1. Show that if X is a compact topological space and $\sim$ is an equivalence relation, then $X/\sim$ is also compact.

2. Show that if X is path-connected then it is connected.

3. Prove, that if A and B are topological spaces with A compact and B Hausdorff, and if $f : A \to B$ is continuous and bijective, then f is a homeomorphism.

4. Show that a homotopy equivalence $f : X \to Y$ induces a bijection between the set of path-components of X and the set of path-components of Y.

5. Show that a complex manifold is orientable.

6. Let $X_1, \ldots, X_n$ be the path-connected components of the space X. Prove that the k-th homology group
$$H_k(X) \cong \bigoplus_{i=1}^{n} H_k(X_i)$$
for all $k \in \mathbb{Z}$.

7. Prove that principal G-bundle is trivial if and only if it has a section.