Hydrodynamic Attractors and Quark-Gluon Plasma

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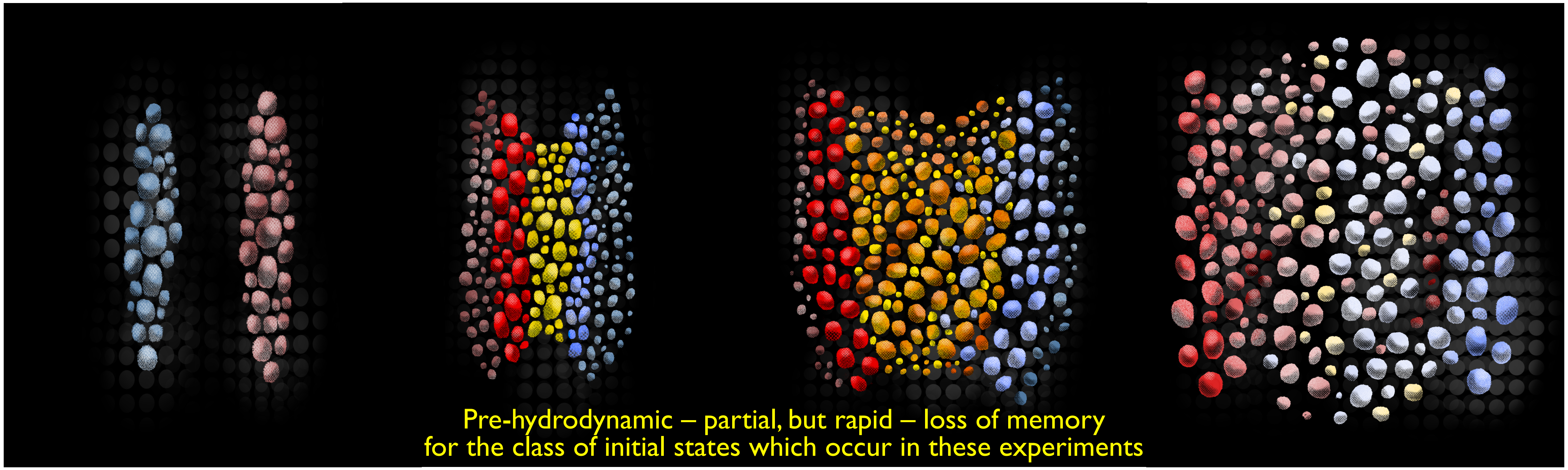
Abstract

The successful application of fluid-dynamical models to the physics of Quark-Gluon Plasma created in ultra-relativistic nuclear collisions has inspired a number of theoretical investigations aimed at understanding how this is possible, given the complexity of the post-collision state. I will describe a possible resolution of this "early-thermalisation puzzle" which is suggested by studies of various idealised models.

Ultra-relativistic heavy-ion collisions



Far from equilibrium | Hydrodynamics | Almost thermal

- Goal: understand the phase structure of QCD
- Novel phase of nuclear matter: Quark-Gluon Plasma
- Initial state: far from equilibrium
- Why does hydrodynamics appear to work at very early times?

Approaching equilibrium in heavy-ion collisions

The role of kinematics

- In toy models, the effectiveness of fluid dynamics at early times could be explained using the concept of hydrodynamic attractors.
- The origin of attractor behaviour at early times has been linked to the longitudinal expansion and as such may be of kinematic nature.
- This suggests that even though this effect was observed in toy models, it may also occur in QCD given the same kinematics.
- Crucial question: how robust are the features attractors in when symmetry assumptions are relaxed?

Relativistic hydrodynamics
for perfect fluids

Focus on the energy-momentum tensor for any ideal fluid:

$$\langle \hat{T}^{\mu\nu} \rangle \equiv T^{\mu\nu} = (\mathscr{E} + \mathscr{P}) u^\mu u^\nu + \mathscr{P} \eta^{\mu\nu}$$

for which we have the conservation law

$$\boxed{\partial_\alpha T^{\alpha\beta} = 0}$$

Four equations, four independent variables:

- local energy density $\mathscr{E}$
- fluid 4-velocity $(u^\mu) = (\gamma, \gamma \overrightarrow{v}), \quad \gamma \equiv 1/\sqrt{1 - v^2}$

Microscopic information enters through the equation of state

$$\mathscr{P} = \mathscr{P}(\mathscr{E}) \approx \mathscr{E}/3$$

For conformal systems

Relativistic hydrodynamics
incorporating dissipation

To account for entropy production, we include dissipative terms:

$$T^{\mu\nu} = \frac{1}{3}\mathcal{E}(\eta^{\mu\nu} + 4u^\mu u^\nu) \boxed{+\, \pi^{\mu\nu}}$$

The simplest possibility (relativistic Navier-Stokes theory)

$$\pi^{\mu\nu} = -\,2\eta\,\sigma^{\mu\nu} \equiv -\,2\eta\,(\partial^\mu u^\nu + \ldots)$$

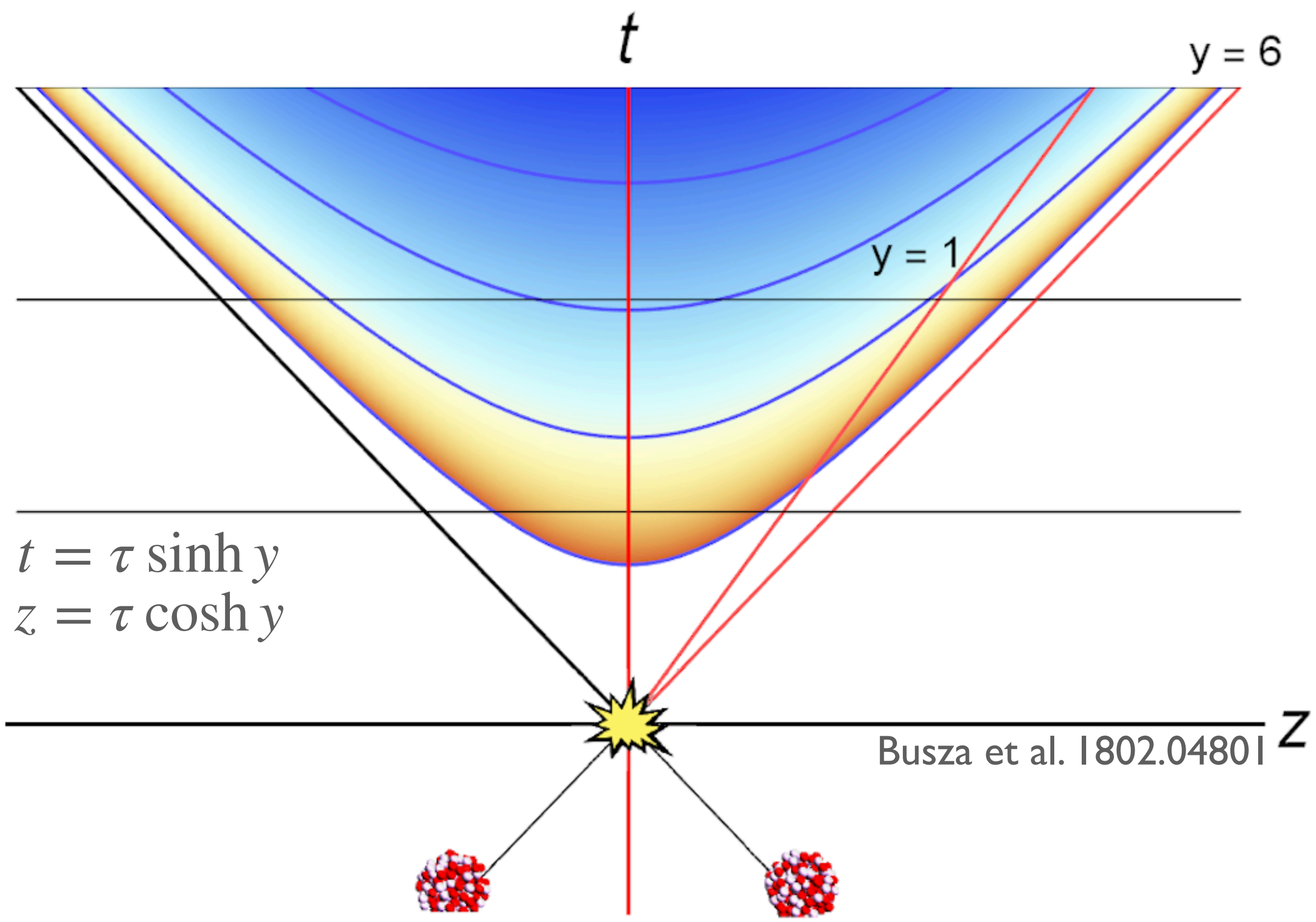
This theory is however not causal, and global equilibrium is unstable.

The simplest causal and stable theory: Mueller-Israel-Stewart (MIS)

$$\tau_\pi D\pi^{\mu\nu} + \pi^{\mu\nu} = -\,2\eta\sigma^{\mu\nu} + \ldots$$

This generates a gradient expansion (starting with the N-S term), which can be used to match the parameters to microscopic theories.

Bjorken flow
in conformal models



$$\boxed{(T^{\mu}_{\nu}) = \mathrm{diag}(-\mathscr{E}, \mathscr{P}_L, \mathscr{P}_T, \mathscr{P}_T)}$$

Expressed in terms of 2 functions of proper time:

$$\mathscr{P}_L \equiv \frac{\mathscr{E}}{3}\left(1 - \frac{2}{3}\mathscr{A}\right), \quad \mathscr{P}_T \equiv \frac{\mathscr{E}}{3}\left(1 + \frac{1}{3}\mathscr{A}\right)$$

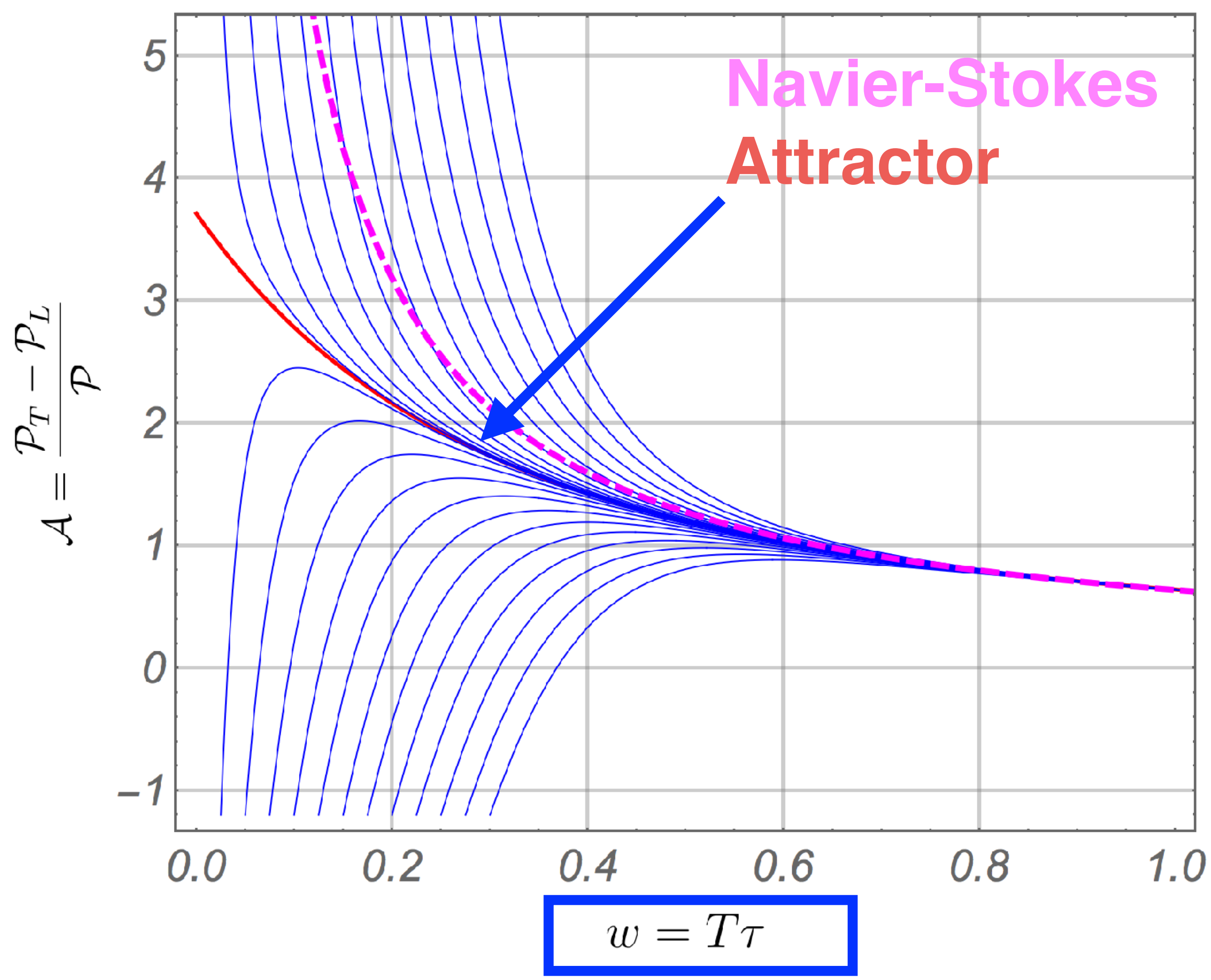
Conservation of the energy-momentum tensor:

$$\frac{d \log T}{d \log w} = \frac{\mathscr{A} - 6}{\mathscr{A} + 12}$$

$$\mathscr{E}(\tau) \sim T(\tau)^4, \quad w \equiv \tau T$$

- Solving the conservation equation introduces a single integration constant and the remaining initial data is contained in $\mathscr{A}(w)$.
- In a number of models there is a universal attractor form of the function $\mathscr{A}(w)$ which is rapidly approached by generic solutions.

Modelling the attractor
in conformal Mueller-Israel-Stewart theory



The Israel-Stewart relaxation equation

$$C_\tau \left(1 + \frac{\mathscr{A}}{12}\right) \mathscr{A}' + \frac{C_\tau}{3w}\mathscr{A}^2 = \frac{3}{2}\left(\frac{8C_\eta}{w} - \mathscr{A}\right)$$

Early time asymptotics

$$\mathscr{A} = 6\sqrt{\frac{C_\eta}{C_{\tau\Pi}}} + O(w) \quad \text{or} \quad \mathscr{A} \sim 1/w^4$$

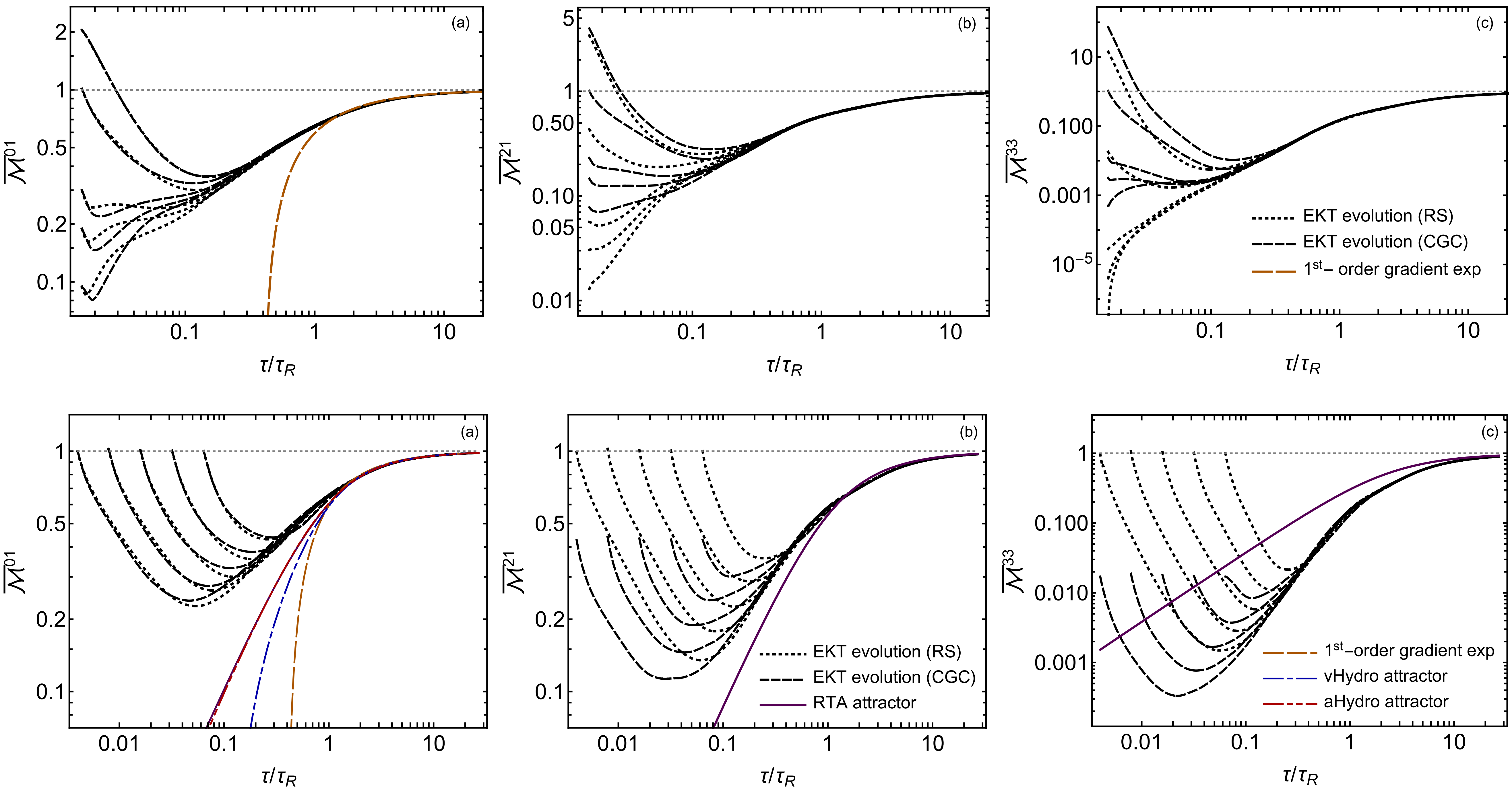
Information about initial conditions is exponentially "dissipated"

Late time asymptotics

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}} + \underbrace{\left(\sigma\, w^{\frac{C_\eta}{C_\tau}} e^{-\frac{3}{2C_\tau}w}\right) \sum_{n\geq 0} \frac{a_n^{(1)}}{w^n}}_{\text{transseries sectors}} + \ldots$$

Modelling attractor behaviour

in kinetic theory



Romatchke 1704.08699; Blaizot, Yan 1712.03856, …, 2106.10508; Strickland 1809.01200; Almaalol et al 2004.05195
Kamata et al. 2004.06751; Du, Schlichting 2012.09068, 2012.09079

Origins of attractor behaviour

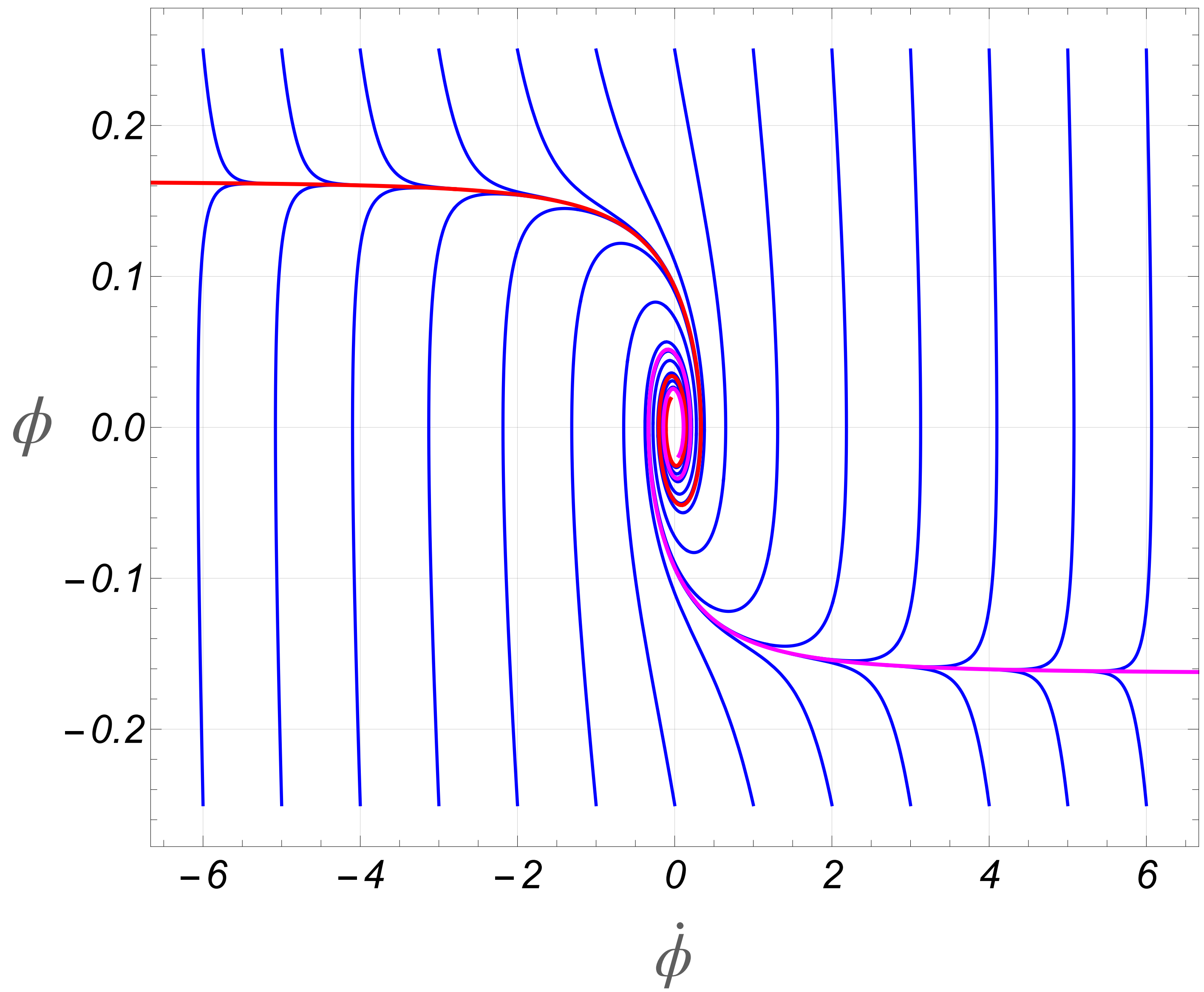
from the kinetic theory perspective

> “The main features of the dynamics of expanding plasmas are determined by the competition between the expansion itself, which is dictated by the external conditions of the collisions, and the collisions among the plasma constituents which generically tend to isotropize the particle momentum distribution functions”.
>
> Blaizot, Yan 1712.03856

- In kinetic theory the expansion always wins at early times and leads to free streaming.
- In hydrodynamic models there is fair competition: free streaming is not automatic, details of the early time attractor depend on the dynamics.
- The general answer is not known.

Blaizot, Yan 1712.03856; Kurkela et al. 1907.08101; Kurkela, Moore 1108.4684

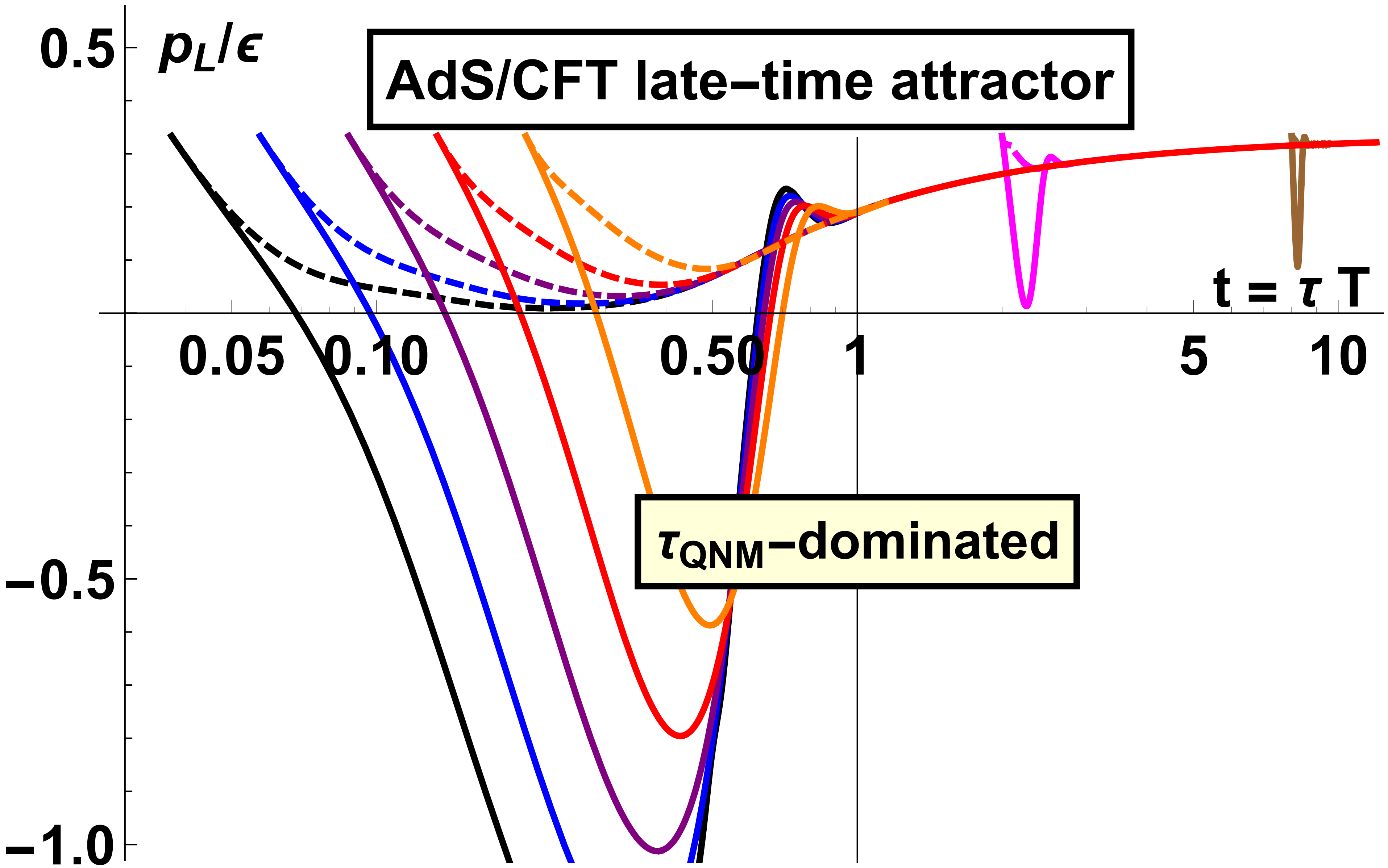
Origins of attractor behaviour
In inflationary cosmology



A wide class of initial conditions quickly end up on the inflationary attractor, where the potential gradient competes with the Hubble expansion of the gravitational background.

See e.g. Mukhanov's "Physical Cosmology", Cambridge University Press 2005

Origins of attractor behaviour
from an AdS/CFT perspective



There seems to be no unique early-time behaviour, only a late-time an attractor attained at anisotropy $\mathscr{A} \approx 0.6$.

Kurkela et al. 1907.08101

An example application

If we approximate the entire history of a heavy-ion collision with a conformal, boost-invariant attractor, one consequence is the formula

$$\frac{< \frac{dN}{dy} >_c}{< \frac{dN}{dy} >_{c'}} = \frac{< \int d^2\mathbf{x}_\perp \mathcal{E}(\tau_0, \mathbf{x}_\perp)^{\frac{2}{4-\beta}} >_c}{< \int d^2\mathbf{x}_\perp \mathcal{E}(\tau_0, \mathbf{x}_\perp)^{\frac{2}{4-\beta}} >_{c'}}$$

Here $\beta = 1$ corresponds to a free-streaming attractor.

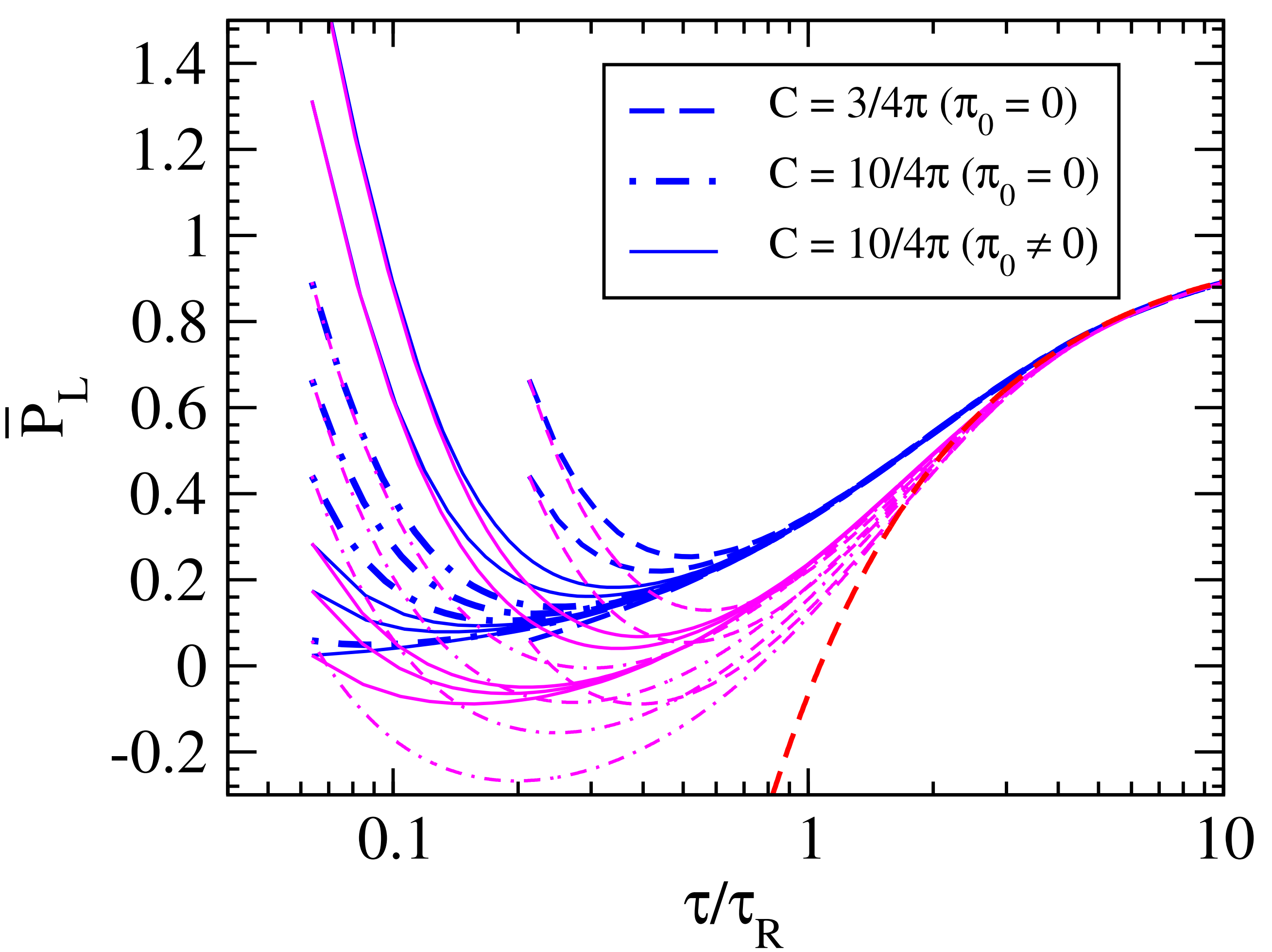
Consistency with experiment suggests that the attractor at early times must match the model of the initial energy deposition.

Could be relevant for current Bayesian studies which vary the initial state models using free-streaming pre-hydro evolution.

Giacalone et al. 1908.02866; Jankowski et al. 2012.02184
Moreland et al. 1808.02106; Nijs et al. 2010.15130, 2010.15134
Du, Schlichting 2012.09068, 2012.09079; Coquet et al. 2112.13876

Beyond Conformal Bjorken Flow

- With transverse flow
- Without conformal symmetry
  - More degrees of freedom
  - In kinetic theory:
    an early-time attractor
    identified in $\mathscr{P}_L$
  - Standard 2nd order hydro does
    not capture this, but a version of aHydro does
  - A challenge for hydro-modellers



Kurkela et al. 1907.08101, 2007.06851; Ambrus et al. 2109.03290
Chattopadhyay et al. 2107.05500; Jaiswal et al. 2107.10248; Alqahtani et al. 2203.14968

Modelling attractors
with hydrodynamic models

Hydro models (MIS, aHydro, general frame, hydro+, …)

- are engineered to mimic near-equilibrium asymptotics
- propagate initial data in a causal and stable way
- capture (more) information about the initial state
- model nontrivial nonhydrodynamic (transient) sectors

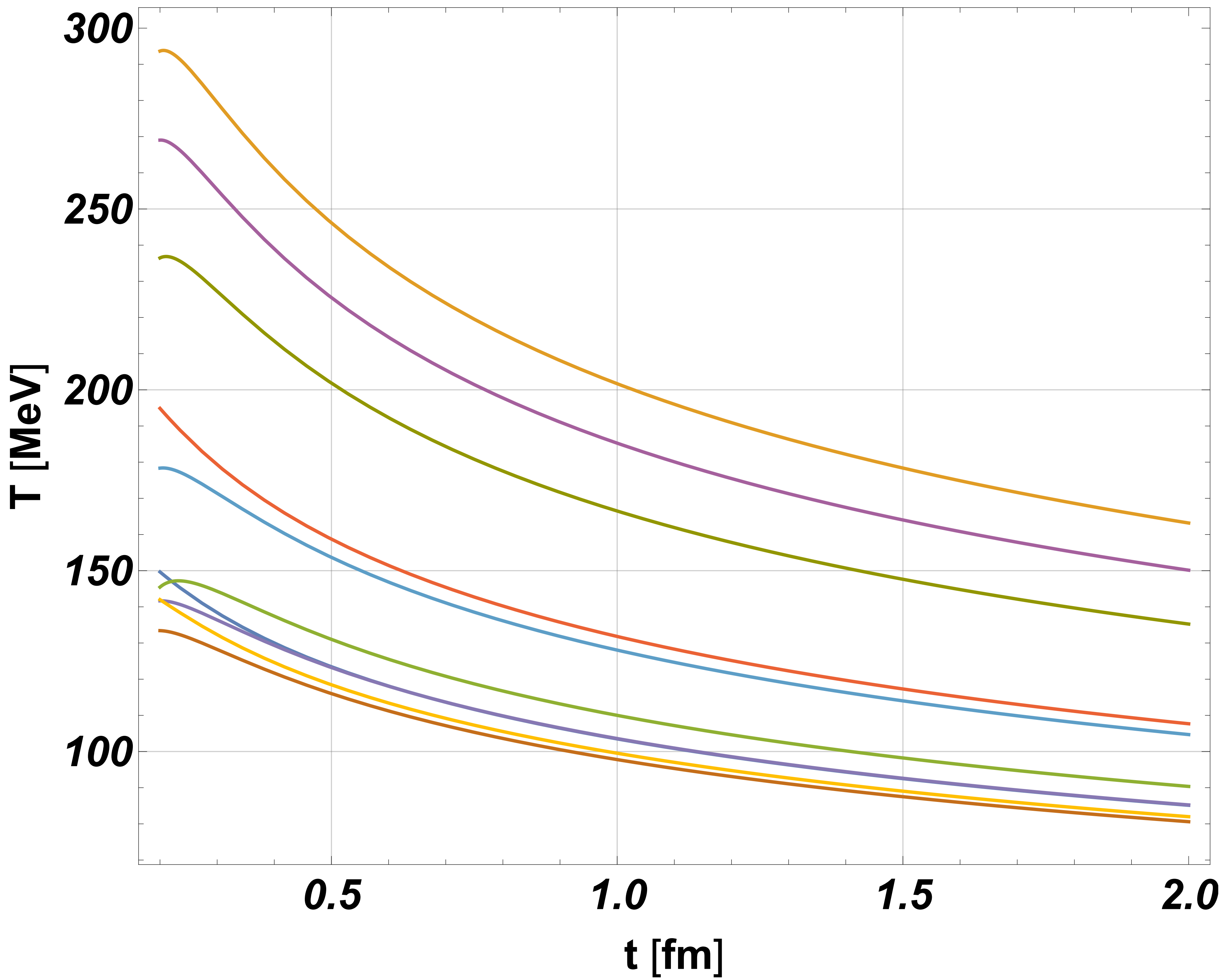
Results coincide only in the asymptotic region, see e.g. the recent comparison of MIS and BDNK.

It looks like matching hydro models to nonconformal kinetic theory presents interesting challenges.

Bantilan et al. 2201.13359; Speranza et al. 2105.01034;

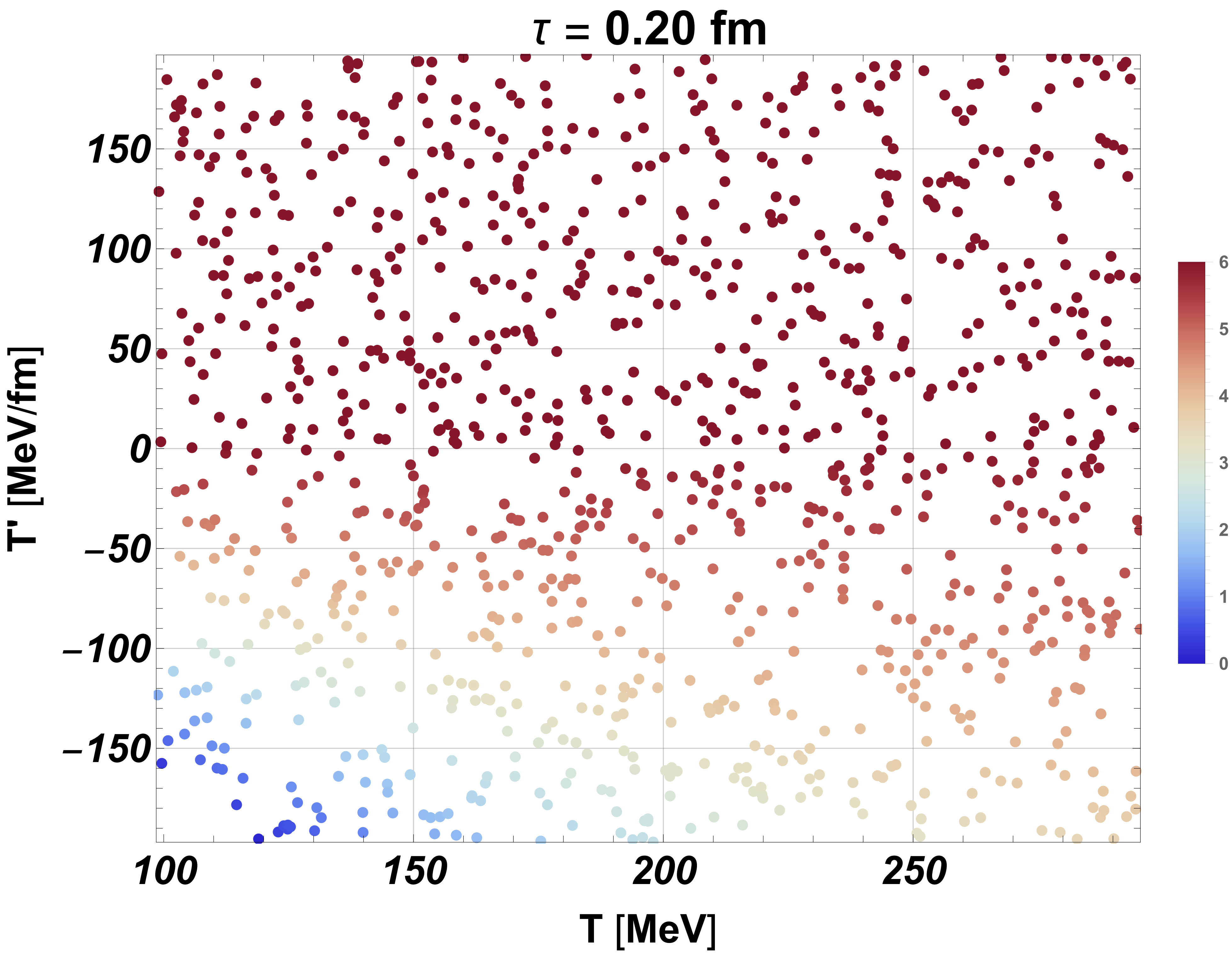
The Phase Space Approach

Can you see the MIS attractor in this picture?



The hydrodynamic attractor in MIS

A view from phase space



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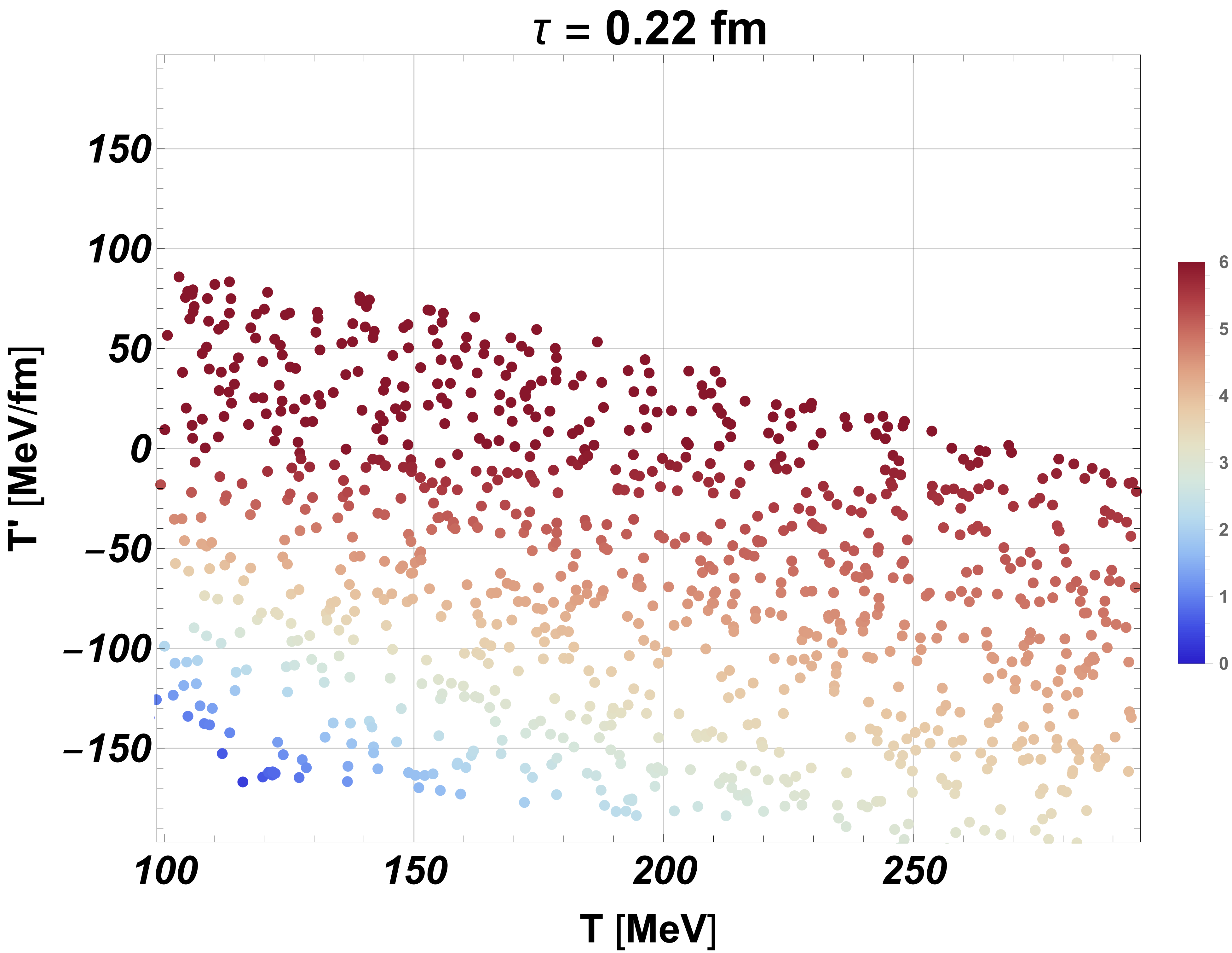

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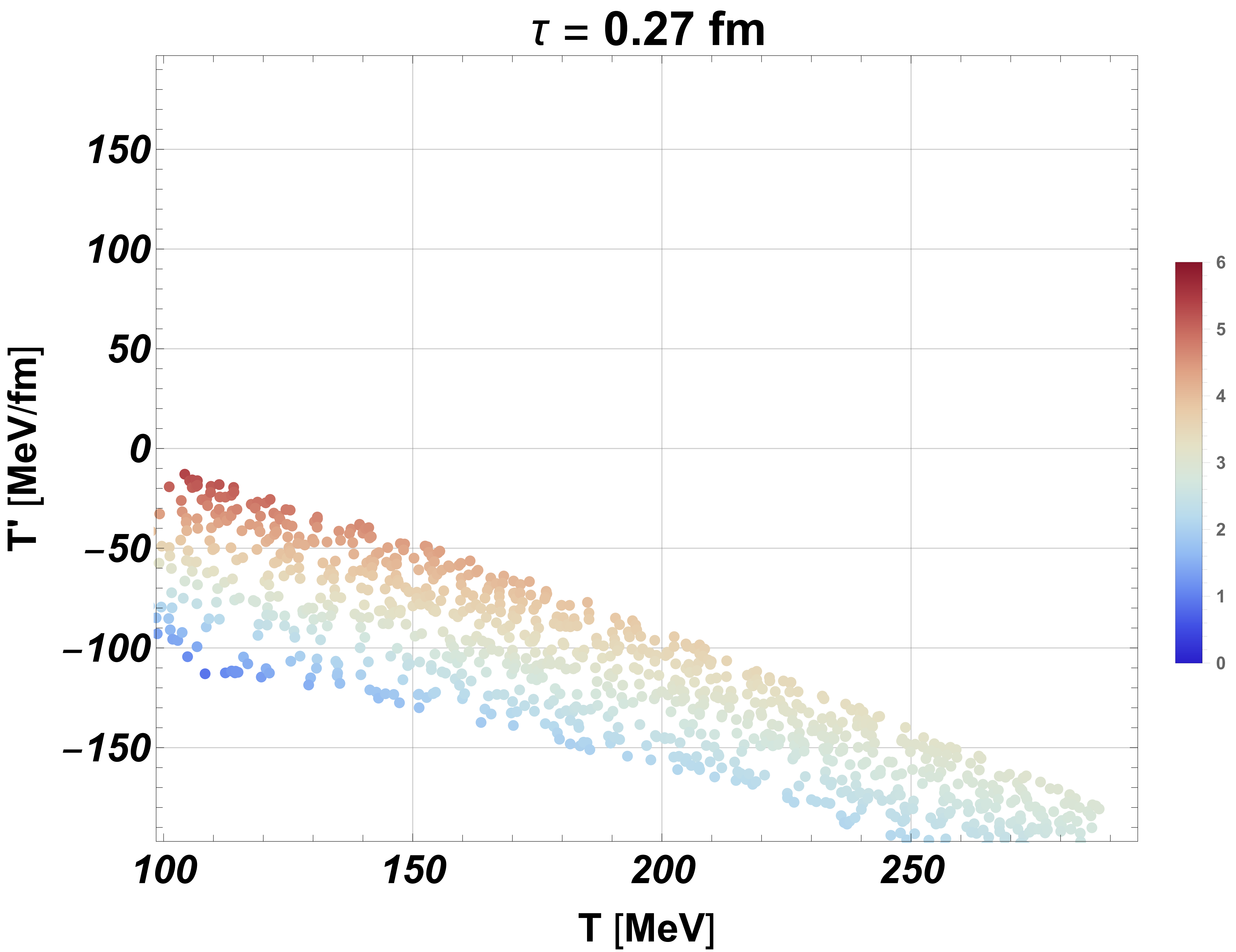

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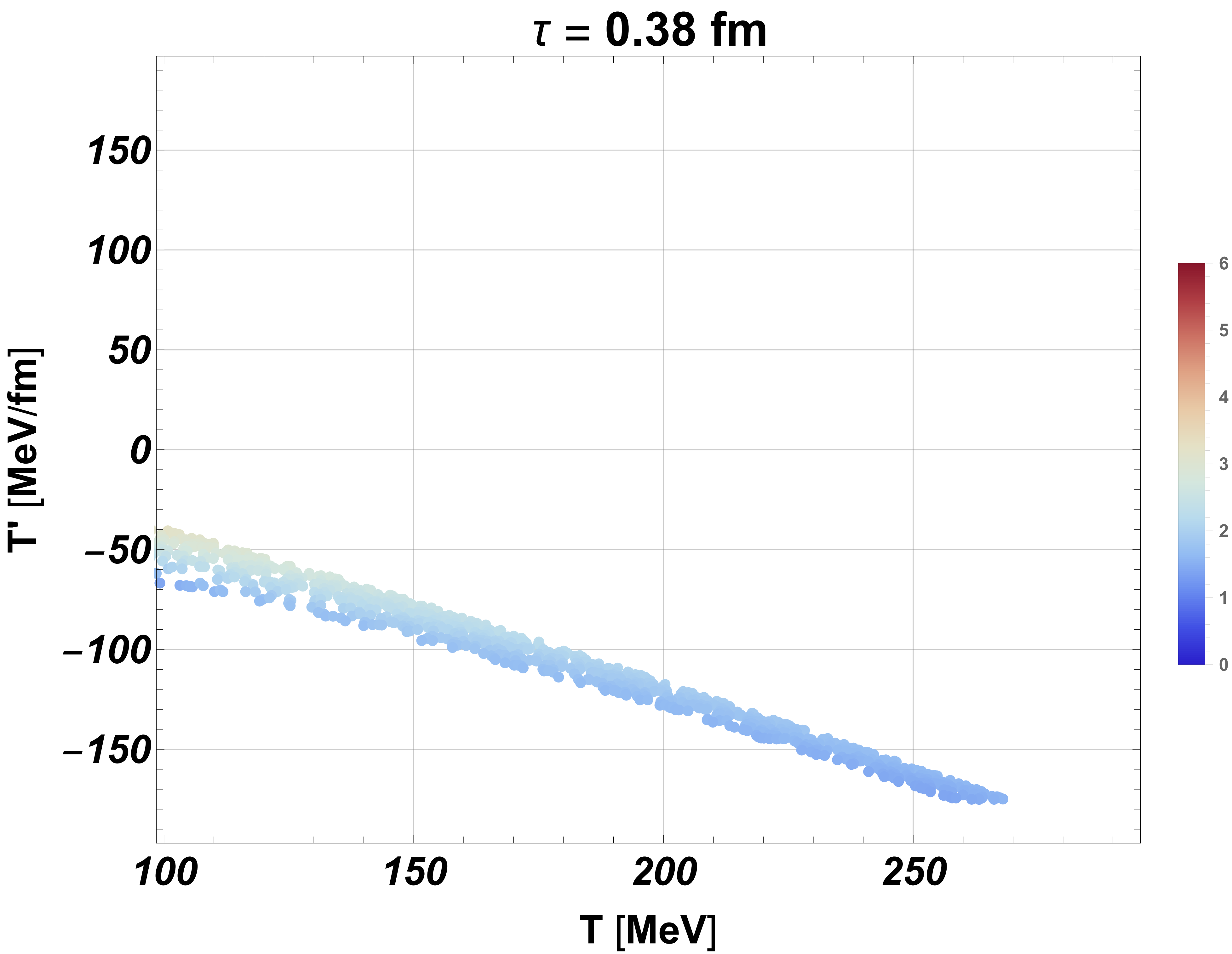

The hydrodynamic attractor in MIS

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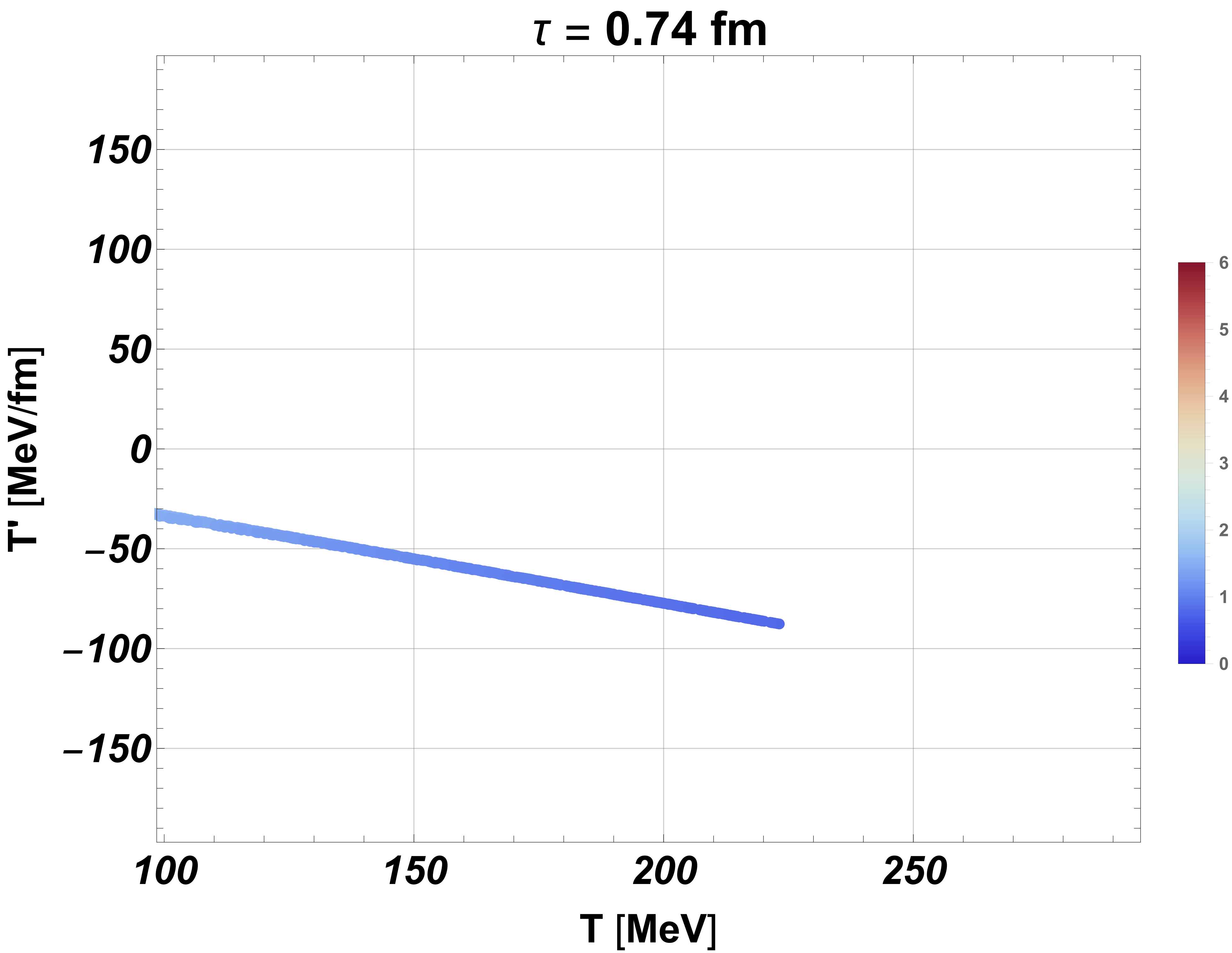


The Phase Space Approach

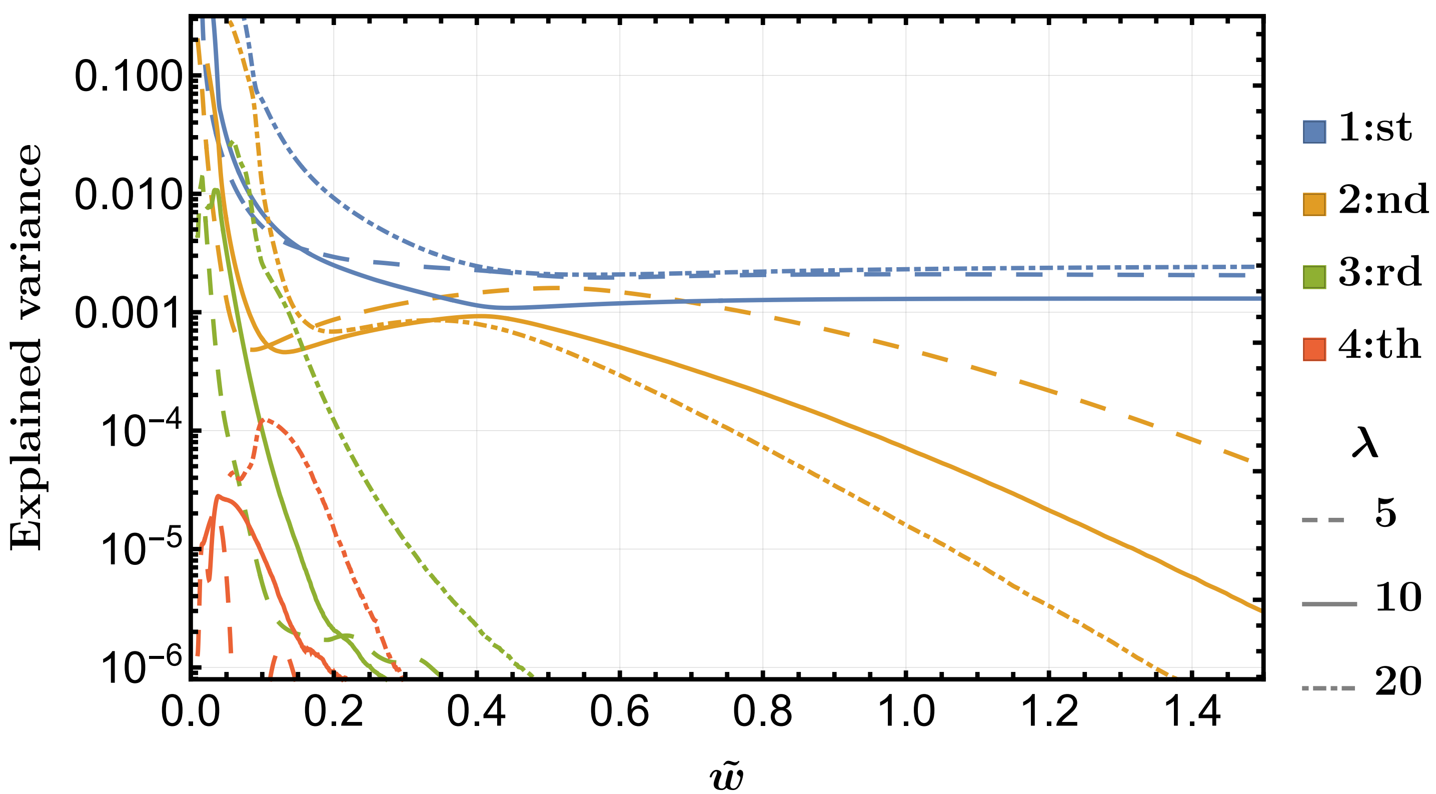
- A set of solutions spanning a D-dimensional region on the initial time slice ends up in a region of lower dimensionality $d<D$ on a subsequent time slice.
- The attractor phenomenon is identified with this reduction of dimensionality of sets of solutions on slices of phase space.
- No special variables are necessary
- No limitations such as boost-invariance or conformal symmetry
- Natural area for techniques of data science/ML.

Heller et al. 2003.07368

The Phase Space Approach

Examples (still boost-invariant), using PCA for the data analysis:

- MIS-type models with 2d or 3d phase space
- BE-RTA: A 16d coarse-grained representation of phase space
- BE-EKT: A 4d coarse-grained representation of phase space



Heller et al. 2003.07368, 2203.16549

Outlook

- Approximate boost-invariance at early times may be a key element of why early-time attractors appear beyond toy models
- Early results suggest that some features of toy models persist when idealisations are relaxed
- New approaches may be useful in identifying and making use of attractors in situations with more degrees of freedom
- More elaborate hydrodynamic modelling may be needed
- Recent progress concerning the asymptotics of more general flows may signal progress on attractors in the near future

Heller et al. 2110.07621, 2112.12794