Quantum Mechanics III, set 11.

Ex. 1. Consider a Dirac field interacting with the external field (not quantum) with the interaction

$$\mathcal{H}_{int} = \bar{\Psi}(x)\Gamma(x)\Psi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find a form of the generating functional $\mathcal{Z}(\xi, \bar{\xi}) = \exp W(\xi, \bar{\xi})$ using the analogy with the case of bosonic field. $\Gamma(x)$ is an arbitrary external matrix function. Consider the case $\Gamma(x) = C$ (constant proportional to a unit matrix) and determine the full form of the fermion propagator in the analogy with the complex scalar field.

Ex. 2. Consider a system containing a complex scalar field $\Phi(x)$, $\Phi^\dagger(x)$ with a mass m interacting with a real scalar field $\phi(x)$ with a mass M through the interaction

$$\mathcal{H}_I = g\Phi^\dagger(x)\phi(x)\Phi(x).$$

Find all connected diagrams to order g^2.

Ex.3. Assume that Dirac field with a mass m interacts with the quantum real scalar field with a mass M and the interaction has a form

$$\mathcal{H}_{int} = g\bar{\Psi}(x)\phi(x)\Psi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find all connected diagrams to order g^2. What are their signs as compared to similar diagrams from ex. 2.

Ex.4. Find all two-body amplitudes for scattering processes (two incoming and two outgoing particles) for processes described by the diagrams from Ex. 2 and Ex. 3.