1. Let $\sim$ be the equivalence relation on $\mathbb{R}$ defined as follows
$$x \sim y \iff x = 2^k y, k \in \mathbb{Z}. \tag{1}$$
Describe $Y = \mathbb{R}/\sim$. Is it Hausdorff?

2. Let $\sim$ be the equivalence relation on $\mathbb{R}^2$ defined as follows
$$(x, y) \sim (x', y') \iff (x - x', y - y') \in \mathbb{Z}^2. \tag{2}$$
Show that $\mathbb{R}^2/\sim$ is homeomorphic to $S^1 \times S^1$.

3. Describe $S^1 \times S^1/(x, y) \sim (y, x)$

4. Show that
   - if X is Hausdorff then any compact subset of X is closed,
   - if X is compact and A is a closed subset of X then A is compact,

5. Check if the following pairs of spaces are homeomorphic, if not name topological invariants which are different, if yes provide an example of homeomorphism:
   (a) $[0, 1]$ and $\mathbb{R}$,
   (b) $(0, 1)$ and $\mathbb{R}$,
   (c) S^1 and $\mathbb{R}$,
   (d) a parabola $y - x^2 = 0$ and hyperbola $y^2 - x^2 = 1$,
   (e) open disc $D^2 = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 < 1\}$ and $\mathbb{R}^2$,
   (f) a circle $S^1 = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = 1\}$ and a square $I^2 = \{(x, y) \in \mathbb{R}^2 | (|x| = 1 |y| \leq 1), (|y| = 1 |x| \leq 1)\}$

6. Introduce an atlas $\mathcal{A} = \{(U_1, \phi_1), (U_2, \phi_2)\}$ on a Möbius strip $[0, 1) \times (-1, 1)$.