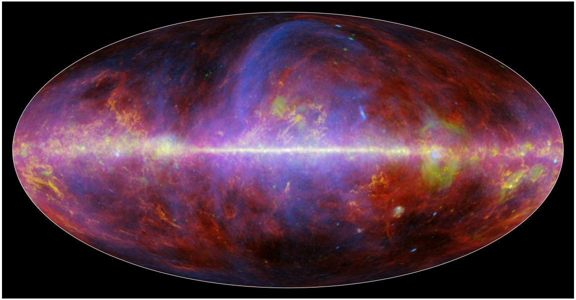
QCD meets gravity — on quantum effects that attract us all

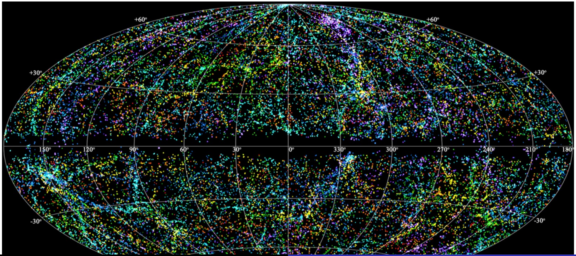
Leszek Motyka

May 26th, 2021

Outline

Mass composition of the Universe





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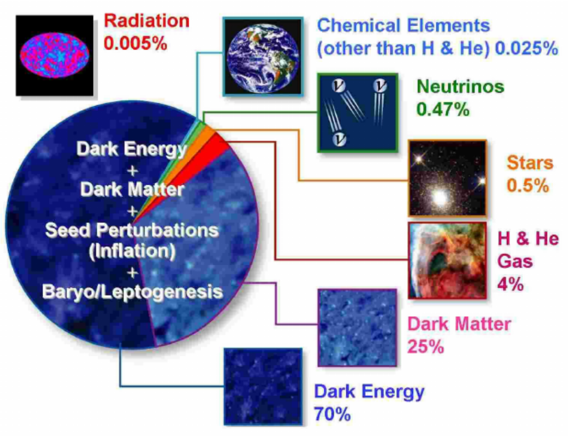
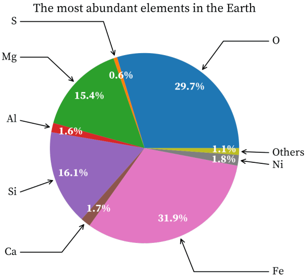
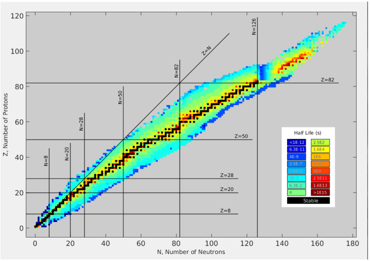


Mass composition of the Earth

Mass composition of the Earth

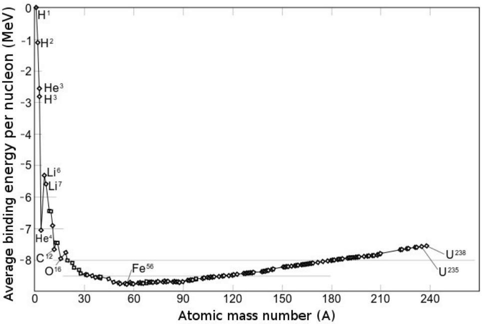




$$A \simeq 2Z, \qquad m_e/2M_p \sim \frac{1}{4000}$$

Mass composition of the Earth

For the mass, the nucleons binding energies in nuclei are more relevant than the electrons:



The gravity of the Earth comes mostly from the baryons, with sub-% corrections due to nuclear interactions — the baryons matter!

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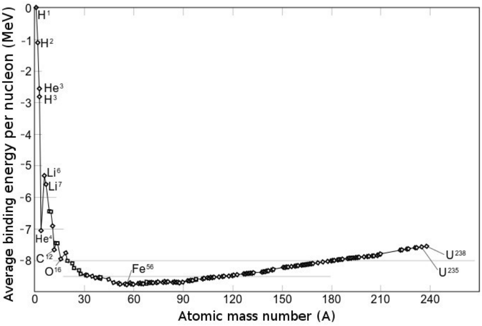

The gravity of the Earth comes mostly from the baryons, with sub-% corrections due to nuclear interactions — the baryons matter! (in total about $3.6 \cdot 10^{51}$ of them...)

Nucleons, electrons, nuclei masses — what do we know?

- The most accurate determinations of the ordinary matter particles: the proton, electron, nuclei are obtained using the Penning trap
- The mass determined in this measurements is by analysis of the motion of a charged particle in the electromagnetic fields — it is the inertial mass
- The mass of the neutron — from the deuterium and radiative capture of the neutron
- fantastic experimental accuracy: mass of the proton: 1.007276466583(15)(29) atomic units (30ppt accuracy)

The three masses

The mass may be defined in one of three ways:

- Inertial
- Gravitational Passive
- Gravitational Active

Inertial equals gravitational passive: tests of the equivalence principle — tested to 10^{-15} accuracy

Gravitational passive equals to gravitional active: symmetry, the IIIrd Newton's law, also the momentum conservation

The origin of particle masses at the electroweak scale

- In the Standard Model all particles, except of the Higgs boson, are primarily massless
- Masses are given by the Higgs mechanism — massless particles in a classical vacuum with a sponteneously broken $SU_L(2) \times U_Y(1)$ symmetry: fermion masses are generated by Yukawa couplings, for the heavy gauge bosons (unstable) — by gauge couplings
- A really BIG puzzle — the mass span problem: $m_\nu \sim 0.1$ eV, $m_e \sim 5 \cdot 10^5$ eV, $m_t \sim 1.75 \cdot 10^{11}$ eV: 12 orders of magnitude span for fundamental parameters?

On the origin of the baryon masses

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... and gluons

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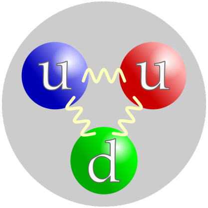
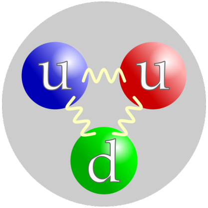


On the origin of the baryon masses

Baryons are made of quarks...



... and gluons

Particle Data Group say:

$2m_u + m_d \simeq 10$ MeV vs. the mass of the proton $M_p \simeq 938$ MeV

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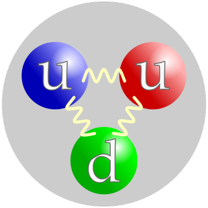

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Completely different from the classical and atomic scale intuitive pictures of almost additive mass

Scale symmetry of massless gauge theories

QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi} i \hat{D}\psi, \qquad \hat{D} = \gamma^{\mu}(\partial_{\mu} - ieA_{\mu})$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

QCD:

$$e \to g,\ \psi \to \psi^{i} \in \mathbb{C}^{3},\ A_{\mu} \to A^{a}_{\mu}T^{a} \in \mathrm{s}u(3),$$

$$F_{\mu\nu} \to F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$$

Rescaling:

$$x \to x' = \lambda x,\ \psi(x) \to \lambda^{3/2}\psi(\lambda x),\ A(x) \to \lambda A(\lambda x)$$

leaves the Lagrangean density unchanged.

The traceless energy momentum tensor

Emmy Noether's theorem:

$$x^\mu \to x'^\mu = \lambda x^\mu = x^\mu + \varepsilon x^\mu,$$

hence the coordinate shift $\delta^\mu = \varepsilon x^\mu$

For a general δ^μ the translational invariance leads to the conserved (canonical) stress–energy–momentum tensor $\theta^{\mu\nu}$. The scale symmetry involves all coordinates, and leads to an additional conserved current:

$$J^\mu_{\mathrm{s}cale} = x_\nu \theta^{\mu\nu}, \qquad \partial_\mu J^\mu_{\mathrm{s}cale} = 0$$

$$0 = \partial_\mu \left[x_\nu \theta^{\mu\nu}\right] = \theta^{\mu\,\cdot}_{\cdot\,\mu} + x_\nu \partial_\mu \theta^{\mu\nu} = 0 \Rightarrow \theta^\mu_\mu = 0.$$

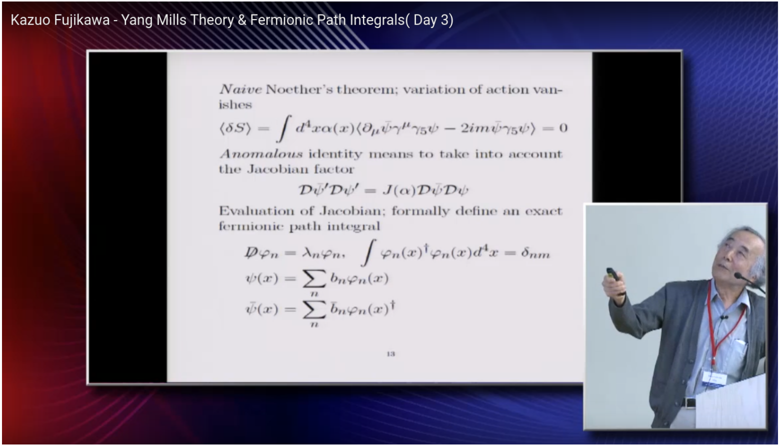
Anomalous symmetry breaking

- The classical symmetries not always survive in Quantum Field Theory
- If they do not, the symmetry is anomalously broken
- In Yang–Mills theories the anomalies were discovered in a pertrurbative analysis of triangle diagrams. The well known example is the Adler–Bell–Jackiw (axial) anomaly, in the massless gauge theory the classically conserved axial current is anomalously broken
- At a more general level, this phenomenon occurs because of the properties of the functional measure for infinite degrees of freedom, related to the ultra–violet divergencies in quantum loops

Digression: the trace / conformal / Weyl anomalies and the gravity

- The trace anomaly was lively discussed in the 1970s. At that time there were serious doubts whether the trace anomaly as found by D. M. Capper, M. J. Duff and L. Halpern (1974) for CFTs on non-trivial geometry was not an artifact of regularization
- The discussion involved great physicists, including B. DeWitt, S. Adler, F. Englert, S. Hawking, R. Kallosh, E. Fradkin and others. Most of them were surprised by the emergence of trace anomaly. [M. J. Duff, hep-th/9308075v1]
- The path integral formulation: K. Fujikawa, *Path Integral Measure for Gauge Invariant Fermion Theories*, Phys.Rev.Lett. 42 (1979) 1195, *Comment on Chiral and Conformal Anomalies*, Phys.Rev.Lett. 44 (1980) 1733, also *Energy Momentum Tensor in Quantum Field Theory*, Phys.Rev.D 23 (1981) 226. Powerful formalism, in particular a beautiful connection of the axial anomaly to Atiyah–Singer index theorem

Anomalies from the functional measure: Kazuo Fujikawa



Trace anomaly — Fujikawa method

The fermion part of functional integration (fermion mass m included)

$$Z[j,A] = \int d\psi d\bar{\psi} \exp\left[i \int d^4x \left(\mathcal{L}(\psi,\bar{\psi},A) + j\tilde{\theta}^{\mu}_{\mu} \right) \right], \quad \tilde{\theta}^{\mu}_{\mu} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{D}_{\mu}\psi$$

$$\psi(x) = e^{-\alpha(x)/2}\psi'(x), \quad \bar{\psi}(x) = e^{-\alpha(x)/2}\bar{\psi}'(x), \qquad \alpha(x) \ll 1$$

$$\int d^4x \left(\mathcal{L}(\psi,A) + \alpha(x)\tilde{\theta}^{\mu}_{\mu} \right) = \int d^4x \, \left(\mathcal{L}(\psi',A) + \alpha(x) m \bar{\psi}'\psi' + i\bar{\psi}'\gamma^{\mu}\psi'\partial_{\mu}\alpha \right)$$

Trace anomaly in QCD using Fujikawa method — path integration over fermions

$$Z[j+\alpha, A] = \int d\psi d\bar{\psi} \exp\left[i \int d^4x \mathcal{L}(\psi, \bar{\psi}, A) + (j+\alpha)\tilde{\theta}^{\mu}_{\mu} \right]$$

$$= \int d\psi' d\bar{\psi}' \, \mathcal{J} \, \exp\left[i \int d^4x \mathcal{L}(\psi', \bar{\psi}', A) + j\tilde{\theta}^{\mu}_{\mu} + \alpha m \bar{\psi}' \psi' \right]$$

$$i \int d^4x \tilde{\theta}^{\mu}_{\mu} \alpha(x) = \log \mathcal{J} + i \int d^4x m \bar{\psi} \psi \alpha(x)$$

$$\mathcal{J} = \left[\det(e^{-\alpha/2}) \right]^{-2} = \lim_{M\to\infty} \exp \mathrm{Tr} \int d^4x \, \langle x | \alpha \exp(-\hat{D}/M)^2 | x \rangle$$

$$\mathrm{Tr} \int d^4x \, \langle x | \alpha \exp(-\hat{D}/M)^2 | x \rangle = \frac{3iM^4}{4\pi^2} + \frac{ig^2}{48\pi^2} F^a_{\mu\nu} F^{a\,\mu\nu} + \dots$$

Trace anomaly — heuristic approach

The full calculation includes the path integral over A^a_μ yields

$$\theta^\mu_\mu = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_q m_q \bar{\psi}_q \psi_q$$

Less formal method: rescale

$$\bar{A}^a_\mu = g A^a_\mu \longrightarrow \hat{\bar{F}}_{\mu\nu} = \partial_\mu \hat{\bar{A}}_\nu - \partial_\nu \hat{\bar{A}}_\mu + i \left[\hat{\bar{A}}_\mu, \hat{\bar{A}}_\nu\right]$$

$$\mathcal{L}_{\mathrm{g}lue} = -\frac{1}{4g^2} \mathrm{T}r[\hat{\bar{F}}_{\mu\nu} \hat{\bar{F}}^{\mu\nu}]$$

$$\lambda = 1 + \delta\lambda, \quad \frac{\delta S}{\delta\lambda} = \int d^4x \frac{\partial}{\partial\lambda} \left(\frac{-1}{4g^2(\lambda)} \right) \mathrm{T}r[\hat{\bar{F}}_{\mu\nu} \hat{\bar{F}}^{\mu\nu}]$$

$$= \int d^4x \frac{\beta(g(\lambda))}{2g(\lambda)} \mathrm{T}r[\hat{F}_{\mu\nu} \hat{F}^{\mu\nu}]$$

Trace anomaly — dimensional regularization

The symmetric stress-energy tensor of the gluonic fields (the trace over color indices neglected):

$$T^{\mu\nu} = -g_{\alpha\beta} F^{\mu\alpha} F^{\nu\beta} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

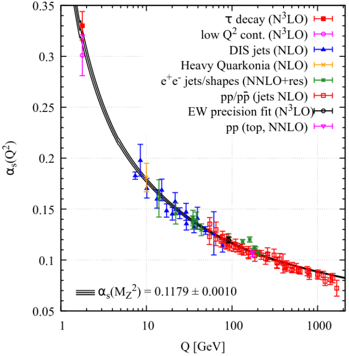
In d dimensions:

$$T^{\mu}_{\mu} = g_{\mu\nu} T^{\mu\nu} = \frac{d-4}{4} F_{\mu\nu} F^{\mu\nu}$$

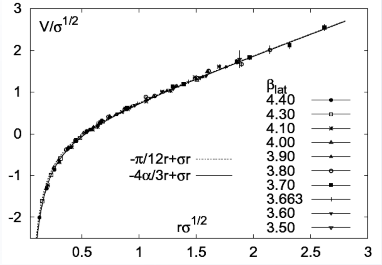
When the operator $F_{\mu\nu} F^{\mu\nu}$ is evaluated at quantum level the ultraviolet divergencies appear. A gauge invariant regularization is necessary e.g. the dimensional regularisation. In this regularization the divergencies appear as poles in $\epsilon = d - 4$. At one loop $\frac{d-4}{4} F_{\mu\nu} F^{\mu\nu}$ receives a prefactor $\sim \beta_0/\epsilon$, and the renormalized T^{μ}_{μ} does not vanish.

Scale generation in QCD

The QCD running coupling



Confinement scale:
QCD string tension:
$\sigma \simeq 1$ GeV / fm



Running quark masses

In perturbative approach the quark masses are subject to quantum corrections

After renormalization they become dependent on the renormalization scale μ, and the scale dependence is goverened by the renormalization group equation:

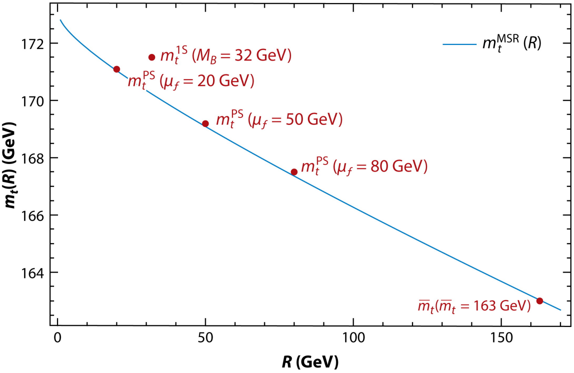
$$\mu^2 \frac{d m_q(\mu)}{d\mu^2} = -\gamma(\alpha_s(\mu)) m_q(\mu)$$

The anomalous dimension γ is known to four loops:

$$\gamma = \sum_{n=1} \gamma_n \left(\frac{\alpha_s(\mu^2)}{4\pi} \right)^n, \qquad \gamma_1 = 4, \quad \gamma_2 = \frac{202}{3} - \frac{20 n_f}{9}, \ldots$$

The masses decrease with the energy scale μ.

The top quark mass



Hoang AH. 2020.
Annu. Rev. Nucl. Part. Sci. 70:225–55

Estimates of quark masses

The most convenient and safe theoretically are the running quark masses (in a given renormalization scheme).
For light quarks u, d, s, the current estimates from the QCD lattice in the $\overline{MS}$ scheme at $\mu = 2$ GeV ($n_f = 2+1+1$ simulations):

$$m_u = 2.32(10)4\,\mathrm{MeV}, \quad m_d = 4.71(9)\,\mathrm{MeV}, \quad m_s = 93.44(0.68)\,\mathrm{MeV}.$$

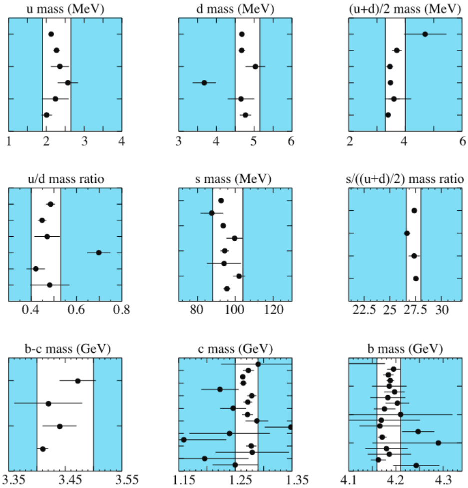
Heavy quarks:

$$m_c(m_c) = 1.280(13)\ \mathrm{GeV}, \qquad m_b(m_b) = 4.198(12)\ \mathrm{GeV}$$

The top quark mass determined directly from the decay products (close to the "pole mass")

$$m_t = 172.9(0.4)\ \mathrm{GeV} \quad \longrightarrow \quad m_t(m_t) \simeq 163\ \mathrm{GeV}$$

Determination of quark massess



Decomposition of QCD trace anomaly

$$\langle P(k)|T^{\mu\nu}|P(k)\rangle = 2k^\mu k^\nu, \qquad g_{\mu\nu}\langle P(k)|T^{\mu\nu}|P(k)\rangle = 2M_P^2$$

$$T^{\mu\nu} = \underbrace{-g_{\alpha\beta}F^{\mu\alpha}F^{\nu\beta} + \frac{1}{4}g^{\mu\nu}\,F_{\alpha\beta}F^{\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum_q \bar{\psi}_q\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}\psi_q}_{T_q^{\mu\nu}}$$

After renormalization

$$T_{g\ \mu}^{\mu} = \frac{\beta(g_R)}{2g_R}(F^2)_R + \gamma_m^R(m\bar{\psi}\psi), \qquad T_{q\ \mu}^{\ \mu} = m(\bar{\psi}\psi)_R$$

Decomposition of QCD trace anomaly

Pionieering works by X. D. Ji using virial theorems, [X. D. Ji, Phys. Rev. Lett.74, 1071 (1995).32. X. D. Ji, Phys. Rev. D52, 271 (1995)].
Different decomposition schemes are possible, depending on the choice of operators.

Recent analyzes: C. Lorce (2017); Y. Hatta, A. Rajan, K. Tanaka (2018); A. Metz, B. Pasquini and S. Rodini (2020), X. D. Ji, Y. Liu, A. Schäfer (2021).

Y. Hatta et al.: separation of independent operators that mix under renormalization:

$$\mathcal{O}_1 = -F^{\mu\alpha}F_{\nu\alpha}, \qquad \mathcal{O}_2 = g^{\mu\nu}F^2,$$

$$\mathcal{O}_3 = \bar{\psi}_q\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}\psi_q, \qquad \mathcal{O}_4 = g^{\mu\nu}\, m\bar{\psi}\psi$$

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

After renormalization is performed the operators mix (in a scheme dependent way):

$$\mathcal{O}_I^R = \mathcal{Z}_I^J\mathcal{O}_J$$

Proton mass composition: general picture

- Schemes of proton mass decomposition are based either on T^{μ}_{μ} or T^{00} decomposition. T^{00} needs to be considered in the nucleon rest frame. Note that in T^{μ}_{μ} there are T^{i}_{i} contributions related to pressure
- The distribution of contributions to the nucleon mass between quarks and gluons is scheme dependent
- In all schemes the major part of the proton mass comes from quantum effects, being proportional to QCD β function or to the quark mass anomalous dimension γ. The current quark mass contributions are at most at few percent level.
- The dominant contribution to the mass comes from the gluon field energy, then from the "dressed" quark mass and quark kinetic energy

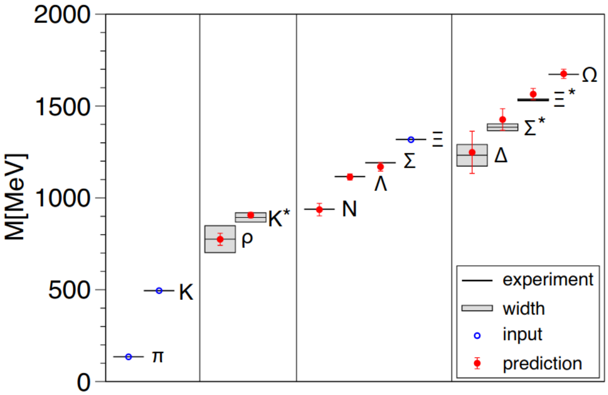
QCD on the lattice

- Discretization of the space-time, the finite lattice spacing $a \ll 1$ fm regularizes UV divergencies
- Wick rotation $x_0 \to -ix_4$, $A_0 \to -iA_4$, $iS \to -S_E$
- Real weights: path integral $\sim$ partition function in statistical mechanics — Monte Carlo evaluation
- Determination of masses of states: from correlation functions of two operators that create a desired state $|h\rangle$ from the vacuum:

$$\langle 0|\mathcal{O}_1(t)\mathcal{O}_2^\dagger(0)|0\rangle \;=\; \langle 0|e^{\hat{H}t}\mathcal{O}_1(0)e^{-\hat{H}t}\mathcal{O}_2^\dagger(0)|0\rangle \;=$$

$$= \sum_n \langle 0|\mathcal{O}_1(0)|n\rangle \frac{\exp(-E_n t)}{2E_n}\langle n|\mathcal{O}_2^\dagger(0)|0\rangle$$

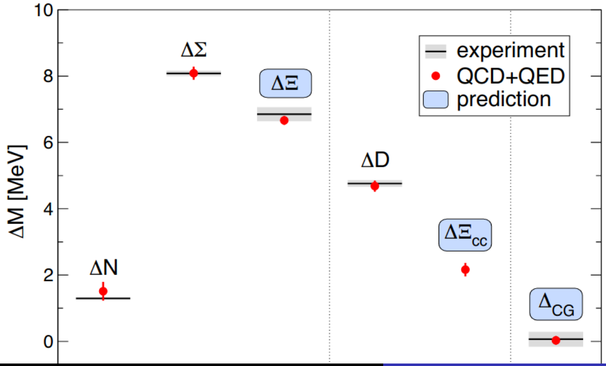
Hadron spectrum from the lattice



Neutron–proton mass splitting

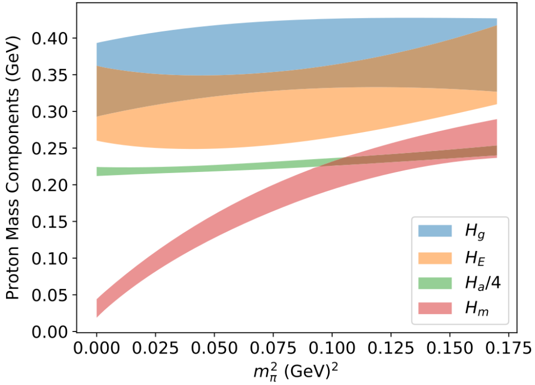
Competition of the quark mass hierarchy and the electromagnetic interaction energy

The neutron mass greater than the proton mass is mostly due to the heavier down quark (+2.5 MeV), the QED contributions are about -1 MeV. [BMW collaboration, 2015]



Proton mass decomposition from the lattice

Lattice calculations from T^{00} in the proton rest frame Yi-Bo Yang et al., Phys. ReV. Lett. 121, 212001 (2018).



H_m – quark mass term
$H_a/4$ – a quarter of the trace anomaly
H_g – gluon field energy
H_E – quark kinetic / potential energy

Some recent and ongoing research

- Gravitational formfactors of the nucleon can be defined with the expectation values of the energy momentum components in the nucleon state
- The expectation values and the formfactors may be computed using lattice QCD. Results found for mass distribution in the proton and the pressure profile
- Extraction of the pressure and shear forces inside the proton from Deeply Virtual Compton Scattering data
- Extraction of proton gravitational radius from the near threshold heavy quarkonia production

Some general open issues

- Coupling of quantum effects to gravity by the famous:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{classical}} = \frac{8\pi G}{c^4} \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

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Conclusions

- We have good understanding of the origin of the nucleon masses: the main source is scale invariance breaking due to trace anomaly. It comes mostly from quantum effects in the gluon field.
- The quantum origin of the nucleon stress energy–momentum tensor does not lead to theoretical problems when gravity is considered — the gravitational formfactors are defined by expectation values
- Current theoretical and experimental developements allow to compute and probe the gravitational properties of nucleons