Fermionic zero modes in baby Skyrme models

Jose Queiruga

Karlsruhe Institute of Technology

June 20, 2016

Motivation

- The baby Skyrme model is a Skyrme-type model in $2+1$ dimensions.
- Static solutions are classified by a winding number.
- Stability of solutions requires a potential term.
- $\mathcal{L}_{bS} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4$.
- It has applications from condensed matter to cosmology.
- SUSY version of the model $\rightarrow$ canonical way of introducing fermions
- We can relate SUSY transformations with the BPS structure of the model (BPS restriction).
- Our goal: how to get some information about the fermionic zero modes in the SUSY model?

The baby Skyrme model

The baby Skyrme model

Baby Skyrme Lagrangian

$$\mathcal{L} = \frac{\lambda_2}{4}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{\lambda_4}{8}\left(\partial_\mu\vec{\phi}\times\partial_\nu\vec{\phi}\right)^2 - \lambda_0\mathcal{V}(\phi^3)$$

$$\vec{\phi}\in\mathbb{S}^2 \quad \vec{\phi}_0 : \mathbb{S}^2\to\mathbb{S}^2$$

The baby Skyrme model

Baby Skyrme Lagrangian

$$\mathcal{L} = \frac{\lambda_2}{4}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{\lambda_4}{8}\left(\partial_\mu\vec{\phi}\times\partial_\nu\vec{\phi}\right)^2 - \lambda_0\mathcal{V}(\phi^3)$$

$$\vec{\phi}\in\mathbb{S}^2 \quad \vec{\phi}_0 : \mathbb{S}^2 \to \mathbb{S}^2$$

After stereographic projection and $\lambda_2 = 0$ BPS submodel $\rightarrow$

$$\mathcal{L}_{BbS} = -\frac{1}{(1+u\bar{u})^4}\left(\left(\partial_\mu u\partial^\mu\bar{u}\right)^2 - \left(\partial_\mu u\right)^2\left(\partial_\nu\bar{u}\right)^2\right) - \mathcal{V}\left(u\bar{u}\right)$$

Energy in terms of holomorphic/antiholomorphic derivatives

$$E = \frac{1}{4}\int d^2x\left(\frac{\left(|\partial u|^2 - |\bar{\partial}u|^2\right)^2}{(1+u\bar{u})^4} + 4\mathcal{V}(u\bar{u})\right)$$

where $\partial = \partial_1 + i\partial_2$. Therefore...

$$E \geq \mp 4\pi \int d^2x \sqrt{V(u,\bar{u})}Q, \quad Q = \frac{1}{4\pi}\frac{\left(|\partial u|^2 - |\bar{\partial} u|^2\right)}{(1+u\bar{u})^2}$$

Q is the degree of u. The energy bound is saturated if

$$\partial u = e^{i\eta}\sqrt{\sqrt{V(u,\bar{u})}\,(1+u\bar{u})^2 + \frac{1}{2}\left(|\partial u|^2 + |\bar{\partial} u|^2\right)}$$

and η a free phase ($\mathcal{L}$ invariant under $u \to e^{i\eta}u$). This is the BPS equation of the model (keep in mind!). Let's move to SUSY BPS baby.

The SUSY baby Skyrme model

$$\mathcal{L}_{BbS}^{\text{SUSY}} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^4\theta H(\Phi, \Phi^\dagger) D^\alpha \Phi D_\alpha \Phi \bar{D}^{\dot{\beta}} \Phi^\dagger \bar{D}_{\dot{\beta}} \Phi^\dagger$$

$$\Phi = u + i\theta\sigma^\mu\bar{\theta}\partial_\mu u + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box u + \sqrt{2}\theta\psi - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi\sigma^\mu\bar{\theta} + \theta\theta F$$

- Integrate w.r.t Grassmann coordinates
- Switch off fermions

The SUSY baby Skyrme model

$$\mathcal{L}_{BbS}^{\text{SUSY}} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^4\theta H(\Phi, \Phi^\dagger) D^\alpha \Phi D_\alpha \Phi \bar{D}^{\dot\beta} \Phi^\dagger \bar{D}_{\dot\beta} \Phi^\dagger$$

$$\Phi = u + i\theta\sigma^\mu\bar{\theta}\partial_\mu u + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box u + \sqrt{2}\theta\psi - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi\sigma^\mu\bar{\theta} + \theta\theta F$$

- Integrate w.r.t Grassmann coordinates
- Switch off fermions

$$\begin{aligned} \mathcal{L}_{BbS}^{\text{SUSY}} &= g(u,\bar{u})\left(\partial^\mu u \partial_\mu \bar{u} + F\bar{F}\right) + \\ &+ h(u,\bar{u})\left((\partial_\mu u)^2(\partial_\nu \bar{u})^2 + 2F\bar{F}\partial^\mu\bar{u}\partial_\mu u + (F\bar{F})^2\right) \end{aligned}$$

- Solve eom's for F
  - $F = 0$
  - $F = e^{i\eta}\sqrt{-\partial_\mu u \partial^\mu \bar{u} - \frac{g(u,\bar{u})}{2h(u,\bar{u})}}$

Take the non-zero solution and substitute (with the choice $h = 1/(1+u\bar{u})^4$) Finally:

$$\mathcal{L}_{BbS}^{\text{SUSY}} = h(u,\bar{u})\left((\partial_\mu u)^2(\partial_\nu \bar{u})^2 - (\partial^\mu \bar{u}\partial_\mu u)^2\right) - \frac{g(u,\bar{u})^2}{4h(u,\bar{u})}$$

- No σ-model term.
- The potential term comes from the Kähler metric g.
- No $N = 2$ baby model (with quadratic term)?
- The BPS equation in the SUSY model is simply

$$\partial u = F$$

Fermions

Taking into account the fermions we get

$$\mathcal{L}_2 = \int d^4\theta K(\Phi, \Phi^\dagger) = g(u, \bar{u}) \Big(\partial^\mu u \partial_\mu \bar{u} - \frac{i}{2} \psi \sigma^\mu \mathcal{D}^\mu \bar{\psi} + \frac{i}{2} \mathcal{D}^\mu \psi \sigma_\mu \bar{\psi} + F\bar{F} \Big) + \mathcal{O}(\psi\bar{\psi})^2$$

Fermions

Taking into account the fermions we get

$$\mathcal{L}_2 = \int d^4\theta K(\Phi, \Phi^\dagger) = g(u, \bar{u}) \left(\partial^\mu u \partial_\mu \bar{u} - \frac{i}{2} \psi \sigma^\mu \mathcal{D}^\mu \bar{\psi} + \frac{i}{2} \mathcal{D}^\mu \psi \sigma_\mu \bar{\psi} + F\bar{F} \right) + \mathcal{O}(\psi\bar{\psi})^2$$

$$\begin{aligned} \mathcal{L}_4^0 \; = \; & \int d^4\theta H(\Phi, \Phi^\dagger) D^\alpha \Phi D_\alpha \Phi \bar{D}^{\dot{\beta}} \Phi^\dagger \bar{D}_{\dot{\beta}} \Phi^\dagger |_{0-\text{der}} = \\ & h(u, \bar{u})((\partial_\mu u)^2 (\partial_\nu \bar{u})^2 + 2F\bar{F} \partial^\mu u \partial_\nu \bar{u} + (F\bar{F})^2 \\ & + \frac{i}{2} \psi \sigma^\mu \bar{\psi} \left(\bar{u}_{,\mu} \Box u - u_{,\mu} \Box \bar{u} \right) \\ & - \frac{i}{2} \psi \sigma^\mu \bar{\psi}^{,\nu} u_{,\mu} \bar{u}_{,\nu} - i \psi^{,\mu} \sigma^\nu \bar{\psi} u_{,\mu} \bar{u}_{,\nu} - \frac{i}{2} \psi \sigma^\mu \bar{\sigma}^\rho \sigma^\nu \bar{\psi}_{,\nu} u_{,\mu} \bar{u}_{,\rho} + \ldots \end{aligned}$$

$$
\begin{aligned}
&\ldots + F\Box u\bar{\psi}\bar{\psi} + \frac{1}{2}Fu_{,\mu}\partial^{\mu}(\bar{\psi}\bar{\psi}) + \bar{F}\Box\bar{u}\psi\psi + \frac{1}{2}\bar{F}\bar{u}_{,\mu}\partial^{\mu}(\psi\psi) \\
&+\frac{1}{2}Fu_{,\mu}\left(\psi\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\psi}_{,\nu} - \bar{\psi}_{,\nu}\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\psi}\right) + \frac{1}{2}\bar{F}\bar{u}_{,\mu}\left(\psi_{,\nu}\sigma^{\nu}\bar{\sigma}^{\mu}\psi - \psi\sigma^{\nu}\bar{\sigma}^{\mu}\psi_{,\nu}\right) \\
&+\frac{3i}{2}F\bar{F}\left(\psi_{,\mu}\sigma^{\mu}\bar{\psi} - \psi\sigma^{\mu}\bar{\psi}_{,\mu}\right) + \frac{i}{2}\psi\sigma^{\mu}\bar{\psi}\left(F\bar{F}_{,\mu} - \bar{F}F_{,\mu}\right) \\
&. + \frac{i}{2}\psi_{,\nu}\sigma^{\nu}\bar{\sigma}^{\mu}\sigma^{\rho}u_{,\mu}\bar{u}_{,\rho} + \mathcal{O}(\psi)^{3})
\end{aligned}
$$

$$
\begin{aligned}
&\ldots + F\Box u\bar{\psi}\bar{\psi} + \frac{1}{2}Fu_{,\mu}\partial^{\mu}(\bar{\psi}\bar{\psi}) + \bar{F}\Box\bar{u}\psi\psi + \frac{1}{2}\bar{F}\bar{u}_{,\mu}\partial^{\mu}(\psi\psi) \\
&+\frac{1}{2}Fu_{,\mu}\left(\psi\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\psi}_{,\nu} - \bar{\psi}_{,\nu}\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\psi}\right) + \frac{1}{2}\bar{F}\bar{u}_{,\mu}\left(\psi_{,\nu}\sigma^{\nu}\bar{\sigma}^{\mu}\psi - \psi\sigma^{\nu}\bar{\sigma}^{\mu}\psi_{,\nu}\right) \\
&+\frac{3i}{2}F\bar{F}\left(\psi_{,\mu}\sigma^{\mu}\bar{\psi} - \psi\sigma^{\mu}\bar{\psi}_{,\mu}\right) + \frac{i}{2}\psi\sigma^{\mu}\bar{\psi}\left(F\bar{F}_{,\mu} - \bar{F}F_{,\mu}\right) \\
&. + \frac{i}{2}\psi_{,\nu}\sigma^{\nu}\bar{\sigma}^{\mu}\sigma^{\rho}u_{,\mu}\bar{u}_{,\rho} + \mathcal{O}(\psi)^{3})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_4^1 \;=\; & \partial_u h(u,\bar{u})\Big(\frac{1}{2}F\bar{F}^2\psi^2 - \frac{1}{2}F\psi^2(\bar{u}_{,\nu})^2 + i\sigma^{\mu}\bar{\psi}\partial_{\mu}\psi \\
& -i\sqrt{2}(\bar{u}_{,\mu})^2\bar{\sigma}^{\nu}\psi^2 u_{,\nu} \\
& -2iF\bar{F}\bar{u}_{,\mu}\psi\sigma^{\mu}\psi - 2F\bar{u}_{,\mu}\psi\bar{\psi}\bar{\sigma}^{\mu}\sigma_{\nu}u_{,\nu} + 2i\psi\bar{\psi}\sigma^{\mu}\sigma^{\nu}\sigma^{\rho}u_{,\mu}\bar{u}_{,\nu}u_{,\rho} \\
& +2F\bar{F}\psi\sigma^{\mu}\bar{\psi}u_{,\mu} - F\bar{\psi}^2\sigma^{\mu}\sigma^{\nu}u_{,\mu}u_{,\mu} + \bar{F}\psi^2\sigma^{\mu}\sigma^{\nu}\bar{u}_{,\mu}u_{,\nu} + \mathcal{O}(\psi)^3\Big)
\end{aligned}
$$

$+$ 19 terms more.

- The auxiliary field F becomes dynamical.
- No trivial solution for F (due to linear terms).
- Even at quadratic order in fermions it is difficult to obtain information from eom's.
- We can use SUSY transformations to obtain fermionic zero modes from bosonic solutions.

$$\delta\psi^\alpha = \partial^{\alpha\dot{\alpha}} u\, \bar{\xi}_{\dot{\alpha}} + F\xi^\alpha$$

Preservation of (a fraction of SUSY) $\Rightarrow \delta\psi^\alpha|_{\text{on-shell}} = 0$. In terms holomorphic/antiholomorphic derivatives

$$\delta\psi^\alpha = \rho^\alpha \partial u + \eta^\alpha \bar{\partial} u + \left(e^{i\eta}\partial u + F\right)\xi^\alpha$$

where

$$\begin{aligned} \eta^\alpha &= -\frac{1}{2}\left(\sigma_1^{\alpha\dot{\beta}} - i\sigma_3^{\alpha\dot{\beta}}\right)\bar{\xi}_{\dot{\beta}} \\ \rho^\alpha &= +\frac{1}{2}\left(\sigma_1^{\alpha\dot{\beta}} + i\sigma_3^{\alpha\dot{\beta}}\right)\bar{\xi}_{\dot{\beta}} + e^{i\eta'}\xi^\alpha. \end{aligned}$$

$$\delta\psi^\alpha = \partial^{\alpha\dot\alpha} u\, \bar\xi_{\dot\alpha} + F\xi^\alpha$$

Preservation of (a fraction of SUSY) $\Rightarrow \delta\psi^\alpha|_{\text{on-shell}} = 0$. In terms holomorphic/antiholomorphic derivatives

$$\delta\psi^\alpha = \rho^\alpha \partial u + \eta^\alpha \bar\partial u + \left(e^{i\eta}\partial u + F\right)\xi^\alpha$$

where

$$\begin{aligned}
\eta^\alpha &= -\frac{1}{2}\left(\sigma_1^{\alpha\dot\beta} - i\sigma_3^{\alpha\dot\beta}\right)\bar\xi_{\dot\beta} \\
\rho^\alpha &= +\frac{1}{2}\left(\sigma_1^{\alpha\dot\beta} + i\sigma_3^{\alpha\dot\beta}\right)\bar\xi_{\dot\beta} + e^{i\eta'}\xi^\alpha.
\end{aligned}$$

Therefore

$$\delta\psi^\alpha|_{\text{on-shell}} = \rho^\alpha\partial u + \eta^\alpha\bar\partial u$$

- If $\partial u \neq e^{i\eta} F$, $\delta\psi^{\alpha}|_{\text{on-shell}} = 0 \Rightarrow \rho^{\alpha} = \eta^{\alpha} = \xi^{\alpha} = 0$ and SUSY in not generically presevrved.
- If $\partial u = e^{i\eta} F$, $\delta\psi^{\alpha}|_{\text{on-shell}} = 0 \Rightarrow \rho^{\alpha} = \eta^{\alpha} = 0$, η^{α} has 1 d.o.f. and ρ^{α} has 2 d.o.f. $\Rightarrow$ 3 SUSY generators are broken $\Rightarrow$ solutions are $1/4$ SUSY (equiv. $\dim \mathrm{Ker}\delta = 1$).
- 3 generators broken $\Rightarrow$ 3 goldstone fermions (fermionic zero modes).

In the background of the BPS solutions we have obtained: $\delta\psi^{\alpha}|_{\text{on-shell}} = \rho^{\alpha}\partial u + \eta^{\alpha}\bar{\partial} u$ and 3 Grassmann parameters leads to three fermionic d.o.f.

$$\left\{ \frac{1}{2}\chi \begin{pmatrix} 1 \\ i \end{pmatrix} \bar{\partial} u, \frac{1}{2} \begin{pmatrix} \zeta_1 \\ 0 \end{pmatrix} \partial u, \frac{1}{2} \begin{pmatrix} 0 \\ \zeta_2 \end{pmatrix} \partial u \right\}$$

In the symmetric *ansatz* $u = f(r)e^{in\varphi}$ with boundary conditions

$$f(r=0) = \infty, \quad f(r=R) = 0 \quad \text{and} \quad f'(r=R) = 0$$

$R = \text{finite}, \infty$ for a compact or non-compact solution respect. The fermionic zero modes can be written as

$$\begin{pmatrix} \frac{1}{2}\chi \begin{pmatrix} 1 \\ i \end{pmatrix} \frac{e^{i(n-1)\theta}(rf'(r)+nf(r))}{r} \\ \frac{1}{2}\zeta_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{e^{i(n+1)\varphi}(rf'(r)-nf(r))}{r} \\ \frac{1}{2}\zeta_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{e^{i(n+1)\varphi}(rf'(r)-nf(r))}{r} \end{pmatrix}$$

- If $R = \infty$ (non compact solution), $\delta\psi^\alpha|_{\text{BPS}} = 0 \Leftrightarrow \rho^\alpha = \eta^\alpha = 0$, 1/4 SUSY in $\mathbb{R}^2$.
- if $R =$ finite, we have $f'(r) = f(r) = 0$ for $r \geq R \Rightarrow 1/4$ SUSY inside the compacton, but 1 SUSY outside.
  - $\Rightarrow$fermionic zero modes only exist inside the compacton

Let us take the following potential:

$$\mathcal{V}(u,\bar{u}) = \left(\frac{u\bar{u}}{1+u\bar{u}}\right)^s, \quad s \in (1,2)$$

The corresponding Kähler potential in the SUSY model is given by

$$K(\Phi,\Phi^\dagger) = 8\frac{(\Phi\Phi^\dagger)^{\frac{s+2}{2}}}{(s+2)^2}\,{}_2F_1\left(\frac{s+2}{2},\frac{s+2}{2},\frac{s+4}{2},-\Phi^\dagger\Phi\right).$$

For the $n=1$ charge solution we get the following profile function:

$$f(r) = \begin{cases} \dfrac{\left(1-\frac{r^2}{R^2}\right)^{\frac{1}{2-s}}}{\sqrt{1-\left(1-\frac{r^2}{R^2}\right)^{-\frac{2}{s-2}}}}, & 0 \le r \le R \\ 0, & r \ge R \end{cases}$$

Compacton radius $R^2 = 4/(2-s)$.

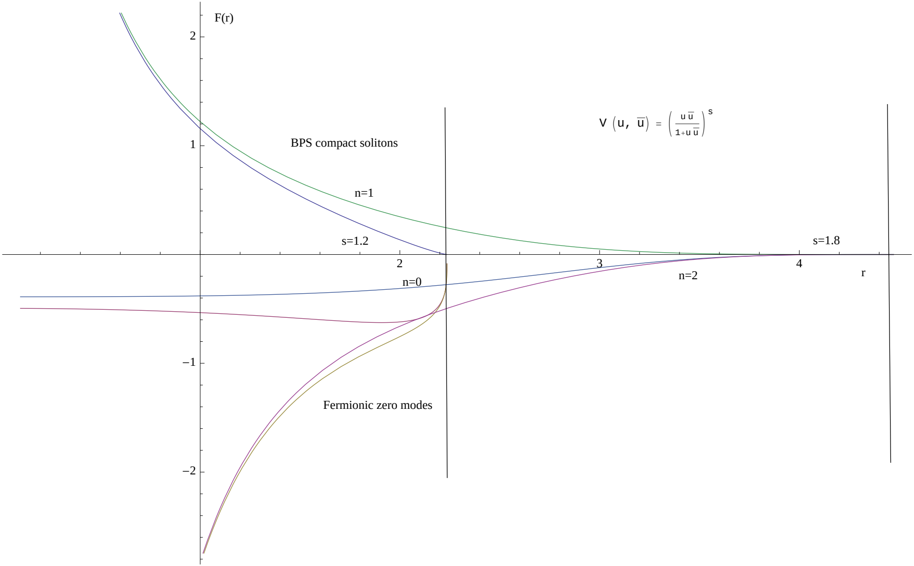

Figure: BPS compact solitons and fermionic zero modes

- Non-compact BPS solutions$\Rightarrow$ fermion zero modes are spread throughout space.
- Compact BPS solutions $\Rightarrow$ fermion zero modes are confined to the compacton.

How general is this statement?

Let us consider the following action:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta K(\Phi,\Phi^\dagger) + \int d^4\theta H(\Phi,\Phi^\dagger) D^\alpha\Phi D_\alpha\Phi \bar{D}^{\dot\beta}\Phi^\dagger \bar{D}_{\dot\beta}\Phi^\dagger \\ &+ \left(\lambda \int d^2\theta W(\Phi) + \text{h.c.}\right)\end{aligned}$$

Expanding for small λ

$$\mathcal{L}^{(1)} = g(u,\bar{u})u_{,\mu}\bar{u}^{,\mu} + h(u,\bar{u})(u_{,\mu}^2)(\bar{u}_{,\nu})^2 + \mathcal{O}(\lambda^2)$$

$$\mathcal{L}^{(2)} = h(u,\bar{u})\left((u_{,\mu})^2(\bar{u}_{,\nu})^2 - (u_{,\mu}\bar{u}^{,\mu})^2\right) - \frac{g(u,\bar{u})^2}{4h(u,\bar{u})} + 2\lambda|W'|\sqrt{-\frac{g(u,\bar{u})}{2h(u,\bar{u})} - u_{,\mu}\bar{u}^{,\mu}} + \mathcal{O}(\lambda^2)$$

$$\mathcal{L}^{(3)} = h(u,\bar{u})\left((u_{,\mu})^2(\bar{u}_{,\nu})^2 - (u_{,\mu}\bar{u}^{,\mu})^2\right) - \frac{g(u,\bar{u})^2}{4h(u,\bar{u})} - 2\lambda|W'|\sqrt{-\frac{g(u,\bar{u})}{2h(u,\bar{u})} - u_{,\mu}\bar{u}^{,\mu}} + \mathcal{O}(\lambda^2)$$

Let us choose $W'(\bar{u}) = m\bar{u}^l +$ higher powers. In a compacton configuration the profile function must behave close to the boundary as

$$f(r) = \beta \left(\frac{r-R}{R}\right)^{\epsilon}$$

Table: SUSY transformation at the boundary

	$\lim_{r\to R} \delta\psi^\alpha$
$l > \min(2 - \frac{2}{\epsilon}, s)$	0
$l < \min(2 - \frac{2}{\epsilon}, s)$	∞
$l = \min(2 - \frac{2}{\epsilon}, s)$	$C\xi^\alpha$

In the third case the fermionic zero modes exist beyond the compacton support!

SUSY baby Skyrme with quadratic term

We will use a trick (M. Nitta, S. Sasaki, Phys.Rev. D90 (2014) no.10, 105001) to construct the $N = 2$ version of the full baby Skyrme model. SUSY Lagrangian again

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi, \Phi^\dagger) + \int d^2\theta d^2\bar{\theta}\, \Lambda\, D^\alpha \Phi D_\alpha \Phi \bar{D}^{\dot{\beta}} \Phi^\dagger \bar{D}_{\dot{\beta}} \Phi^\dagger$$

where Λ is an unknown function of superfields and derivatives. Solving for F

$$F = 0 \quad \text{and} \quad F\bar{F} = -\frac{g}{2\Lambda_0} - \partial_\mu u \partial^\mu \bar{u}$$

Substituting the trivial solution (this procedure works also for the non-trivial one)

$$\mathcal{L}|_{\text{on-shell}} = g\, \partial_\mu u \partial^\mu \bar{u} + \Lambda_0 (\partial_\mu u)^2 (\partial_\nu \bar{u})^2$$

Now force this Lagrangian to be the one corresponding to the full baby Skyrme model and solve for Λ_0.

$$\Lambda_0 = -\frac{V(u,\bar{u}) + h(u,\bar{u})\left(\partial_\mu u \partial^\mu \bar{u}\right)^2 - h(u,\bar{u})(\partial_\mu u)^2(\partial_\nu \bar{u})^2}{(\partial_\mu u)^2(\partial_\nu \bar{u})^2}$$

promoting to superfields

$$\Lambda = -\frac{V(\Phi,\Phi^\dagger) + h(\Phi,\Phi^\dagger)\left(\partial_\mu \Phi \partial^\mu \Phi^\dagger\right)^2 - h(\Phi,\Phi^\dagger)(\partial_\mu \Phi)^2(\partial_\nu \Phi^\dagger)^2}{(\partial_\mu \Phi)^2(\partial_\nu \Phi^\dagger)^2}$$

(the term with four superderivatives saturates the Grassmannian integration for the bosonic sector). So, with this choice of Λ

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi,\Phi^\dagger) + \int d^2\theta d^2\bar{\theta}\, \Lambda\, D^\alpha \Phi D_\alpha \Phi \bar{D}^{\dot{\beta}} \Phi^\dagger \bar{D}_{\dot{\beta}} \Phi^\dagger$$

generates the SUSY baby Skyrme model with quadratic term in the canonical branch ($F = 0$).

Conclusions

- We have studied the SUSY BPS baby Skyrme model.
- Generic BPS solutions preserve 1/4 SUSY.
- We have determined the fermion zero modes in the background of BPS solutions
  - Non-compact BPS solutions $\Rightarrow$ fermion zero modes exist throughout space.
  - Compact BPS solutions $\Rightarrow$ fermion zero modes are confined to the compacton support$\Rightarrow$ restoration of SUSY outside the compacton.
- This behavior was observed for SUSY cosmic strings with higher derivative terms: Phys.Lett. B755 (2016) 498-503.
- The addition of a D-term allows for the "deconfinement" of fermion zero modes even in the background of a compact solution.

$\frac{1}{2}$Thanks