1. Let us define (after the lecture) two topological spaces for $X = [0, 1)$:

$$\tau_1 : \{(a, b) \subset X\} \cup \{[0, a) : 0 < a < 1\}, \qquad X_1 = (X, \tau_1) \tag{1}$$

$$\tau_2 : \{(a, b) \subset X\} \cup \{[0, a) \cup (b, 1) : 0 < a < b < 1\}, \qquad X_2 = (X, \tau_2). \tag{2}$$

   Show that a limit of the sequence $x_n = 1 - \frac{1}{n}$ exits in X_2 but does not in X_1.

2. Check if the following spaces (X, d) are metric spaces and describe the open balls in those metrics:

   (a) $X = \mathbb{R}$, $d(x, y) = |x - y|$,

   (b) X - any set, $d(x, y) = 0$ if $x = y \wedge d(x, y) = 1$ if $x \neq y$ $(0-1$ metric),

   (c) $X = \mathbb{R}^2$, $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ (taxicab),

   (d) $X = \mathbb{R}^2$ (river metric)

$$d(x, y) = \begin{cases} |x_2 - y_2| & \text{when } x_1 = y_1 \\ |x_2| + |x_1 - y_1| + |y_2| & \text{otherwise} \end{cases}$$

   (e) $X = \mathbb{R}^2$, $d(x, y) = \max(|x_1 - y_1|, |x_2 - y_2|)$ (chess knight metric),

   (f) $X = \mathbb{R}^2$ (railway metric)

$$d(x, y) = \begin{cases} \|x - y\| & \text{when } x, y \text{ and } 0 \text{ are inline} \\ \|x\| + \|y\| & \text{otherwise} \end{cases}$$

3. Let $f : X \to Y$ be a continuous function and $x_n \to x$. Show that $f(x_n) \to f(x)$.

4. Write explicit maps from $f_1 : X_1 \to S^1$ and $f_2 : X_2 \to S^1$. Which of these maps and their inversions are continuous and why?

5. Find homeomorphisms:

   (a) $(0, 1) \to \mathbb{R}$,

   (b) $(0, 1) \to (0, \infty)$,

   (c) $\mathbb{R} \times S^1 \to \mathbb{R}^2 \setminus \{0\}$.

6. Find an explicit form of a stereographic projection $S^2 \setminus \{0\} \to \mathbb{C}$.

7. Provide a few examples of sets simultaneously closed and open.

8. Show that if X is arcwise connected it is connected.