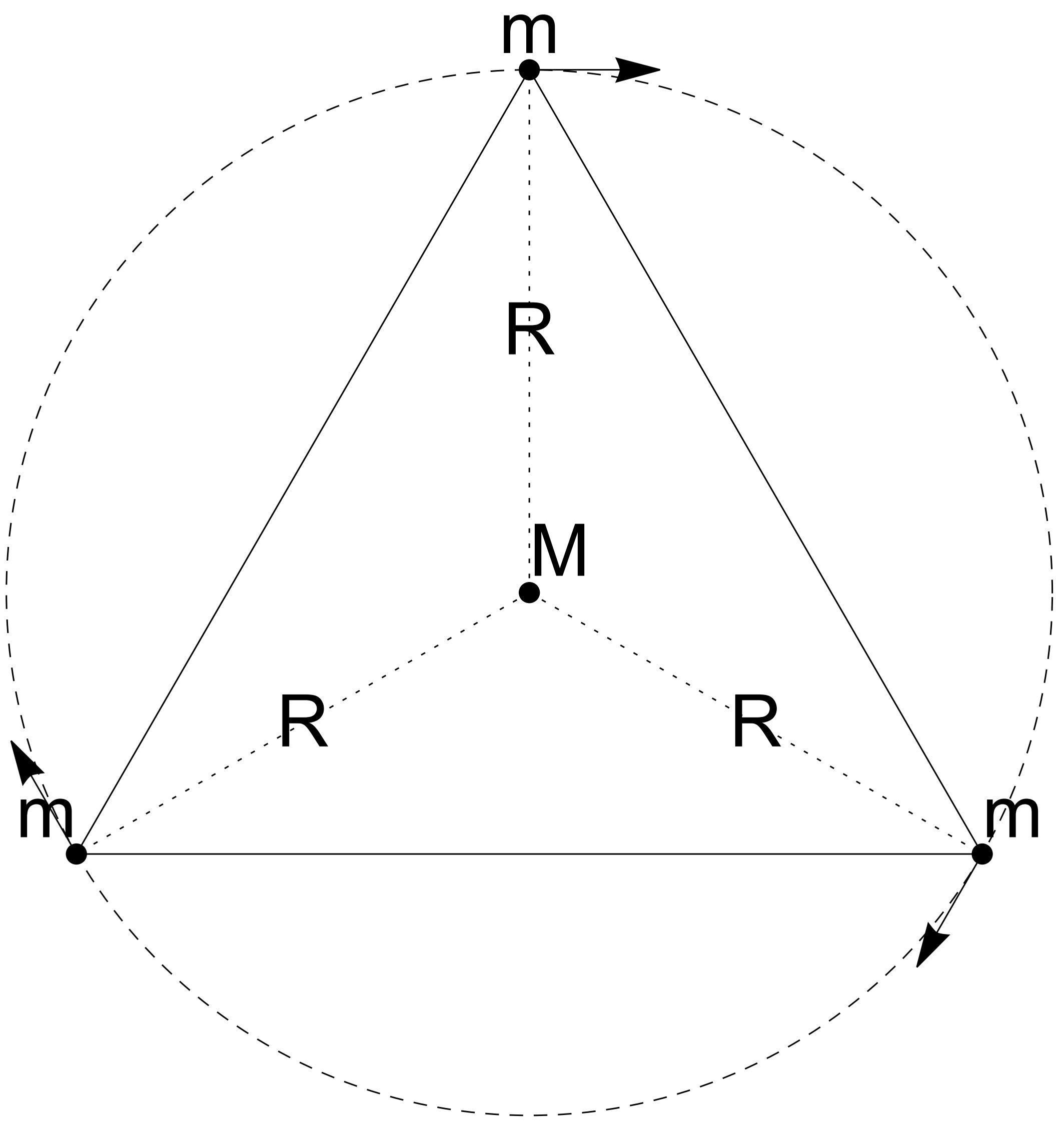
An interesting example from astrophysics is neutron matter. While its density under normal conditions is similar to that of an ideal gas (such as hydrogen or air) at 1 g/cm^3, under the influence of gravity, it can reach densities that are 10^{15} times larger, approaching super-nuclear values.

This project is a thought experiment, as the mass of the planetoid might be astronomical in reality (although this is to be verified), and the elasticity assumptions could quickly fail with minor overdensities ($\bar{\rho} > 2\rho_0$).

The definition of "cheapest" is open to debate and requires some clever guesswork. For instance, there is a significant difference in the cost of materials launched into orbit via a chemical rocket versus those mined in situ from an asteroid.

8. Stabilize triangular N-body solution.

Check stability of 4-body system, including one mass M initially in the center-of-mass, and three masses m initially in the corners of equilateral triangle:



orbiting in circular fashion with radius R and angular velocity:

$$\Omega^2 = \frac{GM}{R^3}\left(1 + \frac{1}{3}\frac{m}{M}\right).$$

In case of $M = 0$ circular solution exists, but is known to be unstable. In case of $M > 0$ and $m = 0$ solution is stable, because no self-gravity operates between masses $m = 0$ - system becomes uncoupled, with three separated circular stable two-body problems.

Main goal of the exercise is to check stability of the above system in case of $M > 0$ and $m > 0$. Both numerical (N-body) and analytical (linear stability) solutions are acceptable. At initial stage, problem could be reduced to planar case, with $M \gg m$, i.e., fixed position of mass M at origin of the coordinate system. Using above assumptions, initial solution is:

$$r_1(t) = r_2(t) = r_3(t) = R = const, \qquad \varphi_1(t) = \varphi_2(t) - 2\pi/3 = \varphi_3(t) + 2\pi/3 = \Omega\, t.$$

See also Lectures 14/2018 and 13/2019, eqns. (6,7) in particular for a hint.

9. Piotrowski Ring Paradoks

Compute sum resulting from orbital velocity of self-gravitating regular n-polygon:

$$\frac{1}{4n}\sum_{i=1}^{n-1}\frac{1}{\sin\left(i\pi/n\right)},$$

exactly, or approximately.

10. Negative mass in positive mass system?

Propose stable N-body system, with one mass being negative (gravitational mass).

11. Future/past solar neutrino spectra

Calculate neutrino spectrum for the Sun at different than present age, or for similar star with slightly different mass.

12. Self-gravitating pile of sand.

Assuming sand with density ρ and friction coefficient μ find non-spherical solution, or prove they cannot exist.

13. MESA binary.

Use MESA to evolve binary star.

14. Realistic gravitational collapse

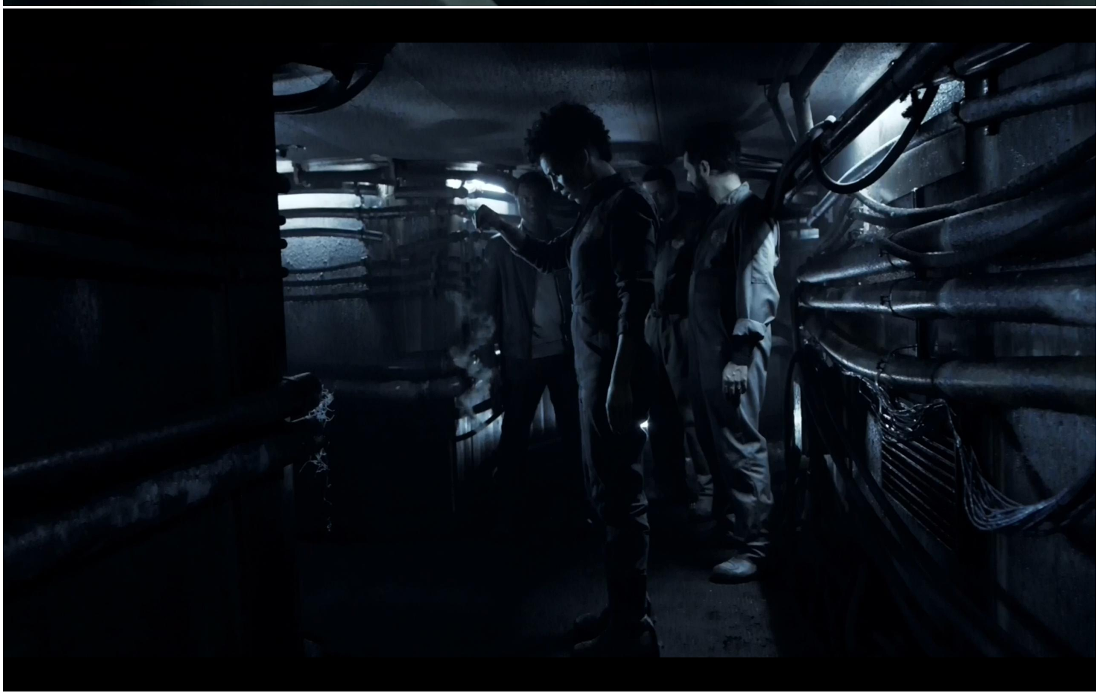
Use GR1D http://www.stellarcollapse.org/codes.html to compute collapse with Hybrid EOS and $\Gamma = 2$ (aka soft EOS) and for $\Gamma = 4$ (hard EOS). Compare evolution of the bounce and shock propagation.

15. Oumuamua as Jacobi ellipsoid

Interstellar planetoid Omuamua is believed to be elongated as much as 1:10. Assuming it was rotating liquid (Jacobi ellipsoid) compute angular velocity, momentum and check stability timescale.

16. Realistic visualization of phenomena in rotating artificial gravity environment.

„The Expanse” TV series http://www.imdb.com/title/tt3230854/ at least twice we have seen naked eye visible Coriolis effects:



Are the effects real, exaggerated, of just fake?

16A. Realistic description of phenomena in the 4-body system.

In the TV series 三 体 (”The Three Body Problem”) based on Cixin Liu’s novel of the same title, the plot revolves around the three-body problem in a fictional version of the triple system of Alpha Centauri. In reality, this system does not seem to be unstable due to its hierarchical structure, which consists of:

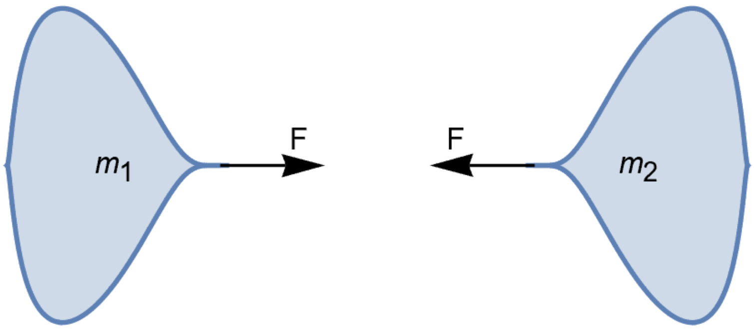
1. the primary component, a “tight” solar-type binary star system, Alpha Centauri A + Alpha Centauri B,
2. the secondary component, a red dwarf in orbit around the primary component, Proxima,
3. a hypothetical planet, Proxima Centauri b.

The aim of the project is to investigate, by solving the N-body equations of motion and projecting onto the celestial sphere of Proxima b, whether the chaotic effects depicted in the TV series were exaggerated or even fictitious. How would the system need to be modified to actually exhibit the described “eras of stability” and chaos? Would there be phenomena such as “flying stars” or a “triple sun day”?

17. Boiling water in a water-based neutrino detector?

The Hollywood disaster blockbuster ”2012” begins with a ridiculous scene in which the water in a Super-Kamiokande type neutrino detector boils due to an increase in solar neutrino flux. Meanwhile, it is known that neutrinos interact with matter so weakly that their detection is very difficult. The aim of the task is to calculate how large the flux and energy of electron antineutrinos or other neutrinos would need to be to boil the oceans. Does such a situation occur anywhere in the Universe?

18. Maximum gravity. UPDATED 7 May 2023.



We have a mass M of incompressible material with density ρ. How should we divide its volume $V = M/\rho$ into two parts with masses $m_1 + m_2 = M$ to maximize the Newtonian gravitational force $\mathbf{F}$ between them? It might be fruitful to assume that distance d between center-of-mass for mass m_1 and mass m_2 is known.

See also Figure illustrating related Project 24. Note that shapes of both parts $m_{1,2}$ are in principle arbitrary.

19. Maximum solar exposure.

We are placing solar panels on a roof tilted at an angle λ to the vertical. The roof faces in the direction of azimuth α and is located at a geographic latitude of φ. The theoretical power of the panel with ideal (perpendicular) alignment is P. Calculate the generated power as a function of time, taking into account the time of day and year, using the following models:

(A) Circular motion with constant angular velocity and the Sun as a point source of radiation;

(B) Accurate orbital and rotational motion (ephemerides, e.g. **AstroPosition** from Mathematica);

(C) Add light absorption in the atmosphere.

In particular, answer the following questions:

(1) When and whether the Sun shines perpendicularly on the panel.

(2) If (1) does not occur, when is the maximum power generated?

(3) What is the average daily, monthly, and annual power output?

Provide specific examples, e.g., φ =50° N, α =115°, λ =30°, and $P = 400$ W. Propose several schemes for optimizing the alignment.

20. Paleorivers during the Ice Age.

During the latest maximum glaciation, the sea level dropped by 200 m, and a significant subcontinent known as Sundaland emerged in the region of present-day Indochina and Indonesia. Rivers that currently flow into the Gulf of Thailand (e.g., Chao Phraya, Mekong) must have flowed further. Determine their paths, for example, by calculating the steepest gradient in a numerical model of the sea floor or by simulating the flow.

[1] Submergence of Sundaland, YouTube

21. Equilateral triangle-shaped tether.

The idea of a space elevator requires lowering a ”line” (a space tether) from geostationary orbit to the surface of Earth (or Mars). One of the engineering problems is the choice of the cross-sectional shape. We assume that it has the form of an equilateral triangle. The aim of the task is to calculate the aerodynamic forces that will be exerted by the wind blowing perpendicular to the tether. In particular, we are interested in whether lift force (perpendicular to the wind and the tether) will be generated on this shape and whether there will be a **moment of force** that causes the tether to twist.

The solution scheme is arbitrary, but one possible approach is to use the ideal fluid approximation and reduce the problem to finding the complex velocity potential by transforming a known solution (e.g., for a cylinder) to an equilateral triangle using the Schwarz-Christoffel formula.

22. RTG generators in space.

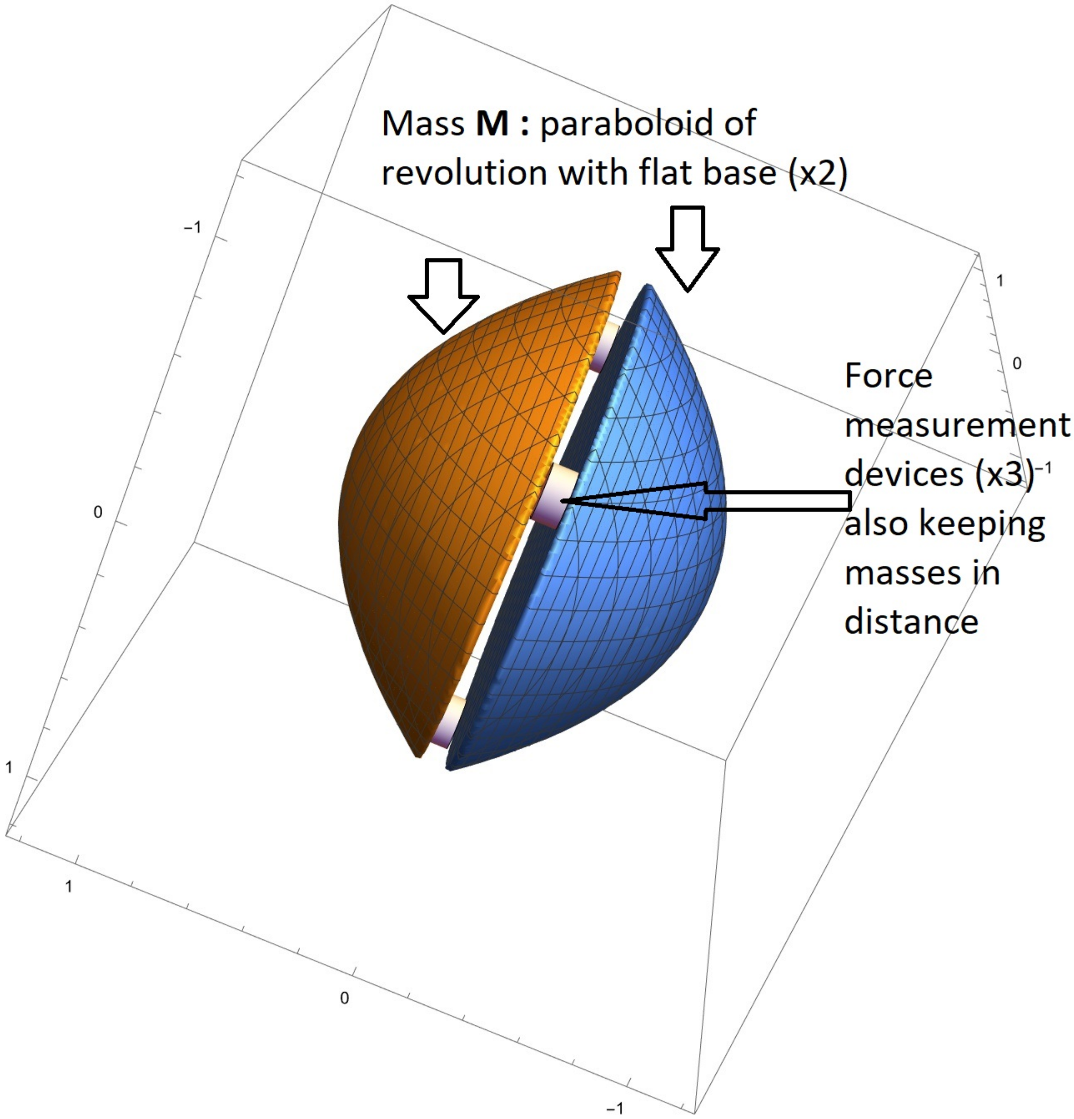
Humanity has launched dozens of radioisotope generators (RTGs) into the Solar System, most of them to its outer regions where energy from photovoltaic panels is insufficient. One of the few exceptions is Ulysses, which travels close to the Sun but on an orbit perpendicular to the ecliptic. There are many experiments demonstrating the effect of "seasons" on radioactive decay (e.g., DAMA), and they are explained by disturbances originating from the stream of dark matter in which the Earth moves. The aim of the task is to gather data on the launched RTG generators and their power generation history as a function of time and position in the Solar System, and then search for correlations in such data. Using broadly defined artificial intelligence methods is a trendy approach nowadays, but due to the expected small amount of data, I rely more on students' ingenuity. The task is aimed at people who have the skill of extracting data from various non-standard sources, such as NASA technical reports.

23. Measurements of G.

This task is intended for people with Benedictine patience, as it involves reviewing and organizing over 100 measurements of the gravitational constant G scattered throughout scientific literature in the last 225 years, starting with the Cavendish experiment of 1798. While collective compilations showing the change in the value of G over the years exist in various articles, the evolution of accuracy or materials used (lead, copper, mercury, water, iron, etc.) is difficult to reconstruct. The aim of the task is to collect and organize all information about measurements of G in the form of a table or database. In particular, we are interested in the place of the experiment, the date, the institution, the materials used, the shape of the source and the test mass, the method (torsion balance, weighing), etc.

24. Innovative experiment for measuring G.

This task is related to the result of Task 18 *Maximum Gravity*. One of the known **approximate** solutions involves two rotating paraboloids with flat bases.



The sketch shows an experiment with two rotating paraboloids with flat bases that attract each other only gravitationally. Three measuring devices are placed between them to maintain a constant distance and measure the force. The goal of the experiment is to verify whether the gravitational force between the paraboloids follows the formula

$$F \simeq 64 M^{4/3} {\rho_{20}}^{2/3},$$

where F is expressed in nanonewtons, M in kilograms, and ρ_{20} is the relative density to tungsten ($\rho \simeq 20$ g/cc). The result probably does not depend on the distance between the paraboloids for a very narrow gap.

Is it possible to perform the experiment and is it feasible in conditions of microgravity outside the laboratory, e.g., on the International Space Station or as a CubeSAT? What forces are acting and what methods are capable of measuring them?

The task consists of analyzing the theoretical conditions for performing the experiment and then assessing its feasibility in practice. In the answer, the following should be taken into account:

1. Calculations of the force for the paraboloids.
2. Proposed measurement method and technical aspects of conducting the experiment.
3. Conditions for conducting the experiment in microgravity outside the laboratory, e.g., on the International Space Station or as a CubeSAT.
4. Possible measurement errors and ways to minimize them.
5. Conclusions regarding the feasibility of the experiment.

Additionally, considerations can be presented regarding the potential impact of external factors, such as a magnetic field or solar radiation, on the experiment's results.