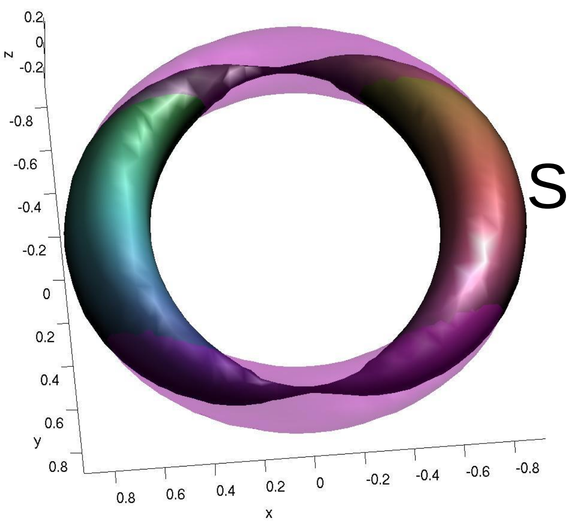
Composite Skyrmions



June 20/2016 @ Krakow

SIG V - 20-23 June 2016

Skyrmions - from atomic nuclei to neutron stars

Keio University

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(IMP, Lanzhou)

Neutron Star Matter

Topological Materials Science

References

Skyrmions

[1] MN Phys.Rev.D87 (2013) 025013 [arXiv:1210.2233]
[2] MN Nucl.Phys.B872 (2013) 62-71 [arXiv:1211.4916]
[3] SBG&MN Phys.Rev.D89 (2014) 025012 [arXiv:1311.4454]
[4] SBG&MN Phys.Rev.D89 (2014) 085022 [arXiv:1403.1245]
[5] SBG&MN Phys.Lett.B747(2015)173-177[arXiv:1407.2822]
[6] SBG&MN Phys.Rev.D90 (2014) 085007 [arXiv:1407.7210]
[7] SBG&MN Phys.Rev.D91 (2015) 045027 [arXiv:1410.8407]
[8] SBG&MN Phys.Rev.D91 (2015) 045018 [arXiv:1412.6995]
[9] SBG&MN Phys.Rev.D91 (2015) 085040 [arXiv:1502.06596]
[10] SBG&MN arXiv:1606.00336

U(*N*) Skyrmions on non-Abelian sine-Gordon soliton

[11] Eto&MN Phys.Rev.D91 (2015) 085044 [arXiv:1501.07038]

Baby Skyrmions

[12] MN Phys.Rev.D86 (2012) 125004 [arXiv:1207.6958]
[13] MK&MN Phys.Rev.D87 (2013) 085003 [arXiv:1302.0989]
[14] MK&MN Phys.Rev.D87 (2013) 125013 [arXiv:1307.0242]

Skyrme model $U(x) \in SU(2)$

$$\mathcal{L} = -\frac{c_2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + c_4\mathcal{L}_4 + c_6\mathcal{L}_6 - V(U)$$

Skyrme term $$\mathcal{L}_4 = -\frac{1}{32}\mathrm{tr}([U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2)$$

6-derivative term $$\mathcal{L}_6 = \frac{1}{144}(\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[U^\dagger\partial_\nu U U^\dagger\partial_\rho U U^\dagger\partial_\sigma U])^2$$

O(4) sigma model

$c_2=c_4=0$: BPS Skyrme, Adam *et.al* (2010-)

$$U = i\sum_{a=1,2,3} n_a\sigma^a + n_4\mathbf{1}_2 \qquad \mathbf{n} = (n_1, n_2, n_3, n_4) \quad \mathbf{n}^2 = 1$$

$$= (\pi_1, \pi_2, \pi_3, \sigma)$$

$$\mathcal{L} = -\frac{c_2}{2}\partial_\mu\mathbf{n}\cdot\partial^\mu\mathbf{n} + c_4\mathcal{L}_4 + c_6\mathcal{L}_6 - V(\mathbf{n})$$

$$\mathcal{L}_4 = -\frac{1}{4}[(\partial_\mu\mathbf{n}\cdot\partial^\mu\mathbf{n})^2 - (\partial_\mu\mathbf{n}\cdot\partial_\nu\mathbf{n})^2]$$

$$\frac{O(4)}{O(3)} \cong S^3 \cong SU(2)$$

$$\mathcal{L}_6 = \frac{1}{36}(\epsilon^{ABCD}\epsilon^{\mu\nu\rho\sigma}n_A\partial_\nu n_B\partial_\rho n_C\partial_\sigma n_D)^2$$

Potential (mass) term

Conventional pion mass term

$$V = m_\pi^2 \mathrm{tr}(2\mathbf{1}_2 - U - U^\dagger) = m_\pi^2 \mathrm{tr}(\mathbf{1}_2 - U)(\mathbf{1}_2 - U^\dagger)$$

$$\sim m_\pi^2 (1 - n_4)$$

Modified mass term

Kudryavtsev, Piette & Zakrzewski,
PRD61 (2000) 025016

$$V = m_\pi^2 \mathrm{tr}(2\mathbf{1}_2 - U - U^\dagger)(2\mathbf{1}_2 + U + U^\dagger) \sim m_\pi^2 (1 - n_4^2)$$

We consider

(1) $V = m^2 (n_1^2 + n_2^2 + n_3^2) = m^2 (1 - n_4^2)$

(2) $V = m^2 (n_1^2 + n_2^2) = m^2 (1 - n_3^2 - n_4^2)$

(3) $V = m^2 n_1^2$

or (2)' $V = m^2 (n_1^2 + n_2^2)(n_3^2 + n_4^2)$

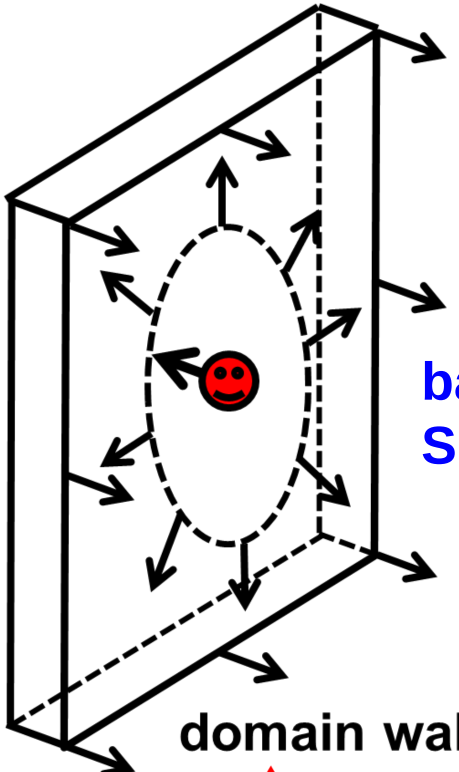
Isospin chemical potential

motivated by Bose-Einstein Condensatations (BEC)

Summary

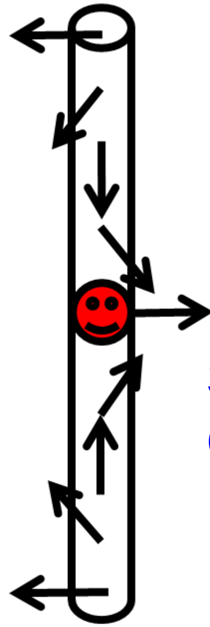
Skyrmion trapped inside

(1) $m^2(n_1^2 + n_2^2 + n_3^2)$ **(2)** $m^2(n_1^2 + n_2^2)$ **(3)** $m^2 n_1^2$



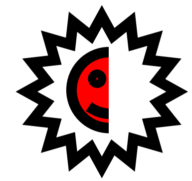
baby Skyrmion

domain wall



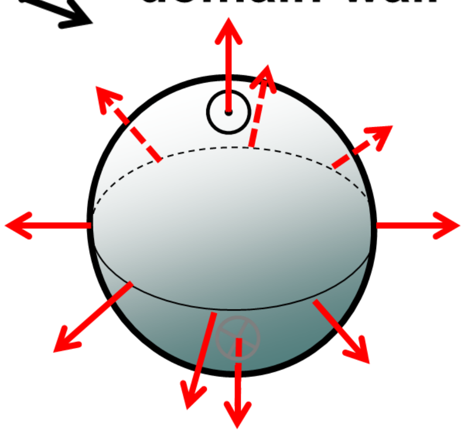
Sine-Gordon

vortex string

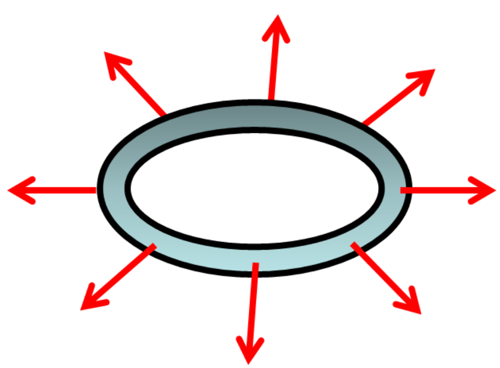


Ising spin

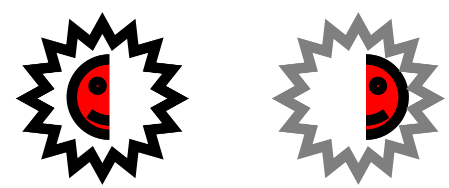
monopole



spherical domain wall



vortex ring



monopole-anti-monopole pair

Summary: Skyrmions

$\mathbf{n} = (n_1, n_2, n_3, n_4)$ $\mathbf{n}^2 = 1$

(1) $V = m^2(1 - n_4^2) = m^2(n_1^2 + n_2^2 + n_3^2)$

Baby Skyrmion on a domain wall

π_2 π_0

Domain wall Skyrmion

(2) $V = m^2(n_1^2 + n_2^2) = m^2(1 - n_3^2 - n_4^2)$

or **(2)'** $V = m^2(n_1^2 + n_2^2)(n_3^2 + n_4^2)$

SG kink on a global vortex

π_1 π_1

Vortex Skyrmion

(3) $V = m^2 n_1^2$

Ising spin on a global monopole

π_0 π_2

Monopole Skyrmion

} **Skyrmion** π_3

π_m on $\pi_n = \pi_{m+n+1}$

Skyrme-like model in d=3+1

$$\mathcal{L} = -\frac{c_2}{2}\partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n} + c_4\mathcal{L}_4 + c_6\mathcal{L}_6 - V(\mathbf{n}) \qquad \mathbf{n} = (n_1, n_2, n_3, n_4)$$

with Skyrme and/or 6-deriv term

$$\mathbf{n}^2 = 1$$

$$\mathcal{L}_4 = -\frac{1}{4}[(\partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n})^2 - (\partial_\mu \mathbf{n}\cdot\partial_\nu \mathbf{n})^2]$$

Target S^3

$$\mathcal{L}_6 = \frac{1}{36}(\epsilon^{ABCD}\epsilon^{\mu\nu\rho\sigma} n_A \partial_\nu n_B \partial_\rho n_C \partial_\sigma n_D)^2$$

Adam *et.al*

(1) Modified mass $V = m^2(1 - n_4^2)$

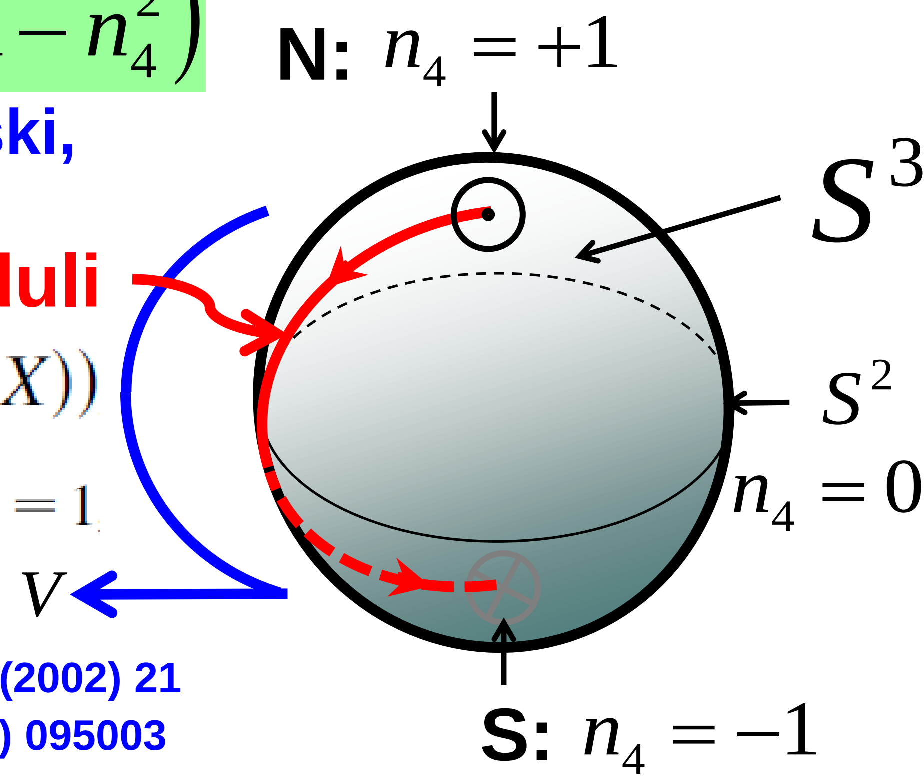
Kudryavtsev, Piette & Zakrzewski,
PRD61 (2000) 025016

NA domain wall with S^2 moduli

$$\theta(x^3) = \arctan\exp(\pm\sqrt{2}m(x^3 - X))$$

$$n_{\hat{i}} = \hat{n}_{\hat{i}}\sin\theta(x) \quad (\hat{i} = 1, 2, 3), \qquad \hat{\mathbf{n}}^2 = \sum_{\hat{i}=1}^{3}\hat{n}_{\hat{i}}^2 = 1$$

$$n_4 = \cos\theta(x)$$



Losev, Shifman & Vainshtein, New J.Phys. 4 (2002) 21
Ritz, Shifman & Vainshtein, Phys.RD70 (2004) 095003

Domain wall effective theory = O(3) NLSM (CP^1)

$$\mathcal{L}_{\text{dw.eff.}} = \frac{\sqrt{2}}{2m}(\partial_a \hat{\mathbf{n}})^2 + \frac{T_{\text{wall}}}{2}(\partial_a X)^2 - T_{\text{wall}}$$

MN, PRD87(2013)025013 [arXiv:1210.2233]

$$\hat{\mathbf{n}}^2 = 1$$

S^2 target space

massive O(3) (CP^1) NLSM

Additional mass term

$$\Delta V = m_3^2(1 - n_3) \rightarrow V_{\text{wall}} = \frac{\sqrt{2}\pi\, m_3^2}{2m_4}(1 - \hat{n}_3)$$

or

$$\Delta V = m_3^2(1 - n_3^2) \rightarrow V_{\text{wall}} = \frac{\sqrt{2} m_3^2}{m_4}(1 - \hat{n}_3^2)$$

$$m_3 << m_4$$

Skyrme term

$$\mathcal{L}^{(4)}(\mathbf{n}) = \frac{1}{2}(\partial_\mu \mathbf{n} \cdot \partial_\nu \mathbf{n})(\partial^\mu \mathbf{n} \cdot \partial^\nu \mathbf{n}) - \frac{1}{2}(\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n})^2,$$

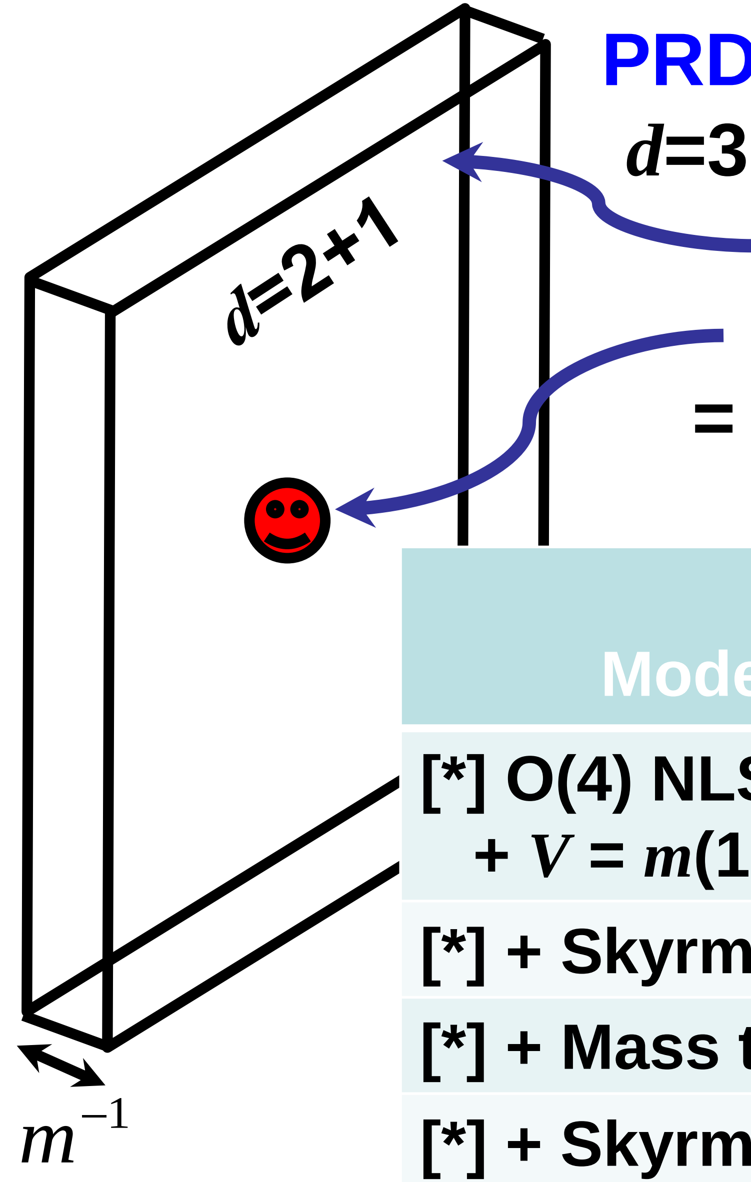
$$\rightarrow \mathcal{L}^{(4)}_{\text{dw.eff}} = -\frac{\sqrt{2}}{3m}[(\partial_a \hat{\mathbf{n}} \times \partial_b \hat{\mathbf{n}})(\partial^a \hat{\mathbf{n}} \times \partial^b \hat{\mathbf{n}})]$$

(and correction to two-deriv term)

SBG&MN, PRD89 (2014)085022[arXiv:1403.1245]

Baby Skyrme term

PRD87(2013)025013[arXiv:1210.2233]



d=3+1 bulk

domain wall (d=2+1 world-volume)

3D Skyrmion in d=3+1 bulk

= lump or baby Skyrmion

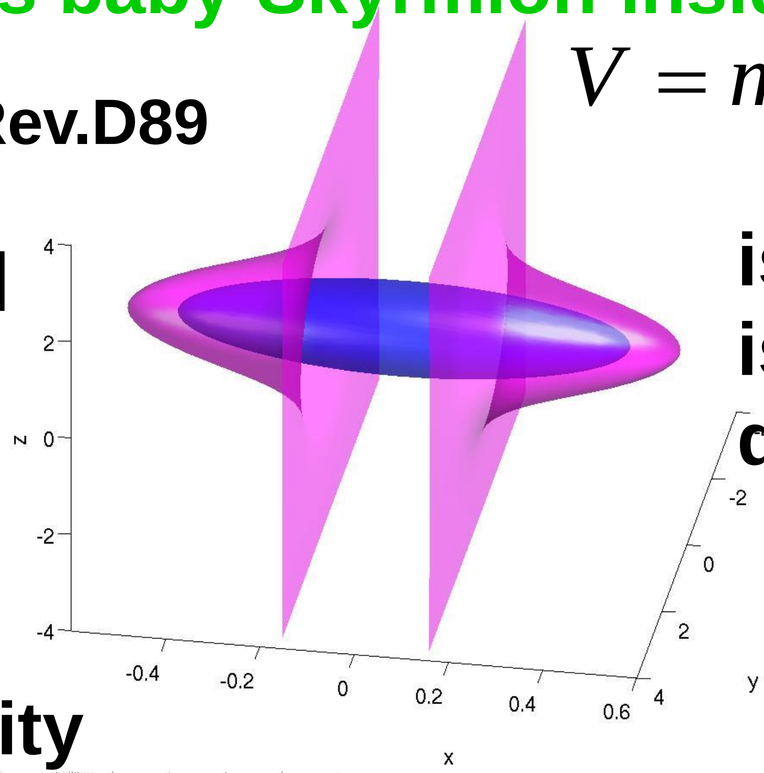
in d=2+1 wall w.v.

Models	Skyrmion in bulk	2D Skyrmion on wall
[*] O(4) NLS model + $V = m(1\text{-}n_4^2)$	Shrink	Marginally stable (BPS lump)
[*] + Skyrme term	Stable	Expand
[*] + Mass term n_3	Shrink	Shrink
[*] + Skyrme term + Mass term n_3	Stable	Stable (baby skyrmion)

(1) Skyrmion as baby-Skyrmion inside domain wall

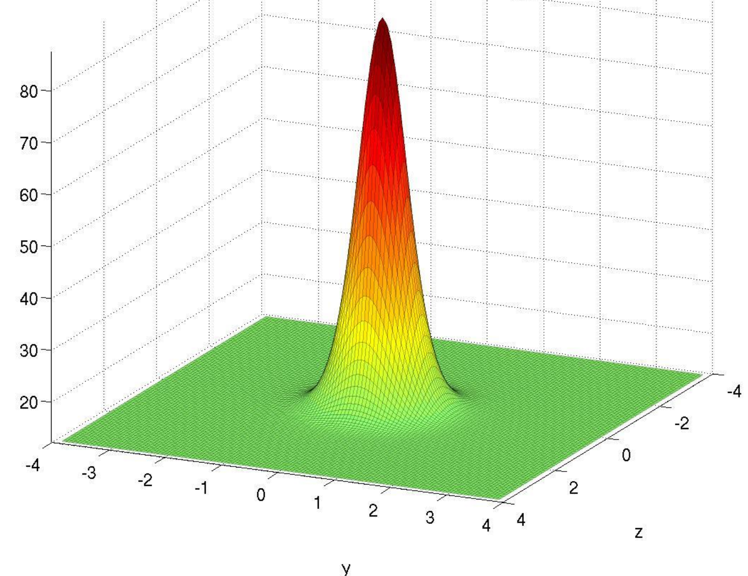
SBG&MN Phys.Rev.D89 (2014) 085022 [arXiv:1403.1245]

$$V = m_4^2(1-n_4^2) + \underline{m_3 n_3}$$

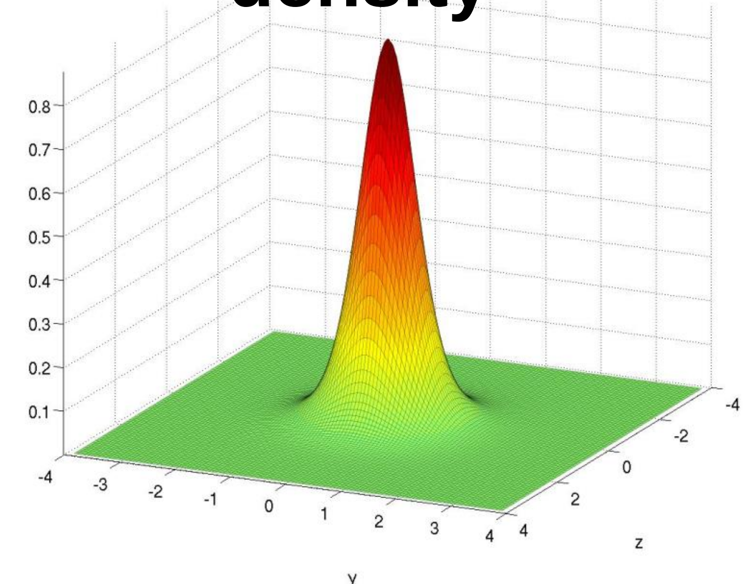


isoenergy and isobaryon charge density surfaces

energy density



baryon charge density

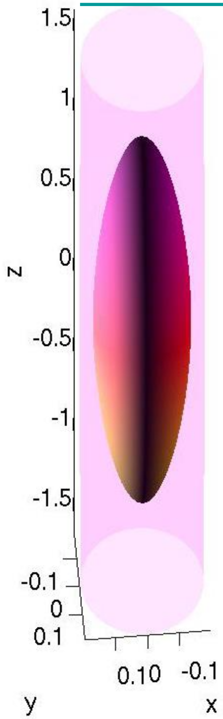


(2) Skyrmion as a SG kink in a vortex

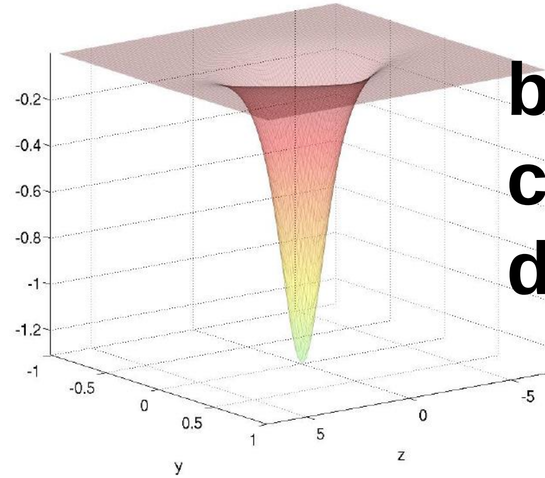
2 model

SBG&MN arXiv:1606.00336

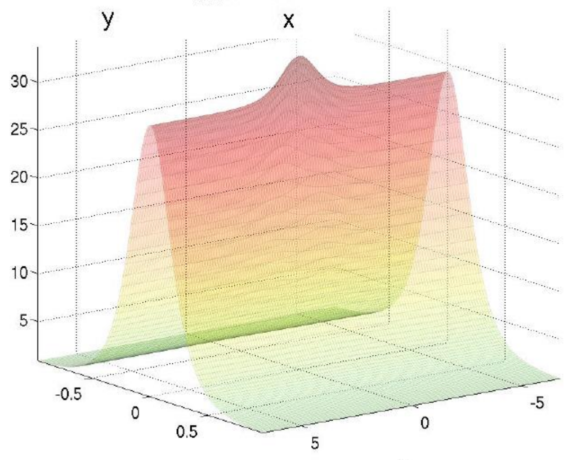
$$V = m^2(1 - n_3^2 + n_4^2) - m_3^2 n_2^2$$



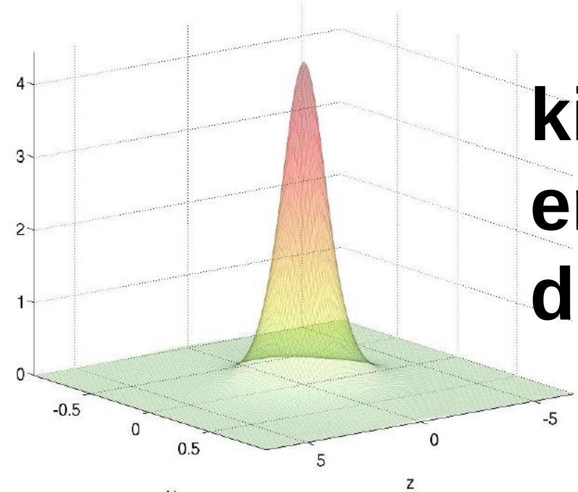
isoenergy and iso-baryon charge density surfaces



baryon charge density



energy density



kink energy density

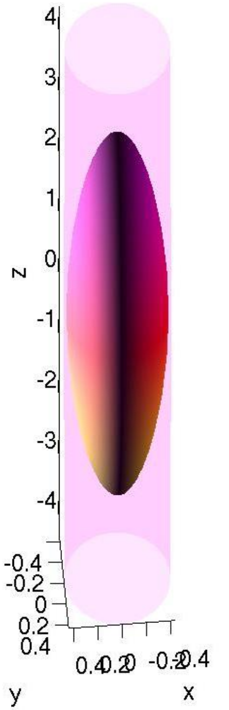
U(1) modulus is twisted half = half Skyrmion

(2) Skyrmion as a SG kink in a vortex

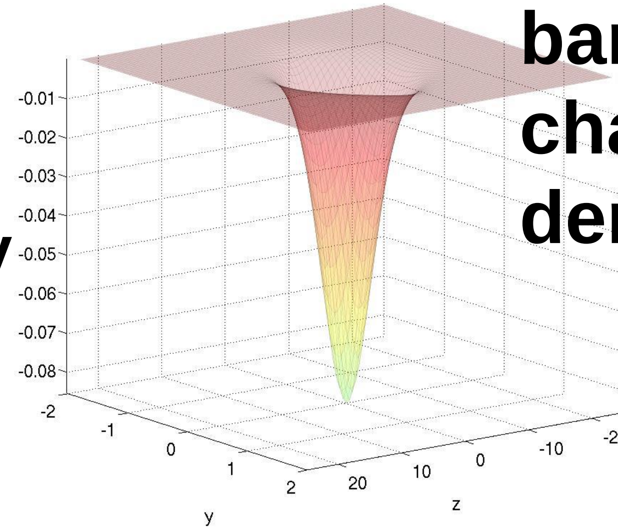
2+4 model

SBG&MN arXiv:1606.00336

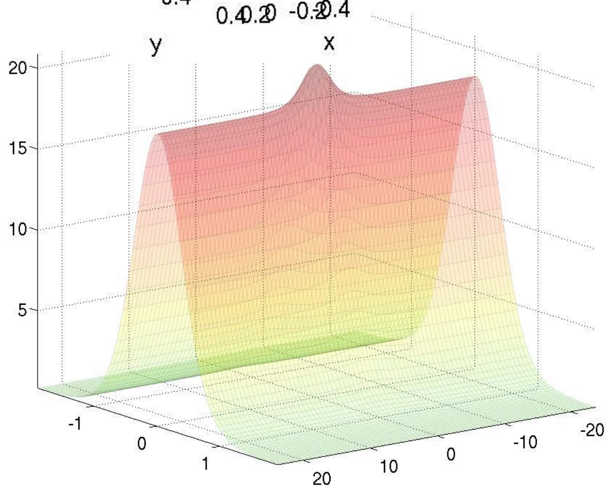
$$V = m^2(1 - n_3^2 + n_4^2) - m_3^2 n_2^2$$



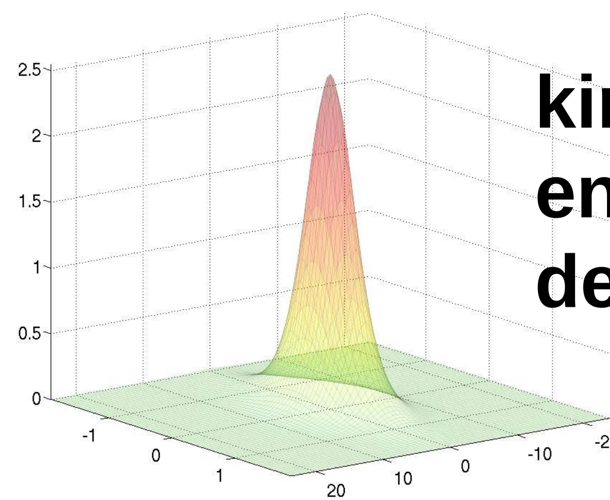
isoenergy and iso-baryon charge density surfaces



baryon charge density



energy density



kink energy density

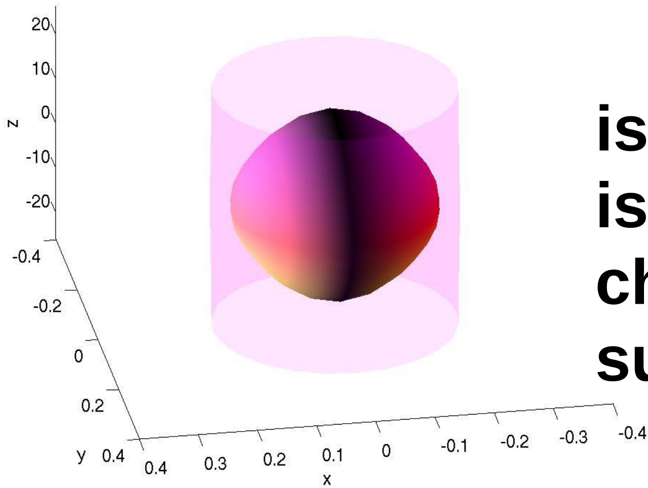
U(1) modulus is twisted half = half Skyrmion

(2) Skyrmion as a SG kink in a vortex

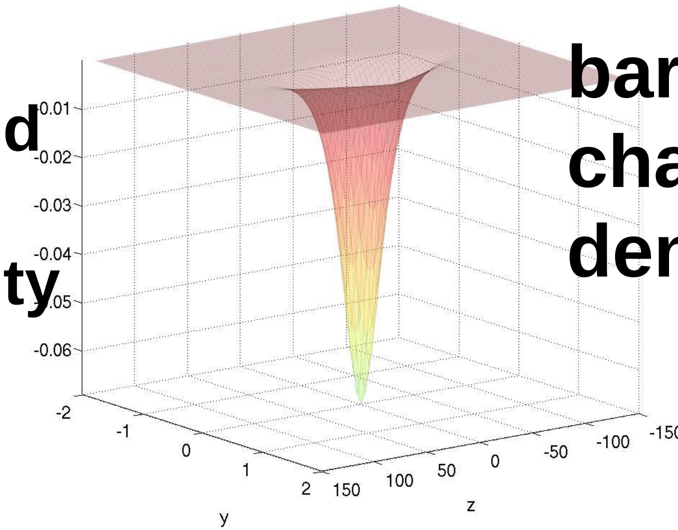
2+6 model

SBG&MN arXiv:1606.00336

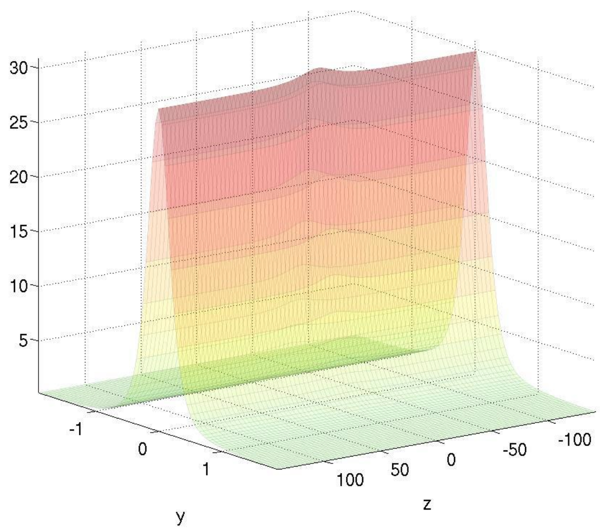
$$V = m^2(1 - n_3^2 + n_4^2) - m_3^2 n_2^2$$



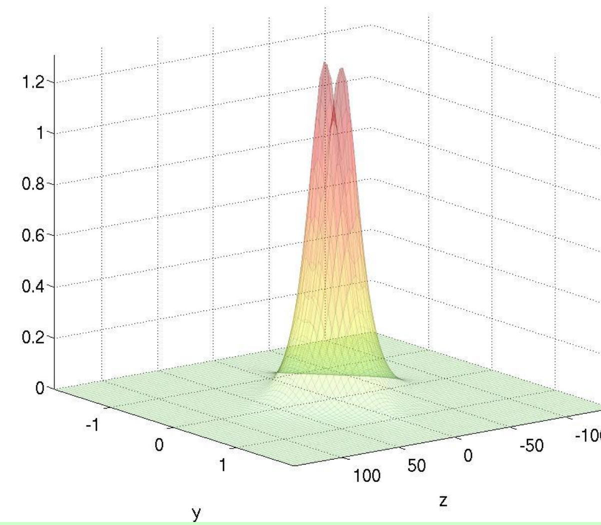
isoenergy and iso-baryon charge density surfaces



baryon charge density



energy density



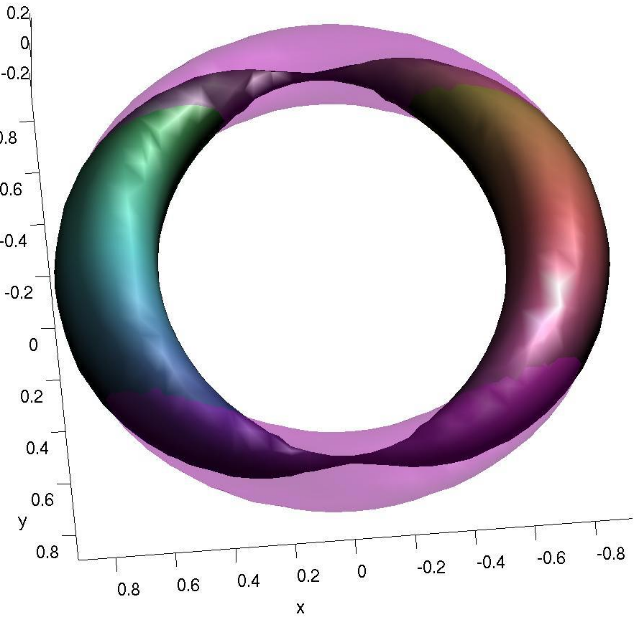
kink energy density

U(1) modulus is twisted half = half Skyrmion

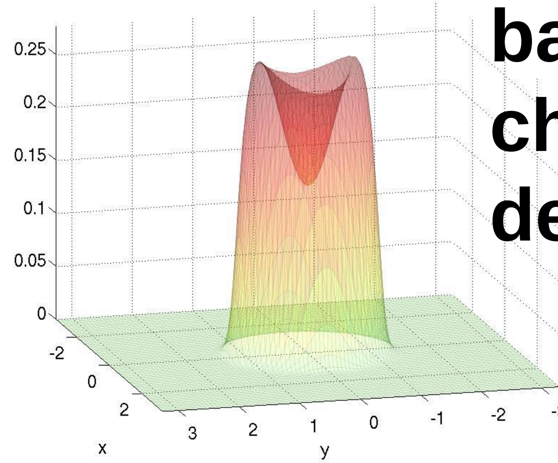
(2) Skyrmion as SG kinks in a vortex ring 2+6 model

SBG&MN arXiv:1606.00336

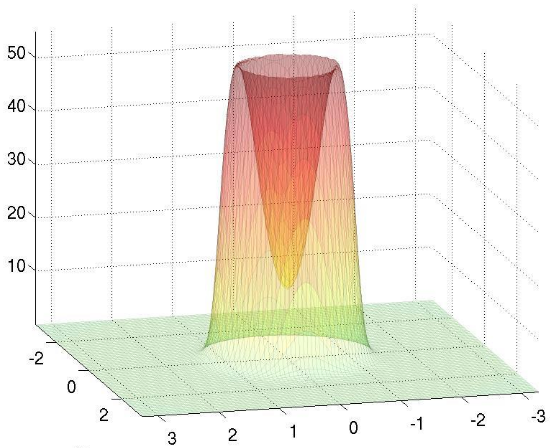
$$V = m^2(1 - n_3^2 + n_4^2) - m_3^2 n_2^2$$



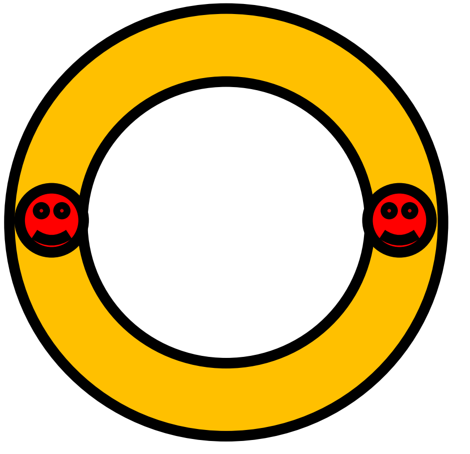
isoenergy and iso-baryon charge density surfaces



baryon charge density



energy density

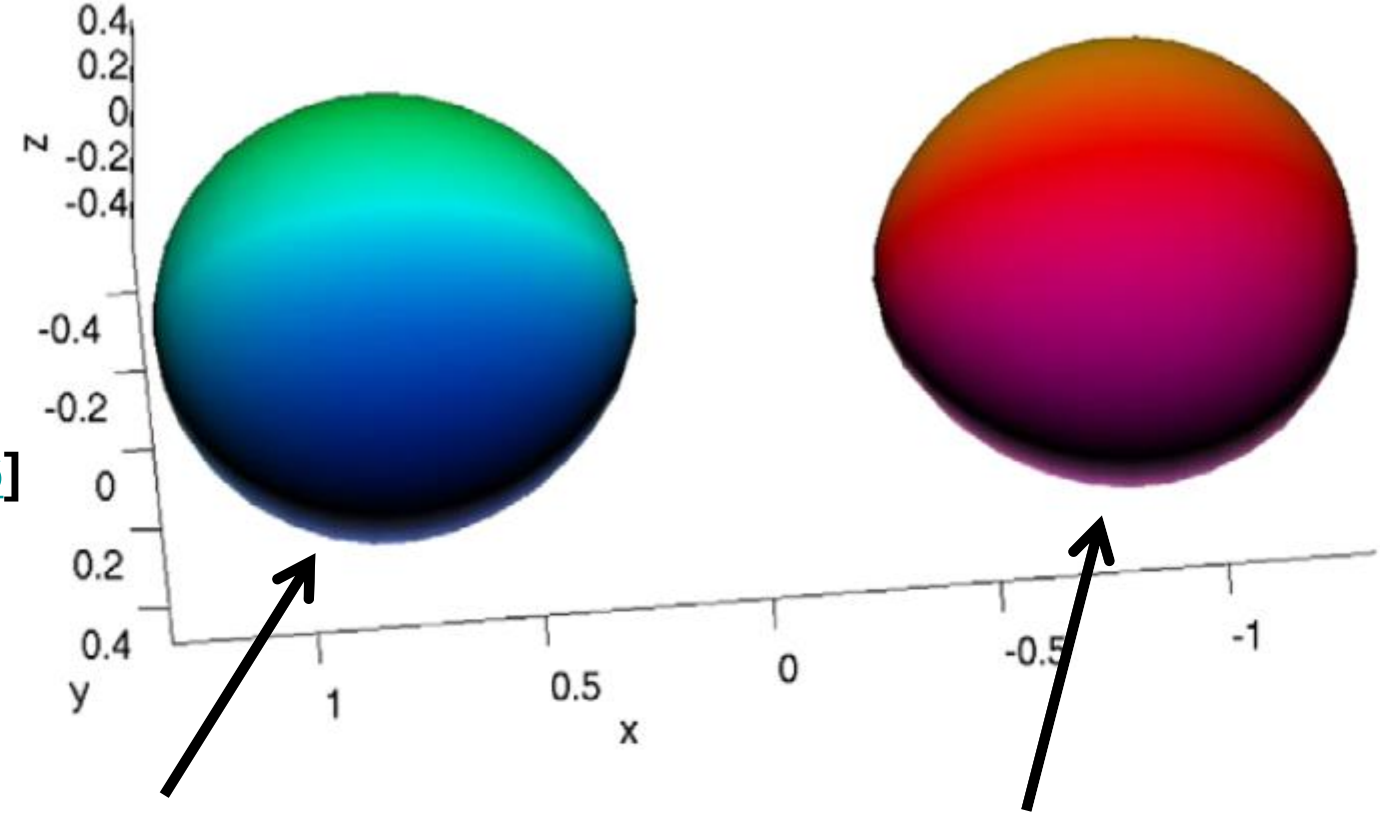


U(1) modulus is twisted half + half = unit Skyrmion

(3) Molecule of half Skyrmions(-global monopoles)

$V = m^2 n_4^2$

SBG&MN
Phys.Rev.D91 (2015) 085040
[arXiv:1502.06596]



global monopole

$1 \in \pi_2 \quad B = 1/2 \in \pi_3$

anti-global monopole

$-1 \in \pi_2 \quad B = 1/2 \in \pi_3$

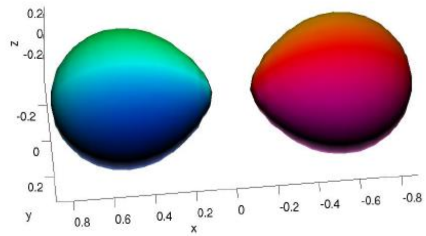
$0 \in \pi_2 \quad B = 1 \in \pi_3$

(3) Molecule of half Skyrmions(-global monopoles)

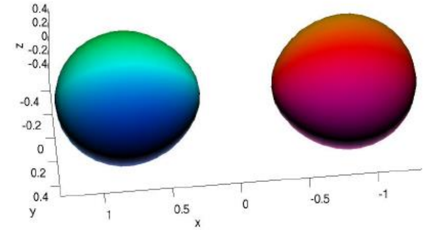
$$V = m^2 n_4^2$$

2+4 model

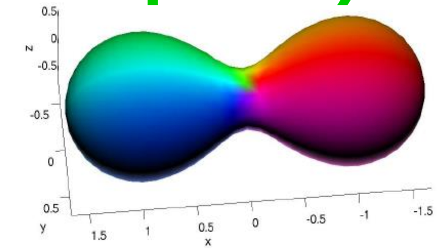
SBG&MN
Phys.Rev.D91 (2015) 085040
[arXiv:1502.06596]



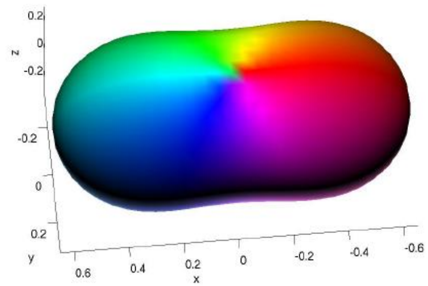
$c_2 = 0, c_4 = \frac{1}{4}$



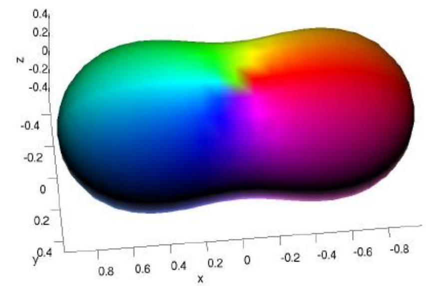
$c_2 = 0, c_4 = 1$



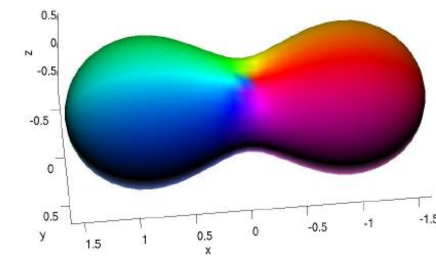
$c_2 = 0, c_4 = 4$



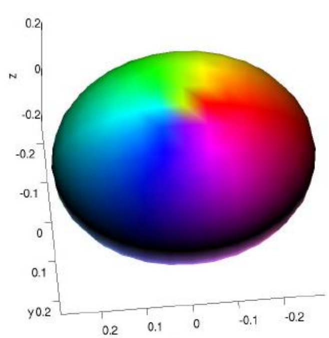
$c_2 = \frac{1}{4}, c_4 = \frac{1}{4}$

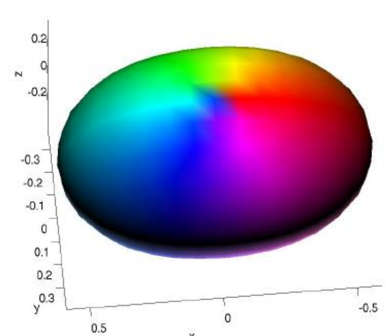


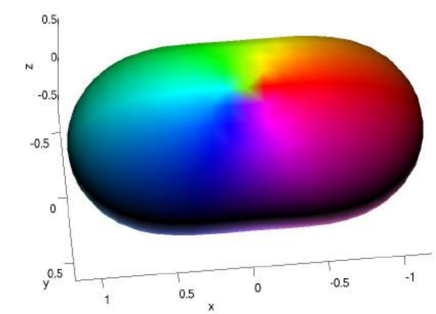
$c_2 = \frac{1}{4}, c_4 = 1$



$c_2 = \frac{1}{4}, c_4 = 4$





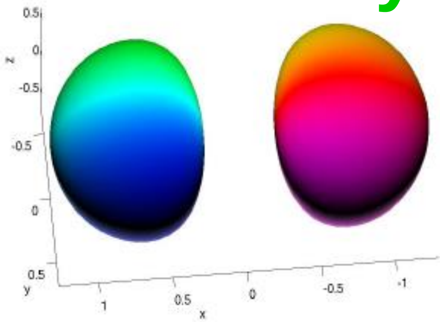


(3) Molecule of half Skyrmions(-global monopoles)

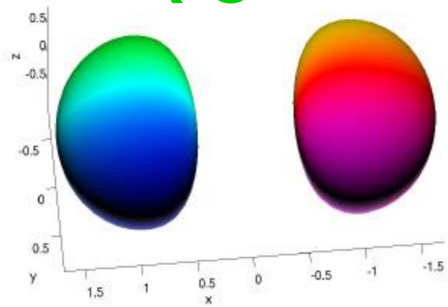
$$V = m^2 n_4^2$$

2+6 model

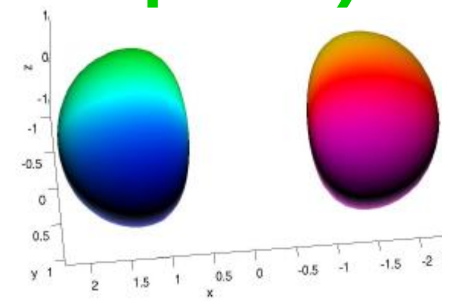
SBG&MN
Phys.Rev.D91 (2015) 085040
[arXiv:1502.06596]



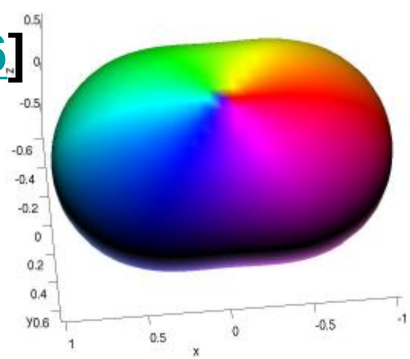
$c_2 = \frac{1}{4}, c_6 = \frac{1}{4}$



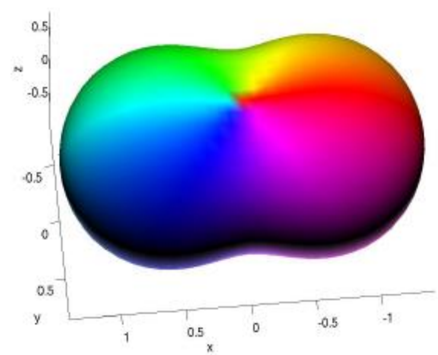
$c_2 = \frac{1}{4}, c_6 = 1$



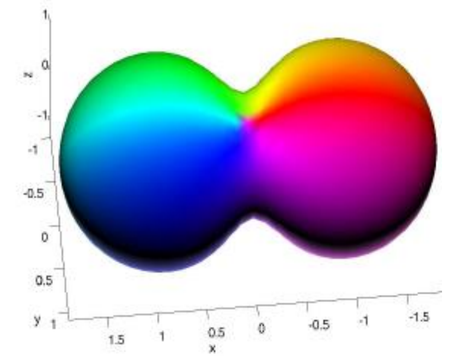
$c_2 = \frac{1}{4}, c_6 = 4$



$c_2 = 1, c_6 = \frac{1}{4}$



$c_2 = 1, c_6 = 1$



$c_2 = 1, c_6 = 4$

$$\mathcal{L}_6 = \frac{c_6}{36 e^4 f_\pi^2} \left(\epsilon^{\mu\nu\rho\sigma} \mathrm{tr}[U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U]\right)^2$$

$$\mathcal{L}_6 = \frac{1}{36} \left(\epsilon^{ABCD} \epsilon^{\mu\nu\rho\sigma} n_A \partial_\nu n_B \partial_\rho n_C \partial_\sigma n_D\right)^2$$

(3) Molecule of half Skyrmions(-global monopoles)

with Higher *B*

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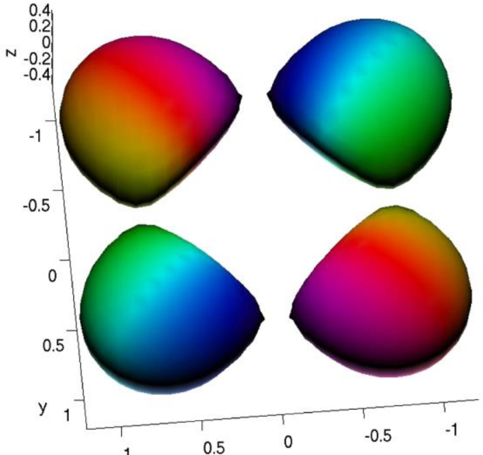
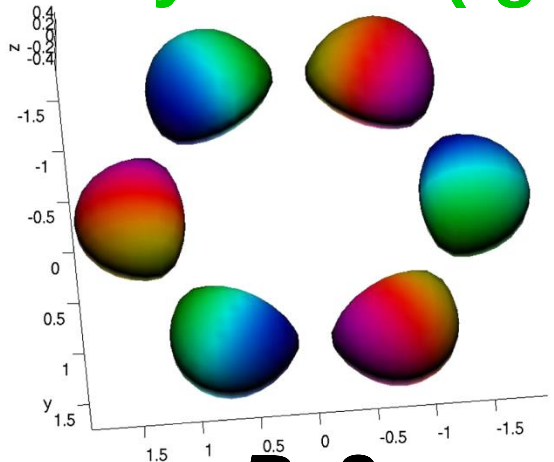
Phys.Rev.D91 (2015) 085040
[arXiv:1502.06596]

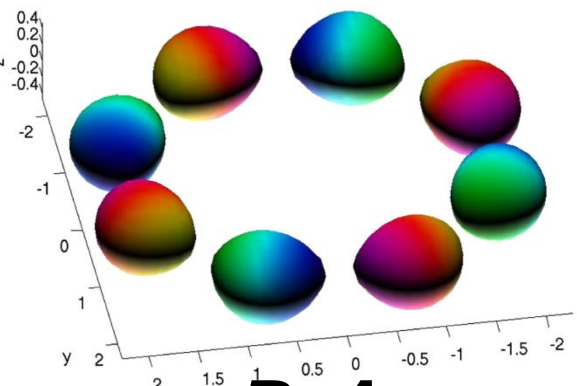


***B*=2**

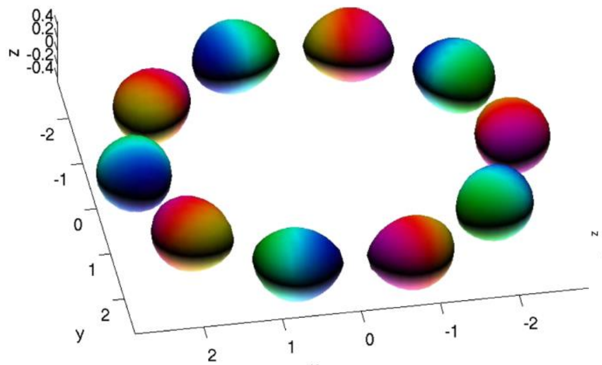


***B*=3**

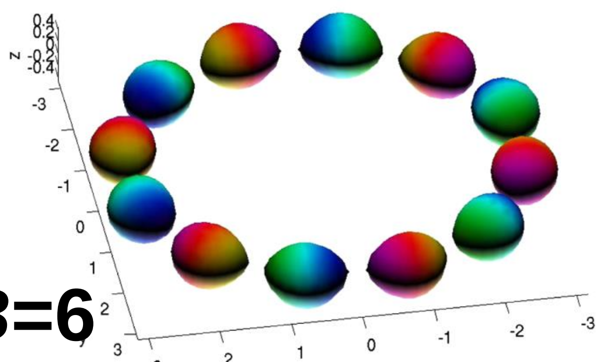
stable



***B*=4**

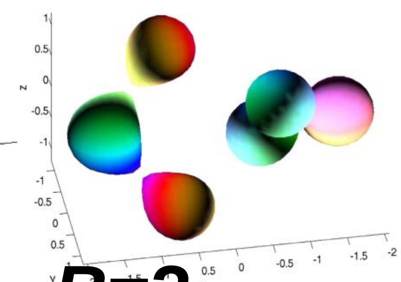


***B*=5**

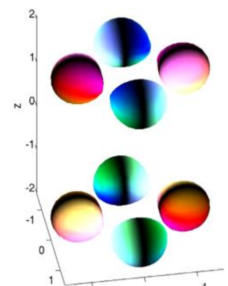


***B*=6**

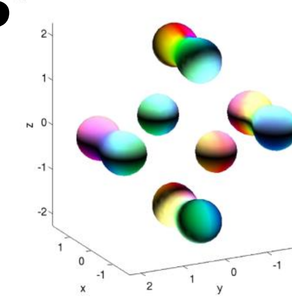
metastable



***B*=3**



***B*=4**



***B*=5**

Summary: Skyrmions $\mathbf{n}=(n_1,n_2,n_3,n_4)$ $\mathbf{n}^2=1$

(1) $V=m^2(1-n_4^2)=m^2(n_1^2+n_2^2+n_3^2)$

Baby Skyrmion on a domain wall

π_2 π_0

Domain wall Skyrmion

(2) $V=m^2(n_1^2+n_2^2)=m^2(1-n_3^2-n_4^2)$

or **(2)'** $V=m^2(n_1^2+n_2^2)(n_3^2+n_4^2)$

SG kink on a global vortex

π_1 π_1

Vortex Skyrmion

(3) $V=m^2 n_1^2$

Ising spin on a global monopole

π_0 π_2

Monopole Skyrmion

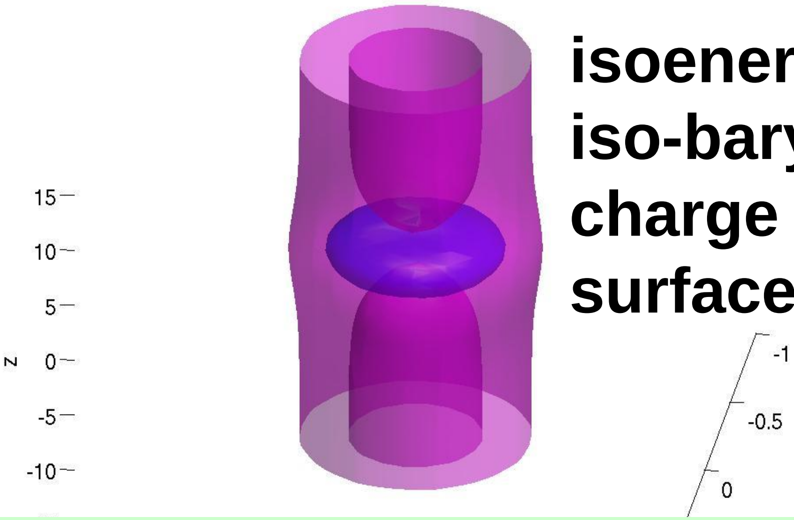
→ **Skyrmion** π_3

π_m on $\pi_n=\pi_{m+n+1}$

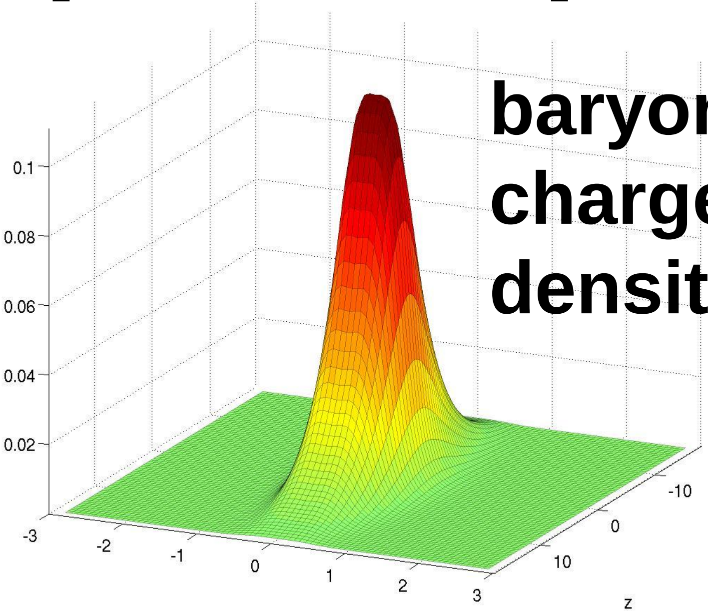
(2)' Skyrmion as SG kink in a vortex

SBG&MN Phys.Rev.D90(2014)085007 [arXiv:1407.7210]

$$V = m^2(n_1^2 + n_2^2)(n_3^2 + n_4^2) + m_3 n_3$$

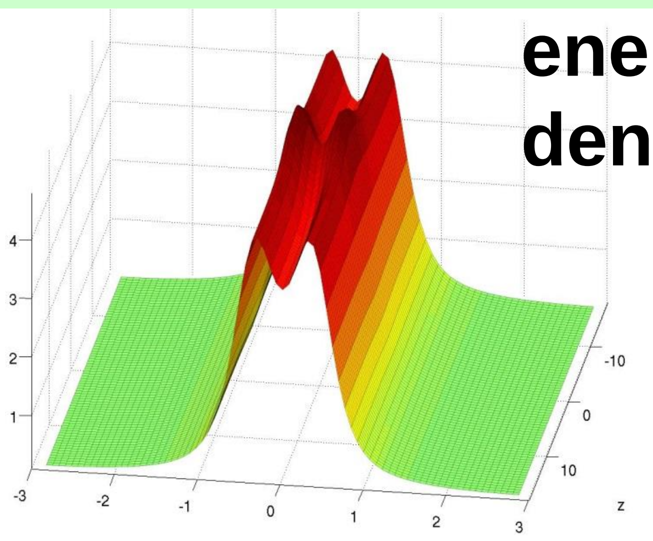


isoenergy and iso-baryon charge density surfaces

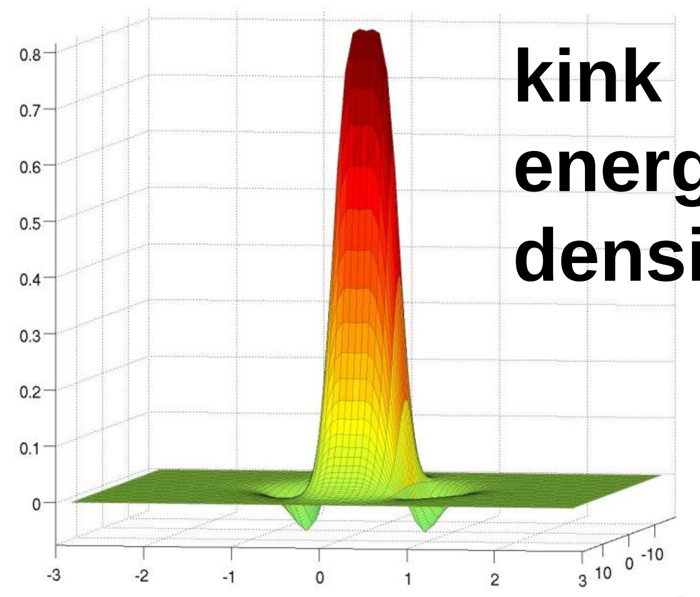


baryon charge density

U(1) modulus is twisted once



energy density

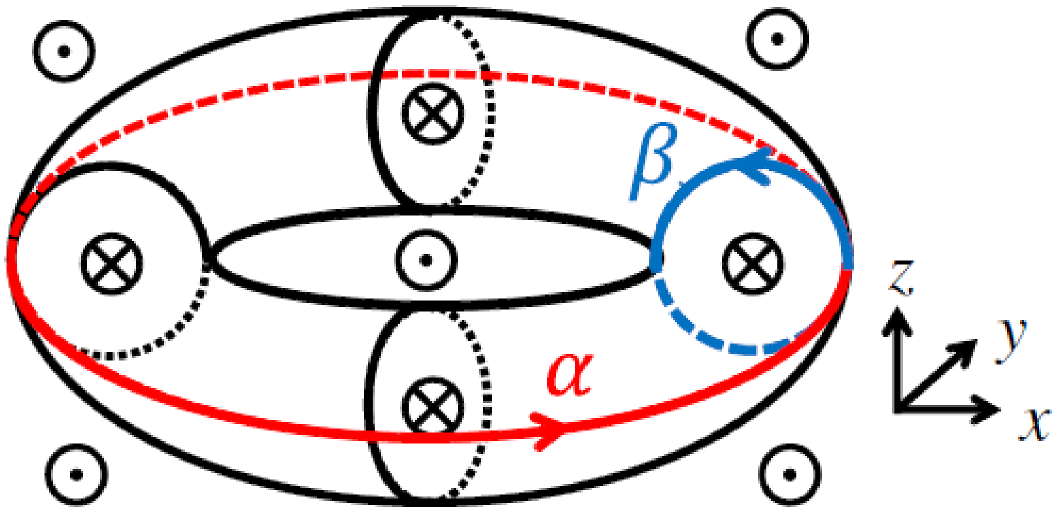


kink energy density

(2)' Baryonic torus

$$V = m^2(n_1^2 + n_2^2)(n_3^2 + n_4^2) + m_3 n_3$$

SBG&MN, Phys.Rev.D91 (2015) 045027 [arXiv:1410.8407]



U(1) modulus is twisted *P* times along *α*
The vortex has the winding number *Q* along *β*

Baryon number $B = PQ$

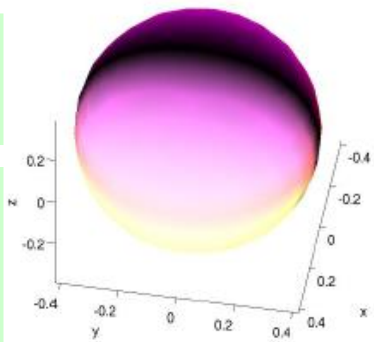
(2)’ Baryonic torus

$$V = m^2(n_1^2+n_2^2)(n_3^2+n_4^2)+m_3n_3$$

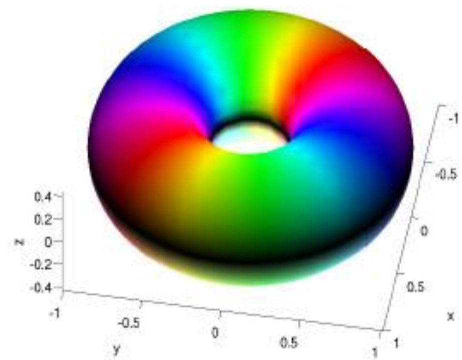
SBG&MN, Phys.Rev.D91 (2015) 045027 [arXiv:1410.8407]

$$B = PQ$$

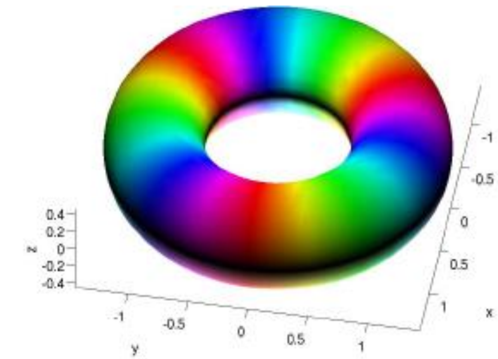
U(1) modulus is twisted *P* times



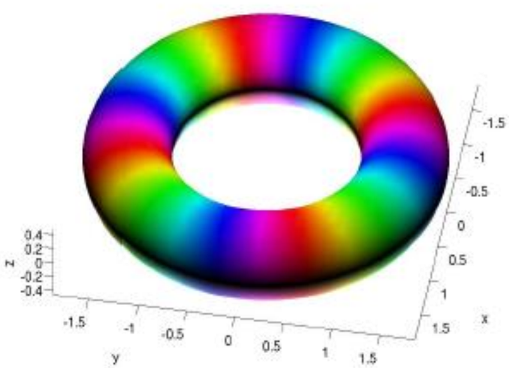
$(P,Q)=(1,1)$



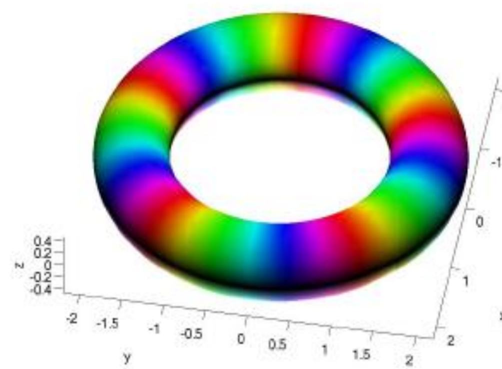
$(P,Q)=(2,1)$



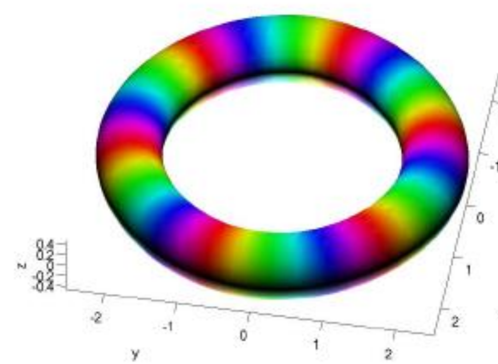
$(P,Q)=(3,1)$



$(P,Q)=(4,1)$



$(P,Q)=(5,1)$



$(P,Q)=(6,1)$