Hoop conjecture in nonspherical self-gravitating systems.

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Abstract

A general framework for the study of the formation of apparent horizons is found. A necessary condition is given in momentarily static nonspherical geometries with non-negative (locally) sectional curvatures. Known results concerning the existence of trapped surfaces are reviewed in a class of conformally flat nonspherical systems.

I. Introduction.

The Cosmic Censorship Hypothesis emerged finally in 1969 [1], more than 50 years after its initial formulation by de Sitter[2]. In retrospect, its existence was provoked by new results on singularities obtained by R. Penrose[3] (in collaboration with S. Hawking[4]) in and after 1965. Before that time, there were known singularities but, in highly symmetric space-times. R. Penrose introduced the notions of apparent horizons and trapped surfaces[3,5], and used them to prove that in generic space-times singularities have to develop, assuming reasonable properties of matter. That meant that, singularities are rather typical phenomena and one had to adjust them into the existing physics, without destroying the harmony of physical laws. The idea of R. Penrose was that, singularities are hidden inside black holes and cannot influence the outside observers.

One of unfortunate problems is that the singularity theorems do not predict the character of a singularity. Also, it was a quick discovery that there are some singularities that are not hidden inside an event horizon; the phrase is that they are "naked"[6]. A way to remedy that situation was a restricted form of cosmic censorship, that should hold for a narrower clas of strong curvature singularities[7]. Israel pointed out that this version was contradicted by the "shell-focusing" singularities [8]; that provoked Israel to remark that *This evidence further suggests that it may not be possible, and is almost certainly not worth the effort, to argue that cosmic censorship applies only to "malignant" singularities and to attempt to formulate an intrinsic distinction betwee "benign" and "malignant" singularities.*[9] Nevertheless, a proof of cosmic censorship for strong curvature singularities was given by Królak[10], under some causality conditions. Israel made distinction between cosmic censorship and the event horizon conjecture[9], that latter resembling the hoop conjecture of Thorne[11]. Hoop conjecture says that an event horizon will form if and only if $4\pi m > C$, where m is a "mass" (in the original formulation[11] suggested to be the ADM mass) and C is the smallest hoop that can be passed around a collapsing body in any direction. Thus, the hoop conjecture could be regarded as a version of cosmic censorship that applies to a class of "malignant" singularities, which are massive in some sense. The hoop conjecture understood as a strict criterion is known to fail, at least for long-range fields[12]. A suggestion, however, that the formation of black holes is possible if and only if a "mass" (of both matter and gravitational field) is compressed in all three directions, looks plausible. We refer later to this as to the "compactness conjecture".

In this review I will review the recent progress made in proving the hoop conjecture. Let me remark that I will discuss the conditions under which an apparent horizon, rather an event horizon, will form. In the ultimate stage of gravitational collapse the apparent horizon would coalesce with the event horizon, but the last one is much more difficult to locate, being a global property of a spacetime.

I study the formation of apparent horizons in the maximal Cauchy surface of the Einstein equations. There is a new point in this work, namely I derive an equation whose derivation requires a calculation that is essentially tangent to the derivation of the second area variation. That equation, which is valid for general, nonspherical maximal Cauchy hypersurfaces, resembles a related equation found earlier in the spherically symmetric case[13].

A quasilocal quantity, called (for brevity) radiation energy is required to formulate conditions for the formation of trapped surfaces. Radiation energy is bounded from below by a linear functional of the Hawking mass.

Assume that we have initial data on a maximal Cauchy slice Σ, i.e., with the traceless extrinsic curvature K^{ij} and a Riemannian metric g_{ij} satisfying the constraint Einstein equations

$$R^3(g_{ij}) = 16\pi\rho + K^{ij}K_{ij} \quad , \tag{1a}$$

$$\nabla_i K^{ik} \;=\; -8\pi j^k \quad , \tag{1b}$$

with a (not necessarily positive) matter density ρ and a momentum density j^k.

A two-dimensional closed surface in Σ will be denoted by S, which may also denote the area of the surface. I assume that the surface is smooth enough to allow for an adapted system of coordinates, such that the line element reads

$$dl^2 = g_{tt}dt^2 + g_{ik}dx^i dx^k, i, j = 1, 2. \tag{2}$$

The variable t numerates leaves of the foliation (hence this is an analogue of the radial coordinate) while the variables x^1 and x^2 correspond to the angle-type coordinates. In the Lemma stated below I demand only the local (close to S) existence of (2), but later I will require that the whole volume inside S is foliated.

A unit normal to S is given, in the above coordinates, by

$$n = (0, 0, n_t), \tag{3}$$

where $n_t = \sqrt{g_{tt}}$. The divergence p of n is called the mean curvature and is given by the formula

$$p = \nabla_i n^i. \tag{4}$$

The surface integral of p may be called later the total mean curvature. The convergence of outgoing null geodesics h of S is given by

$$h = p - n^i n^j K_{ij} \tag{5}$$

and the surface integral of h is called the total convergence, $H(S)$, of S. In the case of momentarily static Cauchy data $H(S)$ coincides with the total mean curvature of S. A surface S with a negative value of $H(S)$ will be called an averaged trapped surface while if in addition $h(S) \leq 0$, then S will be said trapped (or, strictly saying, outer trapped[5]). In physical terms, TS is a closed 2-space with the property that narrow beams of light orthogonal to it at any point decrease in area, at least initially, when propagating outwards; thus the intensity of light may increas, when moving outwards. By R_{xy} I will understand the sectional curvature R^{xy}_{xy} in directions xy tangent to S.

All geometric quantities here are calculated with respect to the three-dimensional geometry of Σ.

The scalar curvature reads, in terms of the sectional and Ricci curvatures

$$R^3(g_{ij}) = 2[R_{xy} + R^t_t]. \tag{6}$$

$M(S)$ and $P(S)$ are total mass (the integral of ρ) inside S and total normal (to the above foliation adapted to S) momentum (the volume integral of $j^i n_i$). ds_t is a surface integral measure.

II. General self-gravitating systems.

The following lemma can be proven.

Lemma.

$$\partial_t \int_S p\ dS = \int_{s_t} [2det(p^k_i) - R^t_t] n_t ds_t\ . \tag{7}$$

.

Proof.

The proof has been outlined in the Appendix of [14], for surfaces in the flat space. By a direct but lengthy calculation one finds that the left hand side of (7) is equal to the right hand side of (7) plus an exact derivative, which vanishes, because S has no boundary. In Riemannian space, an additional term (in comparison with the flat space result) appears, equal to the Ricci tensor in the direction orthogonal to the foliation. See also a (different, as far as I understand) proof of the above Lemma in a work of Bujalo[15].

The identity (8) is related to, but not identical with, the second area variation. As far as I know, the earliest result (in flat space) is proven by Blaschke[16] more than seventy years ago and in general Riemannian spaces by Davies[17], 50 years ago. Now, we can prove the following.

Theorem 1. Assume $S \subset \Sigma$ is a compact 2-surface and that there does exist a foliation s_t $(0 \leq t \leq T)$of the inner part of S with a line element (2). Let $K(s_t)$ be the Gauss curvature of s_t. Then the following equation is true

$$H(S) = H(s_0) + \int_0^T \int_{s_t} dt d^2 s_t n_t 2K(s_t) - 8\pi[M(S) - P(S)] +$$

$$\int_V [-R_{xy} - K^{ij} K_{ij}/2 - K^{ij} \nabla_i n_j] dv, \tag{8}$$

where $H(s_0) = lim_{S \to s_0} \int_S p(S)$.

Proof.

Inserting the well known Gauss formula

$$det(p^j_i) = K - R_{xy}$$

into (7) yields

$$\partial_t \int_{s_t} p\ dS = \int_{s_t} [2(K - R_{xy}) - R_t^t] n_t ds_t\ . \tag{9}$$

Integrating (9) with respect to t, expressing the Ricci curvature R_t^t in terms of the scalar curvature $R^{(3)}$ and the curvature R_{xy} (see (6)), and finally, using the constraint equation (1a), one obtains the following

$$\int_S p(S) d^2 s_t = p(s_0) + \int_0^T \int_{s_t} dt d^2 s_t\ n_t\ 2K(s_t) - 8\pi M(S) - \int_V \left(R_{xy} + K^{ij} K_{ij}/2\right) dv. \tag{10}$$

From (1b) one gets

$$\int_S K_{ij} n^i n^j\ dS\ = \int_V [-8\pi j_i n^i + K_{ij} \nabla^i n^j] dv. \tag{11}$$

Substracting (11) from (10), immediately yields the equation (8). This ends the proof of the Theorem 1.

Remark. The formula (8) follows also directly from the volume integration of the formula (2) in [18].

Let us define the following quantities,

$$8\pi Rad(S) = \int_0^T \int_{s_t} dt d^2 s_t\ n_t\ 2K(s_t) \tag{12}$$

and

$$8\pi E(S) = \int_V [R_{xy} + K^{ij} K_{ij}/2 - K^{ij} \nabla_i n_j] dv. \tag{13}$$

For brevity, I will call $Rad(S)$ and $E(S)$ the radius and the radiation energy, respectively, of the configuration enclosed by a surface S.

A foliation by surfaces of constant mean curvature p is the most interesting. In such a foliation the averaged trapped surfaces are trapped pointwise, which allows one to make connection with the singularity theorems[3,4]. Equation (2) reads now, in terms of the radius $Rad(S)$ and the radiation energy $E(S)$

$$h(S)S = H(s_0) + 8\pi[Rad(S) - (M(S) - P(S))] - 8\pi E(S). \tag{14}$$

If one assumes that the foliation is smooth and locally convex, then $H(s_0) = 0$. **Thus, the surface S is trapped if and only if**

$$h(S)S = 8\pi[Rad(S) - (M(S) - P(S))] - 8\pi E(S) \leq 0. \tag{15}$$

This formula readily demonstrates that a large influx of matter $P(S)$ through the surfaces s_t makes it easier to create a trapped surface and a smaller amount of matter $M(S)$ is needed. And the opposite way, the greater is the outflux $P(S)$, the more matter $M(S)$ is required to form a trapped surface. Also, since no assumption is made about the energy density nor the validity of the weak energy condition, we can draw the conclusion that the negative energy density prevents the formation of trapped surfaces (in suprising accordance with a naive expectation). These conclusions are exactly the same as in the spherically symmetric case[13].

The radiation energy, however, obscures the situation. If $E(S)$ is bounded from below, then the compression of matter always leads to the formation of trapped surfaces, as stated in the preceding remarks. If a part of the raditation energy, $\int_V R_{xy} dV$ is non-negative then the amount of matter inside a volume of a fixed radius $Rad(S)$ is bounded from above, $M(S)/Rad(S) \leq 2$, as predicted by Arnowitt, Deser and Misner[19] and proven in special geometries in [20,21,22]. I will prove this fact elsewhere.

In the case, when the ratio $E(S)/Rad(S)$ is unbounded from below, the above conclusions may stop to be true. In particular, it would be possible to compress an infinite amount of matter inside a fixed volume, without the creation of the apparent horizon. Thus, the radiation energy unbounded from below would imply the appearance of massive naked sigularities, violating cosmic censorship (or the hoop conjecture).

There are reasons to believe that the radiation energy is bounded from below, at least in some classes of foliations. Below I will list examples that support my conviction.

A. In spherically symmetric and momentarily static geometries the radiation energy is given by the following expression

$$8\pi E(S) = -16\pi \int_0^R dr[\; rf'(f + f'r). \tag{16}$$

Above, f is a conformal factor ($g_{ij} = f^4 \hat{g}_{ij}$, where $\hat{g}$ denotes a flat metric). $E(S)$ is manifestly positive, since $(f'r + f) \geq 1$, as shown in [20].

In [13,23] were proven results that imply that in the case of non-zero extrinsic curvature K_{ij} (i.e., nonsymmetric in time initial data) the radiation energy satisfies the following bound

$$8\pi E(S) \geq -L(S)/6, \tag{17}$$

if S does not contain trapped surfaces.

B. In conformally flat geometries and for time symmetric initial data, the radiation energy inside an equipotential surface S reads

$$8\pi E(S) = -\int_0^T \partial_t f L(f) \int_S \hat{p} d\hat{S}/(4\pi). \tag{18}$$

For the notation see the next section. In [24] it is proved, following an outline of E. Flanagan[25], that $L(f) \geq 1$ for a class of convex (in the flat metric) foliations. By construction of the foliation, $\partial_t f \leq 0$.

Let us comment on the validity of the terms "radius" and "radiation energy". Assuming that the Gauss curvature of the foliation s_t adapted to the surface S is positive and applying the Gauss-Bonnet theorem we get

$$4\pi \int_0^T dt\ inf(n_t)\chi(s_t) \leq 8\pi\ Rad(S)\ \leq 4\pi \int_0^T dt\ sup(n_t)\ \chi(s_t)\ , \qquad (19)$$

where $\chi(S)$ is the Euler characteristic of S. The lower bound is certainly not larger than the radius of the largest sphere inscribed into S while the upper bound may well be bounded by the largest proper radius of the configuration.

That point is not obvious; see a discussion in [14]. One may hope that a suitable technical trick, for instance the method of equidistants, will lead to the conclusion, that $Rad(S)$ is an averaged proper radius of the surface S, at least for locally convex foliations (i.e., with the nonnegative Gauss curvature of the foliation s_t) of the spherical topology.

Let me point out that in the case of foliations by metric spheres $Rad(S)$ coincides with the proper radius. Also, in spherically symmetric geometries $Rad(S)$ is equal to the proper radius (since the Euler number of a surface of spherical topology is equal to 2) of a sphere, and in spheroidal geometries it is bounded from above by the largest proper radius[21].

The radiation energy is bounded from below, as shown above, in spherically symmetric geometries and, more generally, in time symmetric conformally flat initial (Cauchy) geometries. In those two cases radiation energy is entirely expressed in terms of quantities related to matter, hence its name is completely unjustified.

But, on the other hand, the radiation energy $E(S)$ certainly can contain a nonzero part related to the gravitational radiation. For instance, in the case of vanishing matter, from (8) follows the possibility to form trapped surfaces by gravitational fields alone. Such gravitational radiation should create a geometry with a prevailing positive sectional curvature, to surmount the first nonzero term of (8). The existence of trapped surfaces due to the pure gravitational radiation has already been proven in [26] and a numerical evidence is given in [27] but, equation (8) gives some information about the geometry.

The radiation energy vanishes in flat space-time. Its value is also finite and manifestly positive, for the Schwarzchild geometry and for any centered ball (with the centere excised) of finite radius.

For foliations by metric spheres, one can derive the following bound:

$$8\pi E(S) \geq 32(\pi)^{3/2} \int_0^T dt m_H(s_t)/(S_t^{1/2}) - 4\pi Rad(S), \qquad (20)$$

where $m_H(S)$ is the Hawking mass[28,29] of a surface S. Unfortunately, the Hawking mass is not proven to be bounded from below, except for special geometries and special surfaces, and in flat space it may be unboundedly negative. That point is discussed in more detail in [30].

III. Hoop conjecture in massive systems with no gravitational waves.

We review quickly the results of [22]. Let a metric of a momentarily static Cauchy slice be conformally flat. The only nontrivial part of the initial data is a conformal factor f defined by $g_{ab} = f^4 \hat{g}_{ab}$, which must satisfy the hamiltonian constraint

$$\hat{\Delta} f = -2\pi \varrho f^5, \tag{21}$$

where ϱ is the matter density, $\hat{g}_{ab}$ is the Euclidean 3-metric and $\hat{\Delta}$ is the laplacian with respect to the flat metric $\hat{g}_{ab}$. The equation (8) reads now

$$H(S)/8\pi = D(S) - M(S), \tag{22}$$

where

$$2\pi D(S) = \int_V \hat{g}^{\sigma\sigma}(\partial_\sigma f)^2 d\hat{V} + \int_S f^2 \hat{p} d\hat{S}/4, \tag{23}$$

$$H(S) = \int_S [f^2 \hat{p} + 4 f \hat{n}^\sigma \partial_\sigma f] d\hat{S}, \tag{24}$$

and

$$M(S) = \int_V \varrho dV. \tag{25}$$

$\hat{n}$ is an orthonormal (in $\hat{g}$-metric) vector to the surface S. Here and below all hat-ted quantities refer to the background metric $\hat{g}_{ab}$ while the remaining quantities use the metric g_{ab}. A surface S is called an **equipotential surface** if the conformal factor f is constant on S.

For a convex surface $\hat{S}$ in a flat space the estimation

$$\int_{\hat{S}} \hat{p} d\hat{S} \geq 4\sqrt{\pi \hat{S}} \tag{26}$$

holds[31]. In the formula (26) and below, S might denote either a surface or area of a surface. Using (26), we obtain for a convex equipotential surface S the estimation

$$D(S) \geq f^2(S) \int_S \hat{p} d\hat{S}/(8\pi) \geq f^2(S)\sqrt{\hat{S}/\pi}/2 = \sqrt{S/\pi}/2. \tag{27}$$

Thus we may infer the following:

Theorem 2. A necessary condition for an equipotential convex surface S to be an averaged trapped surface (and thus: trapped surface pointwise) is that

$$M(S) \geq \sqrt{S/\pi}/2. \tag{28}$$

This inequality is sharp[22]. Theorem 2 proves a part of the "only if " statement of hoop conjecture. If one starts with a given amount of matter and compresses it in at least two directions, then the condition (28) will be always met. But the compression in one dimension most likely will not produce a trapped surface, except in the case when a very long and thin body is compacted along its length. Then its surface becomes significantly smaller and the necessary condition could be satisfied.

Let us point out that Theorem 2 does not require that all equipotential surfaces are convex. It is sufficient that only those surfaces are convex, that are suspected to be trapped (e.g., close to the surface of the compacted object). Also, the statement of Theorem 2 is stable under small deviations either from equipotentiality or convexity.

The following result was postulated in [22], and proven by E. Flanagan[25] (see also [24]).
Lemma . Let f be a solution of the Lichnerowicz equation (2), with a positive energy density and the boundary condition $f(\infty) = 1$. Let $\hat{S}$ be a level surface of f. Then

$$L(f) \equiv f + 2\int_{\hat{S}} \sqrt{\hat{g}^{\sigma\sigma}}\partial_\sigma f d\hat{S} / \int_{\hat{S}} \hat{p} d\hat{S} \geq 1. \tag{29}$$

Having (29), using $\partial_\sigma \leq 0$ and $\partial_\sigma \int_S \hat{p} d\hat{S} = 2\int_S \hat{K}\hat{n}_t d\hat{S}$ we may bound (23) by another expression:

$$D(S(\sigma)) = D(S(0)) + \int_0^\sigma \; dsL(f)\partial_\sigma f \int_{\hat{S}} \hat{p} d\hat{S}/(4\pi) + \int_0^\sigma \; f^2(s)\partial_\sigma \int_S \hat{p} d\hat{S}/(8\pi) ds$$

$$\leq Rad(\sigma). \tag{30}$$

It is an important fact to stress out that the inequalities (29) and (30) hold for moving matter. In contrast to the preceding results, these two estimations remain true also for non-symmetric in time initial data (on the maximal Cauchy slice) for the Einstein equations. We have used only the Lichnerowicz equation (21) and in the case of non-symmetric in time initial data the only change that occurs is the appearence of a new negative term on the right hand side of (21) related to the extrinsic curvature of the Cauchy surface. Thus the maximum principle still can be applied and the whole idea of the proofs remains intact. Using those last estimations, one obtains[22]

Theorem 3. If the content of matter $M(S)$ inside S exceeds $Rad(S)$ then S itself is an ATS.

An ATS can become pointwise trapped if it contains enough matter[22], as we see below.

Proposition If at S the following inequality is satisfied

$$M(S) \geq Rad(S) + \int_S [n^\sigma (n_\sigma p)_{max} - p] dS/(8\pi),$$

(no summation over σ) then S is pointwise trapped.

Thus, if the content of matter inside S is made infinitely large for fixed $Rad(S)$, then S will always become a trapped surface. In this sense massive singularities on a momentarily static Cauchy surface are always trapped, according to Seifert's conjecture[32].

The point is, however, that in the class of geometries discussed the ratio $M(S)/Rad(S)$ is bounded from above.

Theorem 4. If the initial (nonsymmetric in time) data of Einstein's equations are conformally flat on the maximal Cauchy slice, with level surfaces of the conformal factor being convex inside S then

$$M(S) \leq Rad(S) + \int_{\hat{S}} f^2 \hat{p} d\hat{S}$$

$$\leq 2Rad(S). \tag{32}$$

Thus, in conformally flat and momentarily static geometries the formation of trapped surfaces on the surface of initial data can be described completely. One may guess that this success should be attributed to the fact that these spacetimes do not contain gravitational waves; in the absence of matter and assuming trivial topology, a geometry becomes the flat, Minkowskian one. In order to test this supposition we will consider an asymptotically flat spacetime, that can be sliced using the maximal slicing condition. Assume that a time symmetric slice Σ comprises a bounded region Ω) in which the gravitational field is very strong (i.e., three-dimensional curvatures are large) and, in addition, in Ω all three-dimensional sectional curvatures R^{ij}_{ij} are nonnegative. The examination of spherically symmetric geometries shows that the three sectional curvatures can all be nonnegative only in regions comprising a lot of matter and that outside matter the curvatures $R^{r\phi}_{r\phi}, R^{r\theta}_{r\theta}$ must be nonpositive (here ϕ, θ denote the angle directions).

The second condition is not very interesting, if one imposes it on the whole Cauchy slice, since it excludes gravitational waves - a three-dimensional space with no matter and non-negative sectional curvatures is flat. But, we neeed this condition only locally and in regions of high and positive energy density it is expected to hold. The assumption that sectional curvatures in Ω are non-negative probably means that in Ω the influence of gravitational waves is negligible. In such a case one can prove the following.

Theorem 5. The necessary condition that S be a trapped surface is that the amount of matter $M(S)$ within S exceeds the half of the radius rd of the largest metric sphere inscribed into S.

In [15] is proven that

$$8\pi rd(S) \leq H(S) + \int_{V(S)} R^3 dV$$

(see also p. 273 in [31]) which combined with the Einstein constraint (1a) immediately yields the above result.

IV. Concluding remarks.

To summarize, I have set a framework for the discussion of the formation of trapped surfaces on the Cauchy maximal hypersurface. An important role in it is played by a new quantity, "radiation energy"; its status is unclear, but it seems to be crucial in any attempt to formulate physical criteria for the formation of trapped surfaces. In particular, the radiation energy would imply the violation of the cosmic censorship if it is unbounded from below.

Almost complete set of conditions that describe the formation of trapped surfaces is known in geometries that do not contain gravitational waves. If a maximal slice with a conformally flat and convex (in the sense defined above) 3-geometry is considered, then the volume $V(S)$ of a fixed size $Rad(S)$ can contain the amount of matter $M(V)$ not greater than $2Rad(S)$. If momentarily static initial data are considered, then the necessary condition for S to be trapped is that the total rest mass satisfies the condition (28), that is rougly speaking, that the area of S is very small. This result essentially proves the "only if" part of the hoop conjecture. It holds true also in a class of nonspherical geometries that contain highly curved regions with non-negative sectional curvatures, according to Theorem 5.

Theorem 3 gives a sufficient condition for a surface to be trapped in the average, but if the geometry is strongly nonspherical, then an equipotential surface may not be trapped, regardless of the amount of matter inside it. In this sense the geometry of a Cauchy surface might be "singular", but in fact this singularity is very mild, because of the universal bound on the total rest mass. Relating that to the hoop conjecture, we would say that it essentially proves the "if" part of the compactness conjecture, with this restriction that although trapped surfaces will form when matter is compressed in all dimensions but, in addition, *the compression should be sufficiently "round"*. This reservation is related to the existence of the aforementioned bound. All the results are valid for compact and noncompact distributions of matter and for geometries that are convex or almost convex (in a sense that could be precisely defined).

There have been made a few attempts to describe the formation of trapped surfaces in general nonspherical self-gravitating systems[33,34,35]. Schoen and Yau[33] obtained a necessary condition on a maximal Cauchy slice and a sufficient condition on non-maximal slices. They use, however, a torus measure of size, which is not always suitable in strongly curved geometries[23]. Further work would be required to specify those collapsing systems to which the criteria of Schoen and Yau apply and which are of physical interest. Flanagan[34] proposed a new measure of size for static axisymmetric Weyl metrics and attempted to prove a formulation of the hoop conjecture expressed in terms of quantities that do not depend on the interior of a collapsing region.

It is necessary to point out that, since we work on the Cauchy surface, there is no direct connection between the investigation that is reviewed here and that of Shapiro and Teukolsky[36]. They analyzed the evolution of a system that was initially spheroidal and at rest; the problem is that in later evolution it would stop to belong to the class of geometries discussed in Section III.

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