Nuclear natter EoS and neutron stars in the Skyrme models

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based on collaboration with

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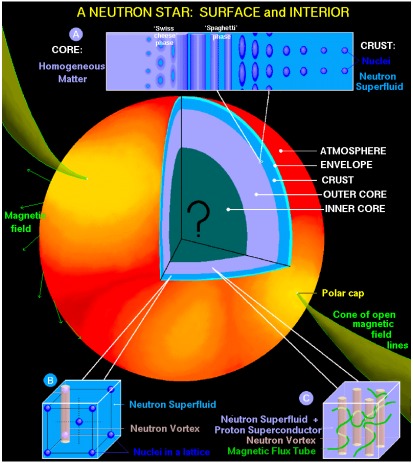
    - non-uniques EoS $\rightarrow$ inverse TOV (?)

Neutron stars in TOV approach

neutron stars in TOV approach

- neutron star structure from nuclear physics

  http://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/



neutron stars in TOV approach

- the canonical approach to neutron stars
  - Einstein eqs

$$G^{\mu\nu} = \frac{\kappa^2}{2} T^{\mu\nu}$$

  - prescribed energy-momentum tensor - **perfect fluid**

$$T^{\rho\sigma} = (p+\rho) u^\rho u^\sigma - p g^{\rho\sigma}$$

- spherically symmetric metric

$$ds^2 = \mathbf{A}(r) dt^2 - \mathbf{B}(r) dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

TOV equations

$$\begin{aligned}
\text{TOV1}: \quad M' &= 4\pi r^2 \rho, \quad \mathbf{B}(r) \equiv \left(1 - \frac{\kappa^2}{8\pi}\frac{M(r)}{r}\right)^{-1} \\
\text{TOV2}: \quad r p' &= (\rho + p)\left(\frac{1}{2}(1-\mathbf{B}) - \frac{\kappa^2}{4} r^2 \mathbf{B} p\right) \\
\left(\frac{\mathbf{A}'}{\mathbf{A}}\right. &= \left.\frac{1}{r}(\mathbf{B}-1) + \frac{\kappa^2}{2} r \mathbf{B} p\right)
\end{aligned}$$

- to close the system: **equation of state** (EoS)

$$p = p(\rho, \ldots)$$

neutron stars in TOV approach

- EoS $\to$ input from nuclear physics
  EFTs for nuclear matter - no perfect fluid
  $\to$ **mean-field** approximation
  examples
  - Walecka model
  - NJL model
- mean-field EoS
  - algebraic EoS $p = \rho(p)$
  - constant densities $\rho = const.$
  - realistic nuclei (nuclear matter) $\rho = \rho(r)$

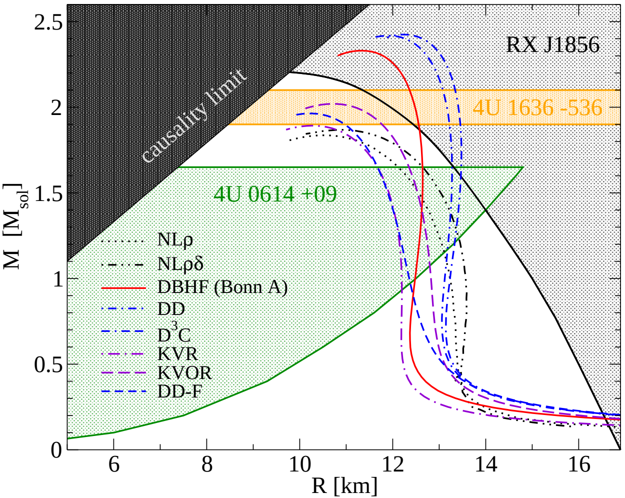
How non-mean-field affects TOV?
Can we do gravitating nuclear matter in a full FT+GR?

$\to$ we need an effective model (action) of nuclear matter
$\to$ couple it to gravity
$\to$ find nuclear stars
$\to$ verify the universality of EoS

solvable nuclear matter action in a **thermodynamical limit (perf. fluid)**

neutron stars in TOV approach

T. Klähn et al, Phys. Rev. C**74** (2006) 035802



Questions

- **neutron stars in the Skyrme model**

  $\rightarrow$ **TOV approach**

  EoS of skyrmionic matter $\rightarrow$ nuclear matter MF EoS
  - thermodynamics of skyrmions
  - role of each term

- **beyond MF approach?**

Mean-Field EoS in the Skyrme model

solitonic Skyrme model

- **the Skyrme framework** Skyrme (61)

  pionic EFT of

  - baryons and nuclei $\rightarrow$ emergent objects: **solitons**
    extended, non-perturbative
  - nuclear matter
  - with applications to neutron stars
    $\rightarrow$ complementary to lattice

- support form $N_c \rightarrow \infty$ limit t'Hooft (83), Witten (84)
  - chiral effective meson/baryon theory
  - primary d.o.f. are mesons
  - baryons (nuclei) are realized as solitons

  - simplest case (two flavors): $U(x) = e^{i\vec{\pi}\vec{\sigma}} \in SU(2)$
  - $\vec{\pi}$ - pions
  - topological charge = baryon number

$$U : \mathbb{R}^3 \cup \{\infty\} \cong \mathbb{S}^3 \ni \vec{x} \rightarrow U(\vec{x}) \in SU(2) \cong \mathbb{S}^3$$

$$\pi_3(\mathbb{S}^3) = \mathbb{Z}$$

solitonic Skyrme model

- the standard (perturbative) Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_0 + \underbrace{\lambda_2 \mathcal{L}_2 + \lambda_4 \mathcal{L}_4}_{\text{massless } \mathcal{L}_{skyrme}}}_{\text{massive } \mathcal{L}_{skyrme}}$$

$$\mathcal{L}_2 = -\lambda_2 \, \mathrm{Tr}\,(L_\mu L^\mu), \quad \mathcal{L}_4 = \lambda_4 \, \mathrm{Tr}\,([L_\mu, L_\nu]^2), \quad L_\mu = U^\dagger \partial_\mu U$$

- successes

    baryon physics Adkins, Nappi, Witten (84)....Praszalowicz, Nowak, Rho...
    deuteron, light nuclei $\rightarrow$ iso-rotational spectra $\rightarrow$ SCQ correct
    Braaten, Carson, Manton, Rho....Halcrow (15)
    ^{12}C and Hoyle states Manton, Liu (14)

- difficulties

    unphysical binding energies Sutcliffe et. al. (97), (02), (05), (06), (10)
    crystal state of matter Klebanov (85), Battye, Sutcliffe et. al. (06)
    very complicated FT - only MF possible

problematic for (heavy) nuclei and nuclear matter $\rightarrow$ **neutron stars**

solitonic Skyrme model

- **physical binding energies** - **the near BPS Skyrme model**
  - Lorentz inv.
  - standard Hamiltonian
  - max. first time derivative squared

$$\mathcal{L} = \underbrace{\tilde{\mathcal{L}}_0 + \lambda_2 \mathcal{L}_2 + \lambda_4 \mathcal{L}_4}_{\text{massive } \mathcal{L}_{skyrme}} \quad + \quad \underbrace{\lambda_6 \mathcal{L}_6 + \mathcal{L}_0}_{\mathcal{L}_{BPS}}$$

$$\mathcal{L}_6 = -\mathbb{B}_\mu \mathbb{B}^\mu, \quad \mathbb{B}^\mu = \frac{1}{24\pi^2} \mathrm{Tr}\left(\epsilon^{\mu\nu\rho\sigma} L_\nu L_\rho L_\sigma\right)$$

- leading BPS part $\lambda_6 \mathcal{L}_6 + \mathcal{L}_0$ Adam, Naya, Sanchez-Guillen, Wereszczynski (13)
  weak impact of $\mathcal{L}_0$ on binding energies
- suitable $\mathcal{L}_0$ Guillard, Harland, Speight (15), Gudnason (15), (16)

use this version to study nuclear matter / neutron stars

MF EoS

MF EoS

generic Skyrme model $\rightarrow$ non perfect fluid form

mean-field $\rightarrow$ perfect fluid

- average densities $\bar{\epsilon}$ and $\bar{\rho}_B$

$$\bar{\epsilon} = \frac{E}{V}, \qquad \bar{\rho}_B = \frac{B}{V}$$

- MF pressure P and baryon chemical potential $\bar{\mu}$

$$\left(\frac{\partial E}{\partial V}\right)_B = -P, \qquad \left(\frac{\partial E}{\partial B}\right)_V = \bar{\boldsymbol{\mu}}$$

note: P is the average pressure

$$P = \frac{\frac{1}{3}\int_\Omega T_{ii} d^3x}{\int_\Omega d^3x}$$

where T_{ij} is the stress tensor and Ω is a set where a Skyrmion is located

- issues:
  $V \rightarrow \infty$ at saturation ($P = 0$)
  Ω (shape)-dependent

MF EoS at high density

use the topological bound to get solution independent insight

$$E_4 = \frac{1}{16}\int d^3x \,\mathrm{Tr}\,[L_i, L_j]^2 \geq 3(2\pi^2)^{4/3}\frac{B^{4/3}}{V^{1/3}}, \quad E_6 = \int d^3x\,(\epsilon^{ijk}\,\mathrm{Tr}L_iL_jL_k)^2 \geq \frac{B^2}{V}$$

proof: Manton's strain tensor formulation

$$D_{jk} = -\frac{1}{2}\mathrm{tr}\left(R_jR_k\right)$$

D_{jk} = symmetric, positive 3×3 matrix with eigenvalues $\tilde{\lambda}_1^2, \tilde{\lambda}_2^2, \tilde{\lambda}_3^2$.
rescale $\lambda_i = \tilde{\lambda}_i/\sqrt[3]{2\pi^2}$

$$E_4 = 3\int_{\mathcal{M}}\Omega_{\mathcal{M}}\frac{1}{3}\left(\lambda_1^2\lambda_2^2+\lambda_2^2\lambda_3^2+\lambda_1^2\lambda_3^2\right) \geq 3\int_{\mathcal{M}}\Omega_{\mathcal{M}}\left(\lambda_1^4\lambda_2^4\lambda_3^4\right)^{\frac{1}{3}}$$
$$= 3\int_{\mathcal{M}}\Omega_{\mathcal{M}}|\mathcal{B}_0|^{\frac{4}{3}} \geq \frac{3}{\mathrm{Vol}_{\mathcal{M}}^{\frac{1}{3}}}\left(\left|\int_{\mathcal{M}}\Omega_{\mathcal{M}}\mathcal{B}_0\right|\right)^{\frac{4}{3}} = \frac{3}{\mathrm{Vol}_{\mathcal{M}}^{\frac{1}{3}}}|B|^{\frac{4}{3}}\ .$$

- E_6 bound - saturated (BPS) at high P
- E_4 bound - not saturated (only for isometries $\to \mathcal{M} = S^3$)
- no bound for E_2

MF EoS at high density

- energy (asymptotic regime)

$$E = \pi^4 \lambda^2 \frac{B^2}{V} + \alpha \frac{B^{4/3}}{V^{1/3}} + o(V^{-1/3}), \quad V \to 0$$

where $\alpha \geq 3(2\pi^2)^{4/3}\lambda_4$

  - average pressure

$$P = \pi^4 \lambda^2 \bar{\rho}_B^2 + \frac{\alpha}{3} \bar{\rho}_B^{4/3} + o(\bar{\rho}_B^{4/3})$$

  - average chemical potential

$$\bar{\mu} = 2\pi^4 \lambda^2 \bar{\rho}_B + \frac{4\alpha}{3} \bar{\rho}_B^{4/3} + o(\bar{\rho}_B^{4/3})$$

  - MF energy density

$$\bar{\varepsilon} = \pi^4 \lambda^2 \bar{\rho}_B^2 + \alpha \bar{\rho}_B^{4/3} + o(\bar{\rho}_B^{4/3})$$

- **leading** sextic term $\quad \bar{\varepsilon} = P$
- **subleading** Skyrme term $\quad \bar{\varepsilon} = 3P$

- kinetic and potential: conjecture

$$o(\bar{\rho}_B^{4/3}) = \tilde{\beta}\, \bar{\rho}_B^{\gamma} + \beta + o(1), \quad \gamma \approx \frac{2}{3}$$

Walecka model

the Walecka model

$\mathcal{L}_W = \mathcal{L}_N + \mathcal{L}_{\sigma,\omega} + \mathcal{L}_{int}$

$$\mathcal{L}_N = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N + \mu\gamma^0\right)\psi$$

$$\mathcal{L}_{\sigma,\omega} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

$$\mathcal{L}_{int} = g_\sigma\bar{\psi}\sigma\psi + g_\omega\bar{\psi}\gamma^\mu\omega_\mu\psi + \dots$$

$\rightarrow$ non perfect fluid form

- **mean-field $\rightarrow$ perfect fluid**

- compute the partition function the thermodynamical limit
$Z = \int e^{\int \mathcal{L}_W}$
- bosonic fields take their vacuum expectation values $\bar{\sigma}$, $\bar{\omega}_0$
- all derivative dependent terms disappear and the interactions are simplified to a mesonic background field seen by nucleons

$$\mathcal{L}_W = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N^* + \mu^*\gamma^0\right)\psi - \frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2$$

where

$$m_N^* = m_N - g_\sigma\bar{\sigma}, \quad \mu^* = \mu - g_\omega\bar{\omega}_0$$

the Walecka model

- MF EoS (at higher densities)
  Walecka

$$\bar{\varepsilon} = \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\bar{\rho}_B^2 + \frac{3}{4}\left(\frac{3\pi^2}{2}\right)^{1/3}\bar{\rho}_B^{4/3} + \frac{1}{2}\frac{m_N^2 m_\sigma^2}{g_\sigma^2} + o(1)$$

  Skyrme

$$\bar{\varepsilon} = \pi^4\lambda^2\bar{\rho}_B^2 + \alpha\bar{\rho}_B^{4/3} + \tilde{\beta}\,\bar{\rho}_B^{2/3} + \beta + o(1)$$

  - the same leading behaviour
    Walecka: ω meson (baryon current) dominating at high ρ, P
    Skyrme: $\mathcal{L}_6$ dominates - based on the baryon current
  - the same subleading behaviour
    Walecka: free fermion gas
    Skyrme: $\mathcal{L}_4$

$\mathcal{L}_6$ unavoidable for high $\rho/P \rightarrow$ obligatory for NS

Beyond MF - the Skyrme model in the BPS limit

the Skyrme model in the BPS limit

- **the BPS Skyrme model**

$$\mathcal{L}_{BPS} = \lambda_6 \mathcal{L}_6 + \mathcal{L}_0$$

- **BPS**
  - classically zero binding energies
  - physical values if SCQ + Coulomb + (iso-spin) Adam, Naya, Sanchez-Guillen, Wereszczynski (13)
- **perfect fluid field theory** for any B
  - not a gas of *weakly* interacting skyrmions Kalbermann (97), Jaikumar et al.(07)

  Physically well motivated idealization of the nuclear matter weakly modified by the (small) non-BPS part
- **solvablity**
  very simple solvable model which covers the main features of nuclear matter $\rightarrow$ hard core of Skyrme-type EFT

- **the near BPS Skyrme model**
  Skyrme theory in a new perspective
  - separation of d.o.f.

$$\mathcal{L} = \underbrace{\tilde{\mathcal{L}}_0 + \lambda_2 \mathcal{L}_2 + \lambda_4 \mathcal{L}_4}_{} + \underbrace{\mathcal{L}_{BPS}}_{}$$

perturbative	**non-perturbative**
explicit pions	coherent d.o.f.
kinetic + two body int.	topological term
	hidden (emergent) ω and σ
surface	**bulk**
shape	SDiff symmetry
	perfect fluid
far attractive int.	BPS: exact $\omega - \sigma$ balance

  - some (not all!) properties/observable of the near BPS action are dominated by the BPS part

$$\Downarrow$$

let's do the BPS model to learn about nuclear matter and neutron stars

Perfect fluid

perfect fluid

- **SDiff symmetries**
- **energy-momentum tensor of a perfect fluid**

$$T^{00} = \lambda^2\pi^2\mathbb{B}_0^2 + \nu^2\mathcal{U} \equiv \varepsilon$$

$$T^{ij} = \delta^{ij}\left(\lambda^2\pi^2\mathbb{B}_0^2 - \nu^2\mathcal{U}\right) \equiv \delta^{ij}P$$

  - *local* thermodynamical quantities
  - BPS eq. = zero pressure condition
  - e-m. conservation: $\partial_\mu T^{\mu\nu} = 0$
    static: $\partial_i T^{ij} = 0 \Rightarrow \partial_j P = 0 \ \Rightarrow P = const.$
  - constant pressure equation is a first integral of static EL eq.

$$\lambda^2\pi^4\mathcal{B}_0^2 - \nu^2\mathcal{U} = P > 0$$

$$\lambda\pi^2\mathcal{B}_0 = \pm\nu\sqrt{\mathcal{U} + \tilde{P}}, \quad \tilde{P} \equiv (P/\nu^2)$$

static non-BPS solutions with $P > 0$

perfect fluid - exact thermodynamics

- **energy density EoS**

$$\varepsilon - P = 2\nu^2 \mathcal{U}$$

  - **non-barotropic chiral fluid** $\varepsilon \neq \varepsilon(P)$

    the step-function potential $\varepsilon = P + 2\nu^2$

    no potential $\varepsilon = P$

    high pressure limit - potential independent

$$\varepsilon = P$$

  - *on-shell* EoS

$$\varepsilon = \varepsilon(P, \vec{x})$$

beyond mean-field thermodynamics:

$P = const.$ but $\varepsilon \neq const.$

perfect fluid - exact thermodynamics

- **particle (baryon) density EoS**

$$\rho_B = \mathcal{B}_0$$

  - generically non-constant (beyond MF)

$$\rho_B = \frac{\nu}{\lambda \pi^2} \sqrt{\mathcal{U} + \frac{P}{\nu^2}}$$

  - *on-shell*

$$\rho_B = \rho_B(P, \vec{x})$$

no universal $\varepsilon = \varepsilon(P)$, $\rho_B = \rho_B(P)$

universal relation - *off-shell* and *non-MF*

$$\varepsilon + P = 2\lambda^2 \pi^4 \rho_B^2$$

- **baryon chemical potential**

definition: $\varepsilon + P = \rho\mu \qquad \Rightarrow \qquad \mu_B = 2\lambda^2\pi^4\rho_B$

  - off-shell
  - universal, potential independent
  - non-MF (local)

perfect fluid - exact thermodynamics

- generically **exact (non-mean field) thermodynamics**
  - ϵ, ρ_B non-constant generically non-constant
  - non-barotropic fluid
  - no universal EoS

- **mean-field limit**
  - MF averages $\bar{\epsilon}$, $\bar{\rho}_B$

$$\bar{\epsilon} = \frac{E_{06}}{V}, \qquad \bar{\rho} = \frac{B}{V}$$

  - *universal* (geometrical) EoS
    - E_{06}, V, $\bar{\epsilon}$, $\bar{\rho}_B$
      - known as functions of P FT pressure
      - no need for solutions!
      - only $\mathcal{U}$ matters

$$E_{06}(P) = B\pi^2\lambda\mu \left\langle \frac{2\mathcal{U} + P/\mu}{\sqrt{\mathcal{U} + P/\mu}} \right\rangle, \qquad V(P) = B\pi^2 \frac{\lambda}{\mu} \left\langle \frac{1}{\sqrt{\mathcal{U} + P/\mu}} \right\rangle$$

- FT pressure is the pressure

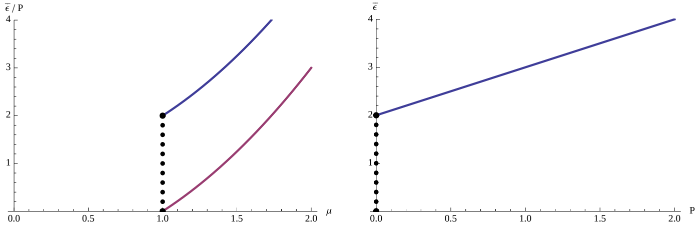
$$P = -\frac{dE_{06}}{dV}$$

micro (FT) thermodynamics = macro thermodynamics

comparison **full** vs. **mean-field**

example: **the step-function potential** $\mathcal{U} = \Theta\left(\mathrm{Tr}\left(1 - U\right)\right)$

- MF = non-MF
  - baryon chemical potential
    $$\mu = 2\pi^4\lambda^2\bar{\rho}_B$$
  - pressure
    $$P = \frac{1}{4\pi^4\lambda^2}\mu^2 - \nu^2$$
  - energy density
    $$\varepsilon = \frac{1}{4\pi^4\lambda^2}\mu^2 + \nu^2$$
  - EoS
    $$\varepsilon = 2\nu^2 + P$$



gas-liquid phase transition

perfect fluid

- the BPS action is equivalent to the action of a field theoretical description of perfect fluid in an **Eulerian formulation**

  - particle trajectories $\vec{X}_n(t) \to$ fluid element trajectories (cont. limit) $\vec{X}(t, \vec{y})$
  $\vec{y}$ comoving fluid coordinates
  - the Eulerian formulation: dynamical variables

$$\rho(t, \vec{x}) = \rho_0 \int d^3y \; \delta^{(3)} \left(\vec{X}(t, \vec{y}) - \vec{x} \right)$$

$$\vec{v}(t, \vec{x}) = \rho^{-1} \vec{j}, \quad \vec{j} = \rho_0 \int d^3y \dot{\vec{X}} \; \delta^{(3)} \left(\vec{X}(t, \vec{y}) - \vec{x} \right)$$

  formulated on phys. space but constrained ($N = const$. etc.)

- FT realisation Brown (93), Dubovski et. al. (03), (13), Jackiw (04) de Boer et. al. (15)
  - y^a promoted to the dynamical fields
  $x^i = X^i(t, y^a) \to y^a = \phi^a(t, x^i)$
  - density $\rho(t, \vec{x}) = \rho_0 D, \quad D = \Omega(\phi^a) \det \left(\frac{\partial \phi^a}{\partial x^i} \right)$
  particle number

$$\mathcal{N}^\mu = \Omega \epsilon^{\mu\nu\rho\sigma} \epsilon_{abc} \partial_\nu \phi^a \partial_\rho \phi^b \partial \sigma \phi^c$$

  - four velocity

$$u^\mu = \frac{\mathcal{N}^\mu}{\sqrt{\mathcal{N}^\nu \mathcal{N}_\nu}} = \frac{1}{\sqrt{6}D} \Omega \epsilon^{\mu\nu\rho\sigma} \epsilon_{abc} \partial_\nu \phi^a \partial_\rho \phi^b \partial \sigma \phi^c$$

$$\mathcal{N}^\mu = \rho u^\mu \quad \Rightarrow \quad \rho = \sqrt{6}D$$

perfect fluid

- a perfect fluid action = chose a Lagrange density $F = F(\phi^a, \partial_\mu \phi^a)$
  - $F = F(\rho, g(\phi^a))$

$$S = \int d^4 x F(\rho, g) \quad \Rightarrow \quad T^{\mu\nu} = (p+\epsilon) u^\mu u^\nu - p\eta^{\mu\nu}$$

where

$$\epsilon = -F(\rho, g), \quad p = \rho \frac{\partial \epsilon}{\partial \rho} - \epsilon$$

  - simplest $F = F(\rho) \Rightarrow$ **barotropic fluid** $\epsilon = \epsilon(p)$
  - general **non-barotropic**
    - interpretation: $g = s(\phi^a)$ entropy and $\mathcal{S}^\mu = s\mathcal{N}$ is entropy current

- **BPS Skyrme model**
  - $\mathcal{B}^\mu = \mathcal{N}^\mu$ i.e., the baryon current
  - fluid Lagrangian

$$F = -\lambda^2 \pi^4 \rho^2 - \nu^2 \mathcal{U}(\phi^a)$$

genuine non-barotropic fluid

$\rightarrow$ thermodynamical interpretation of $\mathcal{U}$?

Complete thermodynamics (at $T = 0$) in a solvable solitonic model

Neutron stars

- **the BPS Skyrme model with gravity** Adam, Naya, Sanchez, Vazquez, Wereszczynski (15)

$$S_{06} = \int d^4x |g|^{\frac{1}{2}} \left(-\lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathbb{B}^\rho \mathbb{B}^\sigma - \mu^2 \mathcal{U} \right)$$

  - energy-momentum tensor

$$\begin{aligned} T^{\rho\sigma} &= -2|g|^{-\frac{1}{2}} \frac{\delta}{\delta g_{\rho\sigma}} S_{06} \\ &= 2\lambda^2 \pi^4 |g|^{-1} \mathcal{B}^\rho \mathcal{B}^\sigma - \left(\lambda^2 \pi^4 |g|^{-1} g_{\pi\omega} \mathcal{B}^\pi \mathcal{B}^\omega - \mu^2 \mathcal{U} \right) g^{\rho\sigma} \end{aligned}$$

  - the energy-momentum tensor of **a perfect fluid**

$$T^{\rho\sigma} = (p+\rho) u^\rho u^\sigma - p g^{\rho\sigma}$$

    where the four-velocity $u^\rho = \mathcal{B}^\rho / \sqrt{g_{\sigma\pi} \mathcal{B}^\sigma \mathcal{B}^\pi}$ and

$$\rho = \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma + \mu^2 \mathcal{U}$$

$$p = \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma - \mu^2 \mathcal{U}$$

  - for a static case with diagonal metric $u^\rho = (\sqrt{g^{00}}, 0, 0, 0)$

$$T^{00} = \rho g^{00} \,, \quad T^{ij} = -p g^{ij}.$$

    flat space case $\Rightarrow$ pressure must be constant (zero for BPS solutions, nonzero for non-BPS static solutions

$$D_\rho T^{\rho\sigma} \to \partial_i T^{ij} = \delta^{ij} \partial_i p = 0$$

  - In general, ρ and p arbitrary functions of the space-time coordinates, $\Rightarrow$ **no universal equation of state** $p = p(\rho)$ **valid for all solutions**

- **Einstein equations**
  - static, spherically symmetric metric (Schwarzschild coordinates)

$$ds^2 = \mathbf{A}(r)dt^2 - \mathbf{B}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

  - axially symmetric ansatz for the Skyrme field with baryon number B

$$U = e^{i\xi\vec{n}\cdot\vec{\tau}}$$

$$\xi = \xi(r), \quad \vec{n} = (\sin\theta\cos B\phi, \sin\theta\sin B\phi, \cos\theta)$$

are compatible with the Einstein equations

$$G_{\rho\sigma} = \frac{\kappa^2}{2} T_{\rho\sigma}$$

FT + GR with *full backreaction* $\leftrightarrow$ TOV: fix EoS

Einstein equations

$$\begin{aligned}
\frac{1}{r}\frac{\mathbf{B}'}{\mathbf{B}} &= -\frac{1}{r^2}(\mathbf{B}-1)+\frac{\kappa^2}{2}\mathbf{B}\rho \\
r\,(\mathbf{B}p)' &= \frac{1}{2}(1-\mathbf{B})\mathbf{B}(\rho+3p)+\frac{\kappa^2}{2}\mu^2 r^2\mathbf{B}^2\mathcal{U}(h)p \\
\frac{\mathbf{A}'}{\mathbf{A}} &= \frac{1}{r}(\mathbf{B}-1)+\frac{\kappa^2}{2}r\mathbf{B}p
\end{aligned}$$

- **A**, **B** and ξ are functions of $r \Rightarrow p$ and ρ are functions of r

$$\rho = \frac{4B^2\lambda^2}{\mathbf{B}r^4}h(1-h)h_r^2+\mu^2\mathcal{U}(h), \quad p=\rho-2\mu^2\mathcal{U}(h)$$

  eliminate $r \Rightarrow$ **on-shell EoS** $p = p(\rho)$
  except the step-function potential $\Rightarrow$ **off-shell EoS** $\rho = p + 2\mu^2$

- axially symmetric ansatz **is the correct one** because gravity straightens out all deviations from spherical symmetry

- **parameters fitting**
  two parameters in the model λ and μ

  - chose a potential

$$\mathcal{U}_\pi^2, \quad \mathcal{U}_\pi = 1 - \cos\xi$$

$$\mathcal{U}_{step} = \begin{cases} \sin^4\xi & \xi \in \left[0, \frac{\pi}{2}\right] \\ 1 & \xi \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

  - $\mathbf{m} \equiv \lambda\mu$ has the dimensions of mass (energy)
    fit to the binding energy of nucleon of infinite nuclear matter

$$E_b = 16.3 \text{ MeV}$$

  - $\mathbf{l} \equiv (\lambda/\mu)^{1/3}$ has the dimensions of length
    fit to the nuclear saturation density

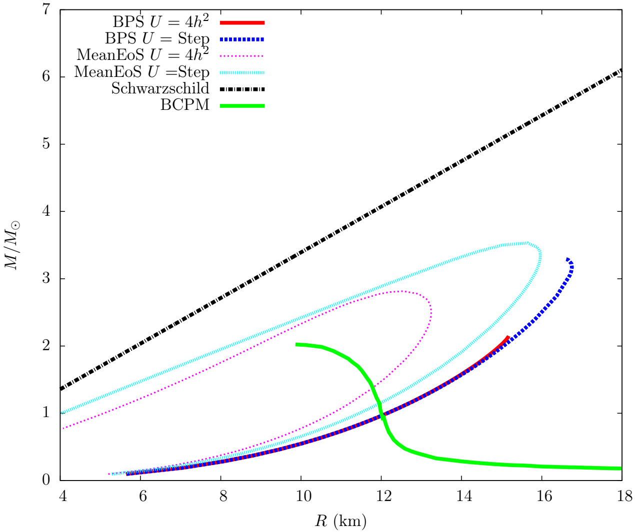
$$n_0 = 0.153 \text{fm}^{-3}$$

particular potentials

$$\mathcal{U}_\pi^2: \quad \lambda^2 = 15.49 \text{ MeVfm}^3, \quad \mu^2 = 141.22 \text{ MeVfm}^{-3}$$

$$\mathcal{U}_{step}: \quad \lambda^2 = 23.60 \text{ MeVfm}^3, \quad \mu^2 = 121.08 \text{ MeVfm}^{-3}$$

- **results: mass-radius relation**



- maximal mass

$$\begin{array}{lcl} \mathcal{U}_{step} & \rightarrow & M_{max} = 3.29 \\ \mathcal{U}_{\pi}^{2} & \rightarrow & M_{max} = 2.15 \end{array}$$

  $\rightarrow$ compatible with exp. data
  $\rightarrow$ compatible with $M = 2.5$ v.difficult for other EFT

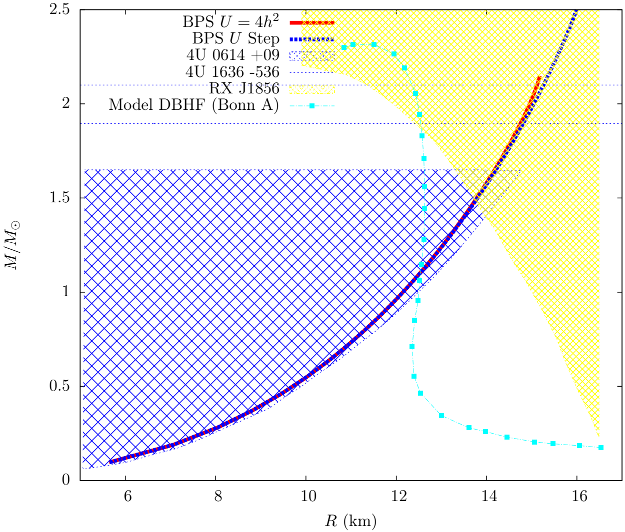
- $M - R$ curve qualitatively different

  $\rightarrow$ EoS approaches the max. stiff EoS
  $\rightarrow$ light NS: surface (low-density nuclear EoS) but academic

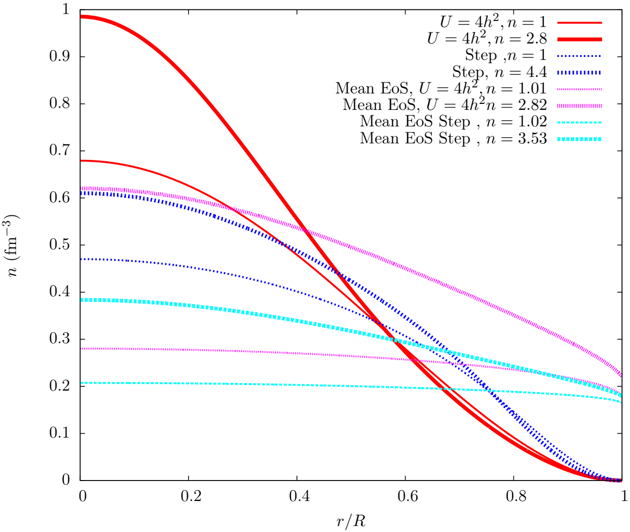
- full FT vs. mean-field

  $\rightarrow$ quantitatively differ
  $\rightarrow$ true M_{max} is lower than predicted by MF

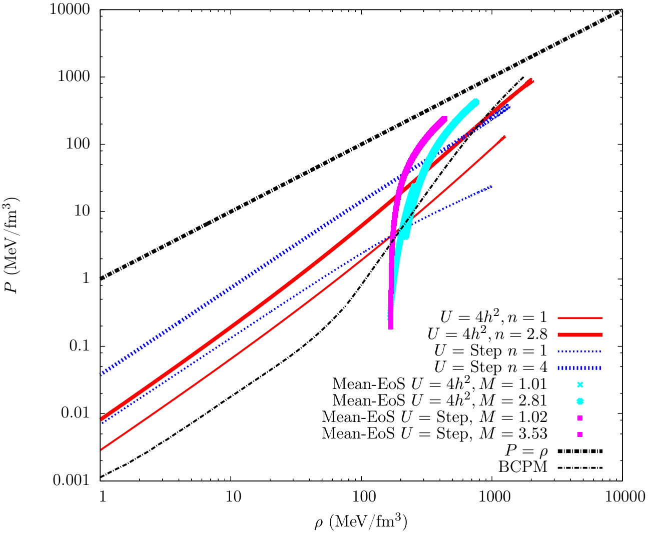
- **results: mass-radius relation**



- **results: local baryon density**



- **results: local Equation of State**



- no unique EoS

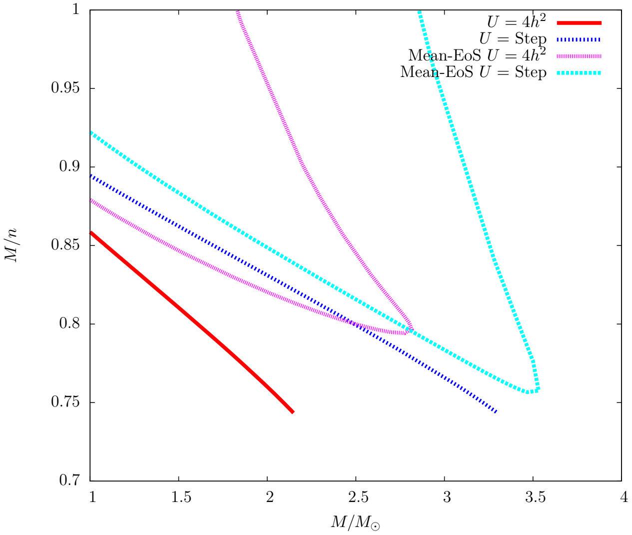
  $\rightarrow$ on-shell EoS: polytropic

$$p \sim a\epsilon^b$$

- inverse TOV questionable

  $\rightarrow$ inhomogeneities important

- **results: gravitational mass loss**



summary = fundamental result

there is a limit of Skyrme type actions in which the model has two properties

- **BPS**

  classically zero binding energies
  physical (small) binding energies: semiclassical quantization
  Coulomb interaction
  isospin breaking

- **perfect fluid field theory**

  $T_{\mu\nu}$ in a perfect fluid form
  SDiff symmetry
  Euler fluid formulation
  FT (micro) d.o.f. = thermodynamical (macro) functions

  Physically well motivated idealization of the nuclear matter
  weakly modified by the (small) non-BPS part

- **solvablity**

  exact solutions
  exact, analytical EoS and thermodynamics
  a way beyond MF limit in nuclear matter