Quantum Mechanics III, set 9.

Ex. 1. Consider a complex scalar field interacting with the external potential $V(x)$.

$$\mathcal{H}_{int} = \lambda \Phi^\dagger(x) V(x) \Phi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Determine the generating functional of Green's functions

$$\mathcal{Z}[J, J^*] = \langle \emptyset | T(e^{\int d^4x (J^*(x)\Phi_H(x) + J(x)\Phi_H^\dagger(x))}) | \emptyset \rangle = e^{W[j,j^*]}$$

using relations derived on the lecture. Find the solution for a special case $V(x) = C$ (constant in space-time).
Hint: Use the derived relation:

- $$\mathcal{Z}[J, J^*] = \lim_{T\to\infty} \frac{\langle 0 | T\left(U_I(T,-T) e^{\int d^4x (J^*(x)\Phi_I(x) + J(x)\Phi_I^\dagger(x))}\right) | 0 \rangle}{\langle 0 | U_I(T,-T) | 0 \rangle}$$

- Use the property
$$\mathcal{Z}[J, J^*] = \exp(W[J, J^*]),$$
where $W[J, J^*]$ can be expressed as a sum of connected Feynman diagrams.

- In the above $|\emptyset\rangle$ is the true (physical) vacuum and $|0\rangle$ the vacuum of the unperturbed Hamiltonian.

Ex. 2. Consider a self-interacting real scalar field ϕ with mass M. Let the interaction Hamiltonian be given by

$$\mathcal{H}_{int} = \frac{g}{3!}\phi^3(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find all connected diagrams of $W[j]$, following the same procedure as in ex. 1 to terms (including) g^2.

Ex. 3. Consider a self-interacting real scalar field ϕ with mass M. Let the interaction Hamiltonian be given by

$$\mathcal{H}_{int} = \frac{g}{4!}\phi^4(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find all connected diagrams of $W[j]$, following the same procedure as in ex. 1 to terms (including) g^2.

Ex. 4. Consider a system containing a complex scalar field $\Psi(x)$, $\Psi^\dagger(x)$ with a mass M interacting with a real scalar field $\phi(x)$ with a mass m through the interaction

$$\mathcal{H}_I = g\Psi^\dagger(x)\phi(x)\Psi(x).$$

Find all connected diagrams of $W[j]$ to order g^2. Find all possible amplitudes, which have two-particle states as the in- and out-states.