Oscillating Hadron and Jet multiplicity moments

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1. Theoretical approach

2. Multiplicity moments for hadrons and jets

3. QCD calculations
   - DLA
   - MLLA
   - Monte Carlo ↔ experiment

4. Conclusions

Recent work with
Matt Buican & Clemens Förster
Motivation

- Simple ansatz for hadronization
  "Local Parton Hadron Duality (LPHD)"
  
  \[ D(\xi) \big|_{\text{hadron}} = K \times D(\xi) \big|_{\text{parton}} \]

  Azimov, Dokshitzer
  Khoze, Troyan

  inclusive spectra in \( \xi = \ln(1/x) \)

- Analytical results for jet structure
  and soft particle production

  Deep principle or accident?

  \[ \downarrow \]

  \[ \rightarrow \text{check more complex observables} \]
  
  - correlations between hadrons
  - jet resolution (\( y_{\text{cut}} \)) dependence

  \[ \rightarrow \text{increase accuracy of QCD calculations} \]
Theoretical approach

Hadronization model

\[ Q_0 \sim 1 - 2 \text{ GeV} \]

Parton Hadron Duality

\[ k_T \geq Q_0 > \Lambda \text{ (few 100 Mev)} \]

QCD cascade evolves towards lower scale results directly compared to hadron final state
Perturbative QCD predictions

Multiplicity generating function

\[ Z(Q, u) = \sum_{n=1}^{\infty} P_n(Q) u^n \]

\[ Q = E \Theta \]

integral evolution equation

\[ \frac{E}{\alpha} \partial_x \xi = \frac{E}{\alpha} + \int d^2 \phi \int_{z_c}^{1} \frac{\alpha_s(\hat{k}_T)}{2\pi} P_{gg}(z) \times \{ Z(Y_c + \ln z, u) Z(Y_c + \ln(1-z), u) \} \]

\[ Z(0, u) = u \]

\[ Z(Y_c, u) = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\hat{k}_T)}{2\pi} P_{gg}(z) \times \{ Z(Y_c + \ln z, u) Z(Y_c + \ln(1-z), u) \} \]

\[ \frac{d}{dY_c} Z(Y_c, u) = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\hat{k}_T)}{2\pi} P_{gg}(z) \times \{ Z(Y_c + \ln z, u) Z(Y_c + \ln(1-z), u) \} \]

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\[ Z(0, u) = u \]

with

\[ Y_c = \ln \frac{E}{Q_c}, \quad \hat{k}_T = \min(z, 1-z)E \]

- non perturbative cut-off \( k_T \geq Q_0 \)
- running coupling \( \alpha_s(\hat{k}_T) \) 1-loop
asymptotic solutions

- DLA: \( P_{gg}^{gg}(z) \sim \frac{1}{z} \) for gluon emission

- MLLA: include next to leading log terms

full solution of evolution equation

- numerical solution of evolution eqn. (Complete up to next to leading log)

- Monte Carlo generation of parton cascade
  ARIADNE-D
  \( \Lambda = 400 \text{ MeV} \quad \lambda = \ln \frac{Q_0}{\Lambda} = 0.01 \)
  only light quarks
  full matrix element one-loop
Multiplicity moments

**hadrons**

\[ n_{\text{had}} \]

**jets**

\[ f_q = \langle n(n-1) \ldots (n-q+1) \rangle \]

\[ F_q = f_q / \langle n \rangle^q \]

**factorial moments**

**kumulant moments** (genuine correlation) \( k_q, K_q \)

\[ F_q = \sum_{m=0}^{q-1} \binom{q-1}{m} K_{q-m} F_m \]

\[ K_2 = F_2 - 1, \quad K_3 = F_3 - 3F_2 + 2, \ldots \]

**H_q-moments**

\[ H_q = \frac{K_q}{F_q} \]

Double log approximation

\[ \bar{n} \sim \exp(2\beta \sqrt{\ln Q/Q_0}) \]

\[ f_q \sim \langle n \rangle^q \quad F_q \to \text{const}(q) \]

\[ H_q \sim 1/q^2 \]

MLLA + higher orders

\[ q_{\text{min}} = \frac{1}{h_1 \gamma_0} + \frac{1}{2} + \mathcal{O}(\gamma_0) \]

\[ \gamma_0 \sim \sqrt{\alpha} \quad h_1 = \frac{1}{24} \]

Dremin et al.
Kumulant multiplicity moments

- Asymptotic LL. expansion in $\sqrt{\alpha_s}$

- Numerical solution of ev.eqn.

OPAL data:
- quark jets

- gluon jets
$H_q$ Moments at $Q = 91$ GeV

Hadrons

but: normalization of $H_q$ far off from asymptotic predictions

Jets at resolution $y_{cut}$
(Durham algorithm $y_{cut} = k_T^2/Q^2$)

$k_T = 100$ MeV

$400$ MeV

$1$ GeV

$3$ GeV

strong variation of oscillation length and amplitude with $y_{cut}$
Double Logarithmic approximation (DLA)

mean multiplicity $\bar{N}$ of partons in jet

$k_T \geq Q_0 \ (\sim \Lambda)$  
$k_T \geq Q_c > Q_0$

partons $\sim$ hadrons  
partons $\sim$ jets ($y_{\text{cut}} = Q_c^2/Q^2$)

energies:  
\[
\uparrow \text{LEP} \quad \uparrow 10^7 \text{GeV} \quad Q_0 = 300 \text{ MeV}
\]

exact solution of DLA ev. eqn.

\[
N(Y_c) = 2\beta \sqrt{Q_c} + \lambda \left[ I_1(2\beta \sqrt{Y_c} + \lambda)K_0(2\beta \sqrt{\lambda}) + K_1I_0 \right] \\
Y_c = \ln \frac{E_{\text{jet}}}{Q_c}, \quad \lambda = \ln \frac{Q_c}{\lambda}, \quad \beta = \beta(n_f, N_c)
\]

high energies $E_{\text{jet}}$  
($Q_c = Q_0 = \text{const.}$)  
$N \sim \exp(2\beta \sqrt{\ln E/Q_c})$  
$N(0) = 1$

limit $Q_c \to Q_0$ (jet $\to$ hadron)  
$E_{\text{jet}} = \text{const.}$  
$\lambda_c \to 0$  
Landau pole  

fixed coupling $\alpha_s$: $N \cong (E/Q_c)\gamma$

Multiplicity moments

- threshold: $F_q = 0$ for $q > 1$, $K_q = (-1)^{q-1}(q - 1)!$
- high energies: $F_q \to \text{const}(q)$  
  $H_q \sim 1/q^2$
DLA results for ratios $H_q = K_q/F_q$

Hadrons vs. $E_{jet}$

$$Ash_{10} = \text{Arsinh}(y/2)/\ln 10$$

$$Ash_{10}(\pm y) \rightarrow \pm \log_{10}(y) \text{ for large } y$$

Jets vs. cut-off $Q_c$

$H_2$, $H_3$, $H_4$, $H_5$
Modified leading log Approx. (MLLA)

mean multiplicity $N$  
factorial moments

N-1

\[ Y_c = \ln \frac{E}{Q_c} \]

numerical solutions of MLLA Ev. Eq.

- **Threshold:** $F_1 = 0$ for $E = q \cdot Q_0$
- **Poissonian transition point** ($F_1 = 1$) at $Y \approx 4.3$

hadrons: $Q_c = Q_0$

- oscillation length increases with $E_{\text{jet}}$
- far away from asymptotic limit
- more minima/maxima than in DLA
- slow approach towards asymptotics (if any)
  \( Y_c = 30 \rightarrow 10^{13} \text{ GeV} \)
Multiplicities of hadrons and jets in $e^+e^-$

hadrons: $Y_{\text{cut}} = 0$  
jets: $Y_{\text{cut}} > 0$

\[
N - 2 = \ln \frac{Q^2}{Q_c^2 + Q_0^2} = \ln \frac{1}{y_{\text{cut}} + \frac{Q_0^2}{Q_c^2}}
\]

- data include b-decays, computations don’t
- splitting of curves due to $\alpha_s(k_T)$

ARIADNE-D  
$\Lambda = 400$ MeV  
$Q_0 = 404$ MeV

$N_{\text{had}} = K_{ch} N_{ch}$  
$K_{ch} = 1.25$
$H_q$ moments

Ariadne - D

$\text{Ash}_{10} \ (H_q \cdot 10^5)$

$\rightarrow \text{rank } q$

- qualitatively similar to MLLA ev. eq. results
$H_q$ moments

Ariadne - D

L3-data
$H_q$ moments
Ariadne - D

\[
Y_{cs} = \ln \left( \frac{1}{y_{cut} + \frac{Q_0^2}{Q_c^2}} \right)
\]
$H_q$ moments

Ariadne - D

\[ Y_{cs} = \ln \left( \frac{1}{y_{cut} + \frac{Q_0^2}{Q_c^2}} \right) \]
$H_q$ moments

Ariadne - D

\[ Y_{cs} = \ln \left( \frac{1}{y_{cut} + \frac{Q_0^2}{Q_c^2}} \right) \]
Conclusions

* Transition jet → hadron for $y_{\text{cut}} \to 0$
  
  - qualitatively in DLA, MLLA
  - quantitatively in parton MC with low cut-off $k_T \geq Q_0 \geq \Lambda \sim 400$ MeV
    ⇒ strong variation of multiplicity $N$ and $H_q$
    near $y_{\text{cut}} \to 0$ from $\alpha_s(k_T) \gtrsim 1$

* Explanation / prediction
  
  - Poissonian transition point $Y_P$
    for particular energy or $y_{\text{cut}}$
  - rapid oscillations below $Y_p$ with length $\Delta q = 2$
  - oscillation length increases with energy
  - secondary extrema of $H_q$ with energy
    in MLLA, MC

* Perturbative approach also applicable to correlations,
  but needs high accuracy
  
  $H_2 > 0$:
  resonance/cluster decay ↔ gluon Bremsstrahlung

* Possible improvements:
  include b-quark, 2-loop results

* Normalization $K \approx 1$

  ⇒ A hadron looks like a parton at scale $Q \sim \Lambda$