1. Introduce parametrization and describe manifolds corresponding to
   - Möbius strip,
   - S^2,
   - $O(n) \subset GL(n, \mathbb{R}) : \{A \in Mat(n, n) : AA^T = id.\}$ for $n = 1, 2, 3$,
   - $O(1, 3) = \{M \in GL(4, \mathbb{R}) : M\eta M^T = \eta\}$, where $\eta = diag(1, -1, -1, -1)$.

   Which of them are connected? Which are orientable?

2. Given smooth $f : \mathbb{R}^n \to \mathbb{R}^m$, show that the graph of f is a regular submanifold of $\mathbb{R}^m \times \mathbb{R}^n$.

3. For real number a, define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = x^3 - 3ax - y^2$. Find all values of b so that $f^{-1}(b)$ is a manifold. Graph $f^{-1}(b)$ for a variety of a and b including the critical points of b.

4. Show that every injective immersion of a compact manifold is an embedding.

5. For given real field theory on a line $\phi : \mathbb{R} \to T \subset \mathbb{R}$ with energy defined as

$$E = \int_{\mathbb{R}} dx\ \frac{1}{2}\phi_x^2 + U(\phi),$$

   where $U(\phi) \geq 0$ and $V = U^{-1}(0)$ is a disjoint vacuum manifold, show that

$$E \geq \left| \int_{\phi_i}^{\phi_j} d\phi\ \sqrt{2U(\phi)} \right|,$$

   where $\phi_i, \phi_j \in V$. Consider two cases $U = \frac{1}{2}(\phi^2 - 1)^2$ and $U(\phi) = \sin^2(\phi/2)$.

6* Can we repeat the same for $(\phi, A_i) :^2 \mathbb{R} \to \mathbb{C} \times \mathbb{R}^2$ for energy density given by

$$E = \int_{\mathbb{R}^2} \frac{1}{2}F_{12}^2 + \frac{1}{2}(D_i\phi)^*(D_i\phi) + \frac{\kappa^2}{8}(\phi^*\phi - 1)^2$$

   where $D_\alpha = \partial_\mu - iA_\mu$.