Supersymmetric Skyrme Model

June 20/2016 @ Krakow

SIG V - 20-23 June 2016: Skyrmions - from atomic nuclei to neutron stars

Muneto Nitta
(Keio U.,Jp)

Keio University
1858
CALAMVS
GLADIO
FORTIOR

Neutron Star Matter

Shin Sasaki (Kitasato U., Jp)
Sven Bjarke Gudnason
(Lanzhou, China)

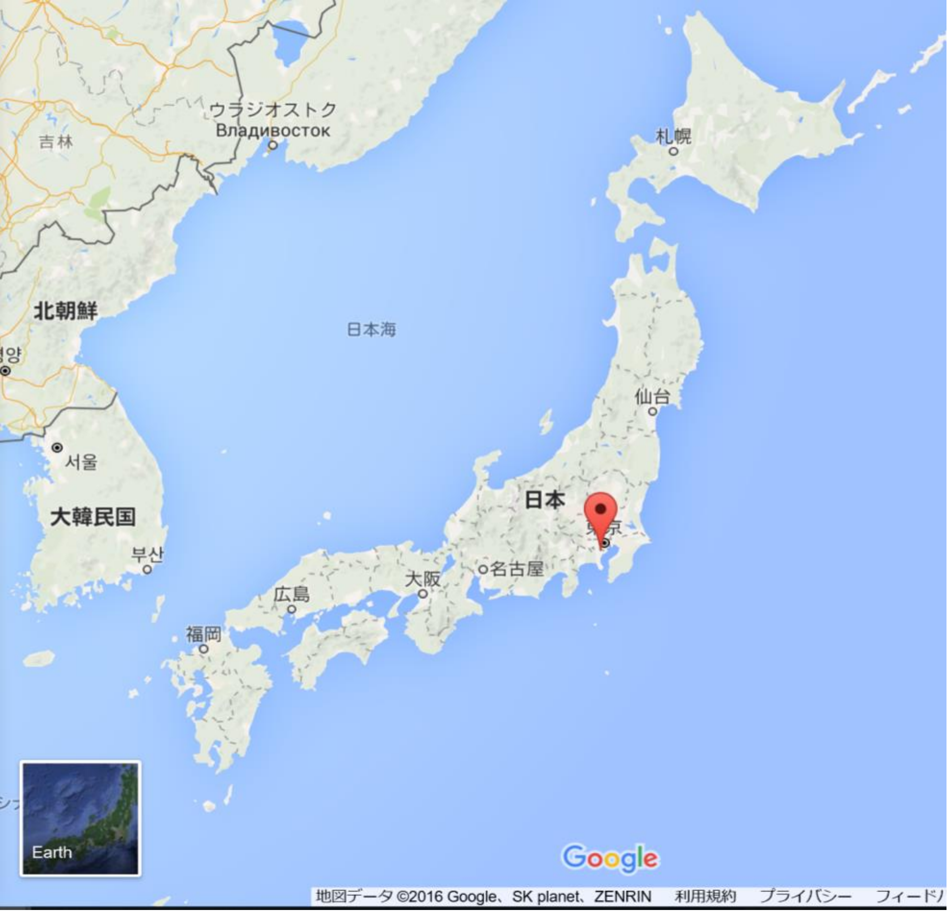
Topological Materials Science

Refs

[1] MN, S.Sasaki,
Phys.Rev. D90 (2014) 105001 [arXiv:1406.7647 [hep-th]]
[2] MN, S.Sasaki,
Phys.Rev. D90 (2014) 105002 [arXiv:1408.4210 [hep-th]]
[3] MN, S.Sasaki,
Phys.Rev. D91 (2015) 125025 [arXiv:1504.08123 [hep-th]]
[4] S.B. Gudnason, MN, S.Sasaki,
JHEP 1602 (2016) 074 [arXiv:1512.07557 [hep-th]]

----

[5] MN, Mod.Phys.Lett. A15 (2000) 2327-2334 [hep-th/0101166]



Keio U.
@ Yokohama
in greater Tokyo

Topological Science Project

Aimed to understand all subjects of physics in terms of Topology

5 years, 11 postdocs

Anyone here is very welcome to visit!!

Motivation

Skyrme model: Skyrme('61)

Low-energy effective theory(LEET) of QCD

Bayons = Solitons (Skyrmions)

+

LEET of $\mathcal{N}$=1, $\mathcal{N}$=2 SUSY QCD ('90s --)

Exact results Seiberg, Seiberg-Witten ('94)

⇓

LEET of SUSY QCD in chiral symm breaking sector

SUSY extension of Skyrme model

SUSY Skyrmions = SUSY baryons? Exact?

Notorious Problem in SUSY higher derivatives

Progress in baby-Skyrme model by

Adam, Queiruga, Sanchez-Guillen, Wereszczynski ('12-)

Motivation

Low-energy effective theories are usually derivative expansion.
Chiral Lagrangian, SUGRA as low-energy string etc

1. **LEEA of SUSY field theories**:
   Derivative corrections in LEEA, SUGRA
2. **SUSY extension of HD theories**
   Chiral Lagrangian, Wess-Zumino-Witten,
   Skyrme model, Faddeev-Skyme model
3. **LEET of BPS p-branes, BPS topological solitons**:
   Dirac-Born-Infeld, Nambu-Goto *etc*
4. **Cosmology**: k-inflation, Galileon inflation, ghost condensate

Notorious Problem: Auxiliary Field Problem

Chiral superfield $y^m = x^m + i\bar{\theta}\sigma^m\theta$ **Auxliary field**

$$\Phi(y,\theta) = \varphi(y) + \theta\psi(y) + \theta\theta F(y)$$

H.D terms→ ∂_m **acts on**

In general, F cannot be eliminated algebraically.
F may become dynamical DOF.
Is SUSY OK? Introduce of further fermion partner?

Is it always the case?
If not, how can we construct higher derivative term without this problem?

SUSY Wess-Zumino-Witten term

Nemechansky & Rohm('85)

Nucl. Phys. B249, 157 (1985).

$$i\int d^4\theta\, \beta_{ij\bar{k}}(\Phi,\Phi^\dagger) D^\alpha \Phi^i \sigma_{\alpha\dot{\beta}}{}^m \partial_m \Phi^j \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{k}} + \text{c.c.}$$

(2,1) tensor

If one sets $F = 0$

$$\mathcal{L}_{\rm boson}|_{F^i=0} = -4i\varepsilon^{\mu\nu\rho\sigma}\lambda_{ijk^*l^*}(A,A^*)\partial_\mu A^i \partial_\nu A^j \partial_\rho A^{*k}\partial_\sigma A^{*l}$$ **WZW**

$$-8\chi_{ijk^*l^*}(A,A^*)\partial_\mu A^i \partial^\mu A^{*k}\partial_\nu A^j \partial^\nu A^{*l}\,.$$ **Additional**

$$\lambda_{ijk^*l^*} = \beta_{ijk^*,l^*} - \bar{\beta}_{k^*l^*i,j} \quad \chi_{ijk^*l^*} = \beta_{ijk^*,l^*} + \bar{\beta}_{k^*l^*i,j}$$

However, explicitly

$$-2\theta\theta\bar{\theta}\bar{\theta}\Big[\{i(F^i\partial_\mu F^{*k} + F^{*k}\partial_\mu F^i)\partial^\mu A^j \beta_{ijk^*}(A,A^*) + \text{c.c.}\}$$

MN('00)

$$+F^{*k}\partial_\mu F^j(\bar{\psi}^l\bar{\sigma}^\mu\psi^i)\{\beta_{ij[k^*,l^*]}(A,A^*) - \bar{\beta}_{k^*l^*[i,j]}(A,A^*)\}\Big]$$

SUSY Faddeev-Skyrme term

Nucl. Phys. B249, 93 (1985).

CP^1 model + 4 deriv Bergshoeff,Nepomechie&Schnitzer('85)

$$\int d^4\theta\, \overline{D}^{\dot{\alpha}} \Phi^{\dagger \bar{i}} \overline{D}_{\dot{\alpha}} \Phi^{\dagger \bar{j}} D^{\alpha} \Phi_i D^{\alpha} \Phi_j$$

$$\int d^4\theta\, \overline{D}^{\dot{\alpha}} \overline{D}_{\dot{\alpha}} \Phi^{\dagger \bar{i}} D^{\alpha} D_{\alpha} \Phi_i$$

$\partial_m F$ **but put** $F = 0$

L. Freyhult, Nucl. Phys. B681, 65 (2004).

1. **4 deriv term must contain** $\partial_t^{\,4}$ **---- no Hamiltonian (Skyrme term** $\mathrm{Tr}\left[U_\mu^\dagger U_\nu U^{\mu\dagger} U^\nu - U_\mu^\dagger U^\mu U_\nu^\dagger U^\nu\right]$ **deos not)**
2. **One cannot introduce superpotential** W
3. $F = 0$ **is a solution, but dynamical? Quantization?**

The unique term free from the AF problem found so far

$$\int d^4\theta\, \Lambda_{ik\bar{j}\bar{l}}(\Phi,\Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger \bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger \bar{l}}$$

$\Lambda_{ik\bar{j}\bar{l}}$ **(2,2) tensor** 4 derivative term

I.L. Buchbinder, S. Kuzenko, and Z. Yarevskaya, NPB411, 665 (1994) **LEEA of susy**

A. T. Banin, I. L. Buchbinder, and N. G. Pletnev, PRD74, 045010 (2006) **WZW**

J. Khoury, J.-L. Lehners, and B. Ovrut, PRD83,125031 (2011) **Ghost condensate**

J. Khoury, J.-L. Lehners, and B. Ovrut, PRD84,043521 (2011) **Galileon**

M. Koehn, J.-L. Lehners, and B. A. Ovrut, PRD86, 085019 (2012) **SUGRA**

C. Adam, J.M. Queiruga, J. Sanchez-Guillen, and A.Wereszczynski, JHEP05 (2013)108 **SUSY baby Skyrmion**

C. Adam, J.M. Queiruga, J. Sanchez-Guillen, and A.Wereszczynski, Phys. Rev. D 86, 105009 (2012) **SUSY k-field theory**

S. Sasaki, M. Yamaguchi, and D. Yokoyama, PLB718, 1(2012) **suerpotential**

The unique term free from the AF problem found so far

$$\int d^4\theta \, \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger \bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger \bar{l}}$$

$\Lambda_{ik\bar{j}\bar{l}}$ **(2,2) tensor** 4 derivative term

$\Lambda_{ik\bar{j}\bar{l}}$ **can contain derivatives too:**

$$\Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger, \partial_m \Phi, \partial_m \Phi^\dagger)$$

Derivatives more than 4

Can be used to construct derivative corrections to arbitrary order

MN, S.Sasaki, Phys.Rev. D90 (2014) 105002 [arXiv:1408.4210 [hep-th]]

BPS lumps & baby Skyrmions

MN, S.Sasaki,
Phys.Rev. D90 (2014) 105001 [arXiv:1406.7647 [hep-th]]

$$\mathcal{L} = \int d^4\theta K(\Phi^i, \Phi^{\dagger\bar{j}}) + \left(\int d^2\theta W(\Phi^i) + \text{H.c.}\right) + \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha\Phi^i D_\alpha\Phi^k \bar{D}_{\dot{\alpha}}\Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}}\Phi^{\dagger\bar{l}}$$

Bosonic part

$$\mathcal{L}_b = \frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}}(-\partial_m\varphi^i\partial^m\bar{\varphi}^{\bar{j}} + F^i\bar{F}^{\bar{j}}) + \frac{\partial W}{\partial\varphi^i}F^i + \frac{\partial\bar{W}}{\partial\bar{\varphi}^{\bar{j}}}\bar{F}^{\bar{j}} + \Lambda_{ik\bar{j}\bar{l}}(\varphi, \bar{\varphi})[(\partial_m\varphi^i\partial^m\varphi^k)(\partial_n\bar{\varphi}^{\bar{j}}\partial^n\bar{\varphi}^{\bar{l}}) - \partial_m\varphi^i F^k\partial^m\bar{\varphi}^{\bar{j}}\bar{F}^{\bar{l}} + F^i\bar{F}^{\bar{j}}F^k\bar{F}^{\bar{l}}]$$

No derivative on F

EOM $$\frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}}F^i - 2\partial_m\varphi^i F^k\Lambda_{ik\bar{j}\bar{l}}\partial^m\bar{\varphi}^{\bar{l}} + 2\Lambda_{ik\bar{j}\bar{l}}F^iF^k\bar{F}^{\bar{l}} + \frac{\partial\bar{W}}{\partial\bar{\varphi}^{\bar{j}}} = 0$$

EOM $$\frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}} F^i - 2\partial_m \varphi^i F^k \Lambda_{ik\bar{j}\bar{l}} \partial^m \bar{\varphi}^{\bar{l}} + 2\Lambda_{ik\bar{j}\bar{l}} F^i F^k \bar{F}^{\bar{l}} + \frac{\partial \bar{W}}{\partial\bar{\varphi}^{\bar{j}}} = 0$$

We call this canonical branch

For $W = 0$, $F = 0$ is *one* solution

$$\mathcal{L}_b = -\frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}} \partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{j}} + \Lambda_{ik\bar{j}\bar{l}} (\partial_m \varphi^i \partial^m \varphi^k)(\partial_n \bar{\varphi}^{\bar{j}} \partial^n \bar{\varphi}^{\bar{l}})$$

But this is not so good, because it contains $\partial_t{}^4$

(Skyrme term, Faddeev-Skyrme term do not.)

Cannot be applied for $W \neq 0$,

For a single component, we can go further!

(Multi-component is still difficult.)

EOM for a single component

$$K_{\varphi\bar{\varphi}}F - 2F(\partial_m\varphi\partial^m\bar{\varphi} - F\bar{F})\Lambda(\varphi,\bar{\varphi}) + \frac{\partial\bar{W}}{\partial\bar{\varphi}} = 0.$$

$$\boxed{W = 0}$$

Canonical branch $F = 0$

$$\mathcal{L}_{1b} = -K_{\varphi\bar{\varphi}}\partial_m\varphi\partial^m\bar{\varphi} + (\partial_m\varphi\partial^m\varphi)(\partial_n\bar{\varphi}\partial^n\bar{\varphi})\Lambda \qquad \partial_t{}^4$$

Non-canonical branch $F\bar{F} = -\frac{K_{\varphi\bar{\varphi}}}{2\Lambda} + \partial_m\varphi\partial^m\bar{\varphi}$

$$\mathcal{L}_{2b} = (|\partial_m\varphi\partial^m\varphi|^2 - (\partial_m\varphi\partial^m\bar{\varphi})^2)\Lambda - \frac{(K_{\varphi\bar{\varphi}})^2}{4\Lambda}$$

Baby Skyrme without $\partial_t{}^4$

No usual kinetic term

Potential

even without W

Remarks on non-canonical branch

No $\Lambda \to 0$ limit, because of the potential $\frac{(K_{\varphi\bar{\varphi}})^2}{4\Lambda}$ **.**

$F\bar{F} = -\frac{K_{\varphi\bar{\varphi}}}{2\Lambda} + \partial_m \varphi \partial^m \bar{\varphi} \geq 0$ **for consistency.**

$K_{\varphi\overline{\varphi}}$ **does not have a meaning of the Kahler metric.**

SUSY baby Skyrme model for CP[1]

C. Adam, J.M. Queiruga, J. Sanchez−Guillen, and A. Wereszczynski, J. High Energy Phys. 05 (2013) 108.

EOM for a single component

$$K_{\varphi\bar{\varphi}}F - 2F(\partial_m\varphi\partial^m\bar{\varphi} - F\bar{F})\Lambda(\varphi,\bar{\varphi}) + \frac{\partial\bar{W}}{\partial\bar{\varphi}} = 0.$$

$W \neq 0$

$$2\Lambda(\varphi,\bar{\varphi})\frac{\partial W}{\partial\varphi}F^3 + \frac{\partial\bar{W}}{\partial\bar{\varphi}}(K_{\varphi\bar{\varphi}} - 2\Lambda(\varphi,\bar{\varphi})\partial_m\varphi\partial^m\bar{\varphi})F + \left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)^2 = 0$$

3 solutions (branches)

$$F = \omega^k\sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \omega^{3-k}\sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \qquad \begin{array}{c} k = 0, 1, 2 \\ \omega^3 = 1 \end{array}$$

$$p = \frac{1}{2\Lambda(\varphi,\bar{\varphi})}\left(\frac{\partial W}{\partial\varphi}\right)^{-1}\left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)(K_{\varphi\bar{\varphi}} - 2\Lambda(\varphi,\bar{\varphi})\partial_m\varphi\partial^m\bar{\varphi})$$

$$q = \frac{1}{2\Lambda(\varphi,\bar{\varphi})}\left(\frac{\partial W}{\partial\varphi}\right)^{-1}\left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)^2$$

SUSY transformation

$$\delta_\xi^{\text{off}} \psi_\alpha = i\sqrt{2}(\sigma^m)_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_m \varphi + \sqrt{2}\xi_\alpha F$$

Conditions on ξ ⟺ BPS equations

BPS lumps

$F = 0$ $z = \frac{1}{2}(x^1 + ix^2)$ $\partial = \partial_1 - i\partial_2$

½ SUSY **On-shell BPS equation**

$\xi^1 = 0$ ⟺ $\bar{\partial}\varphi = 0$ Λ **cancelled out**

Bogomol'nyi bound (the same with $\Lambda = 0$)

$$\mathcal{E} = K_{\varphi\bar{\varphi}} |\partial_i \varphi|^2 - |\partial_i \varphi \partial_i \varphi|^2 \Lambda$$
$$= |\bar{\partial}\varphi|^2 (K_{\varphi\bar{\varphi}} - |\partial\varphi|^2 \Lambda) + iK_{\varphi\bar{\varphi}} \varepsilon_{ij} \partial_i \varphi \partial_j \bar{\varphi}$$
$$\geq +iK_{\varphi\bar{\varphi}} \varepsilon_{ij} \partial_i \varphi \partial_j \bar{\varphi},$$

$$\pi_2(M) = \mathbf{Z}$$

BPS baby Skyrmion

¼ SUSY

$$\frac{1}{2}(\sigma^1 + i\sigma^2)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} = 0, \quad \frac{1}{2}(\sigma^1 - i\sigma^2)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} = ie^{-i\eta}\xi_\alpha,$$

⟺

Off-shell BPS equation

$$\bar{\partial}\varphi = e^{i\eta}F$$

Same with d.w.junction

On-shell BPS equation

$$\bar{\partial}\varphi = e^{i\eta'}\sqrt{-\frac{K_{\varphi\bar{\varphi}}}{2\Lambda} + \frac{1}{2}(|\partial\varphi|^2 + |\bar{\partial}\varphi|^2)}$$

$$\frac{1}{2}(|\partial\varphi|^2 - |\bar{\partial}\varphi|^2) = \frac{K_{\varphi\bar{\varphi}}}{2\Lambda}$$

Bogomol'nyi bound

Compacton

Adam *et.al*

$$\mathcal{E} = -(|\partial_i\varphi\partial_i\varphi|^2 - (\partial_i\varphi\partial_i\bar{\varphi})^2)\Lambda + \frac{(K_{\varphi\bar{\varphi}})^2}{4\Lambda}$$

$$\geq +iK_{\varphi\bar{\varphi}}\varepsilon_{ij}\partial_i\varphi\partial_j\bar{\varphi}$$

$$\pi_2(M) = Z$$

The same with ½ BPS lump

SUSY Skyrme model

S.B. Gudnason, MN, S.Sasaki,
JHEP 1602 (2016) 074 [arXiv:1512.07557 [hep-th]]

Chiral symmetry breaking

$$G = \mathrm{SU}(N)_{\mathrm{L}} \times \mathrm{SU}(N)_{\mathrm{R}} \to H = \mathrm{SU}(N)_{\mathrm{L+R}}$$

Nambu-Goldstone(NG) boson (pions)

$$U = e^{i\pi^A T_A} \in G/H = \frac{\mathrm{SU}(N)_{\mathrm{L}} \times \mathrm{SU}(N)_{\mathrm{R}}}{\mathrm{SU}(N)_{\mathrm{L+R}}} \simeq \mathrm{SU}(N)$$

Not Kahler

Complexified for $\mathcal{N}$=1 SUSY

$$M = \exp\left(i\Phi^A t^A\right) \in \mathrm{SU}(N)^{\mathbb{C}} = G^{\mathbb{C}}/H^{\mathbb{C}} \simeq \mathrm{SL}(N,\mathbb{C}) \simeq T^*\mathrm{SU}(N)$$

$$\Phi^A(y,\theta) = \pi^A(y) + i\sigma^A(y) + \theta\psi^A(y) + \theta^2 F^A(y)$$

NG ($\pi^A(y)$)

Quasi-NG ($\sigma^A(y)$)

W. Lerche, NPB238(1984)582
G.M. Shore, NPB248(1984)123

Theorem (Lerche, Shore '84)

1. There must appear at least one QNG boson for symmetry breaking with preserving SUSY (in the absence of gauge interaction).

2. For real representation, the same number of QNG bosons as the number of NG bosons appear (fully doubled).
   (eg: chiral symmetry breaking)

G-transformation law

$$M \to M' = g_{\rm L} M g_{\rm R}, \qquad (g_{\rm L}, g_{\rm R}) \in {\rm SU}(N)_{\rm L} \times {\rm SU}(N)_{\rm R}.$$

The most general G-invariant Kahler potential

$$K = f\left[{\rm Tr}\left[MM^\dagger\right], {\rm Tr}\left[\left(MM^\dagger\right)^2\right], \cdots, {\rm Tr}\left[\left(MM^\dagger\right)^{N-1}\right]\right]$$

The simplest $K_0 = f_\pi^2\,{\rm Tr}\left(MM^\dagger\right)$ G.M.Shore('89)

Setting QNG=0

$$U = M|_{\sigma^A=0} \in {\rm SU}(N),$$

$$\mathcal{L}^{(2)}_{0,b}\Big|_{\sigma^A=0} = -f_\pi^2\,{\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)$$ **Usual principal chiral model**

SUSY Skyrme term

$$\mathcal{L}_0^{(4)} = \frac{1}{16}\int d^4\theta\, \Lambda_{ik\bar{j}\bar{l}}\left(\Phi, \Phi^\dagger\right) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\bar{j}\dagger} \bar{D}^{\dot{\alpha}} \Phi^{\bar{l}\dagger}$$
$$= \frac{1}{16}\int d^4\theta\, \Lambda\left(M, M^\dagger\right) \mathrm{Tr}\left[D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger\right]$$

$$\Lambda(M, M^\dagger) = g\left[\mathrm{Tr}\left[MM^\dagger\right], \mathrm{Tr}\left[\left(MM^\dagger\right)^2\right], \cdots, \mathrm{Tr}\left[\left(MM^\dagger\right)^{N-1}\right]\right]$$

Bosonic part

$$\mathcal{L}_{0,b}^{(4)} = \Lambda\left(M, M^\dagger\right) \mathrm{Tr}\left[M_\mu^\dagger M_\nu M^{\mu\dagger} M^\nu + \left(F^\dagger F\right)^2 - M_\mu^\dagger M^\mu F^\dagger F - M_\mu M^{\mu\dagger} F F^\dagger\right]$$

Canonical branch $F = 0$

$$\mathcal{L}_{0,b} = -f_\pi^2\, \mathrm{Tr}\, M_\mu M^{\mu\dagger} + \Lambda\left(M, M^\dagger\right) \mathrm{Tr}\, M_\mu^\dagger M_\nu M^{\mu\dagger} M^\nu .$$

including ${\partial_t}^4$ ***Not*** **Skyrme term**

Non-canonical branch (SU(2))

$$\mathcal{L}_{0,b}^{(4)} = \frac{\Lambda(M,M^\dagger)}{2}\left\{ \mathrm{Tr}\left[2M_\mu^\dagger M_\nu M^{\mu\dagger}M^\nu - \frac{1}{2}M_\mu^\dagger M^\mu M_\nu^\dagger M^\nu - \frac{1}{2}M_\mu M^{\mu\dagger}M_\nu M^{\nu\dagger}\right] \right.$$
$$- \frac{1}{2}\left(\mathrm{Tr}[M_\mu M^{\mu\dagger}]\right)^2$$
$$\left. \mp\sqrt{\left(\mathrm{Tr}\left[M_\mu^\dagger M^\mu M_\nu^\dagger M^\nu\right] - \frac{1}{2}(\mathrm{Tr}[M_\mu M^{\mu\dagger}])^2\right)\left(\mathrm{Tr}[M_\mu M^{\mu\dagger}M_\nu M^{\nu\dagger}] - \frac{1}{2}(\mathrm{Tr}[M_\mu M^{\mu\dagger}])^2\right)}\right\}.$$

Setting QNG=0

$$U = M|_{\sigma^A=0} \in \mathrm{SU}(N),$$

$$\boxed{\mathcal{L}_{0,b}^{(4)} = \Lambda(U,U^\dagger)\,\mathrm{Tr}\left[U_\mu^\dagger U_\nu U^{\mu\dagger}U^\nu - U_\mu^\dagger U^\mu U_\nu^\dagger U^\nu\right]}$$

without $\partial_t^{\,4}$ **Skyrme term!!**

Inclusion of Kahler potential

$$\mathcal{L} = f_\pi^2 \int d^4\theta \ \mathrm{Tr}\left[M^\dagger M\right] + \frac{1}{16}\int d^4\theta \, \Lambda\left(M, M^\dagger\right) \mathrm{Tr}\left[\bar{D}_{\dot{\alpha}} M^\dagger D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger D^\alpha M\right]$$

Setting QNG=0 $U = M|_{\sigma^A=0} \in \mathrm{SU}(N),$

$$\mathcal{L}_b|_{M=U} = \Lambda\left(U, U^\dagger\right) \mathrm{Tr}\left[U_\mu^\dagger U_\nu U^{\dagger\mu} U^\nu - U_\mu^\dagger U^\mu U_\nu^\dagger U^\nu\right] - \mathrm{Tr}\left[\frac{f_\pi^2}{4\Lambda\left(U, U^\dagger\right)} \mathbf{1}_2\right]$$

Skyrme term

No usual kinetic term

"Potential term"

Summary

1. SUSY baby Skyrme model
2. SUSY Skyrme model
3. (Higher derivative terms without the auxiliary field problem)
4. (Classified BPS equations)

Future works

1. BPS Skyrmions in the SUSY Skyrme model?
2. Inclusion of superpotential
3. $\mathcal{N}=2$ SUSY extension
4. Can't we really introduce usual kinetic term?
5. LEET of SUSY QCD? SUSY WZW?
6. SUSY extension of BPS Skyrme model?