Quantum Mechanics III, set 8.

Ex. 1. Using Wick's theorem calculate:

$$\text{a)} \quad \langle 0| \left(\int d^4x (J^*(x)\Phi_H(x) + J(x)\Phi_H^\dagger(x) \right)^n |0\rangle$$

where $\Phi_H(x)$ is a complex scalar field in the Heisenberg picture and $J(x),\ J^*(x)$ are arbitrary functions of space-time.

$$\text{b)} \quad \langle 0| \left(\int d^4x (j(x)\phi_H(x) \right)^n |0\rangle$$

where $\phi_H(x)$ is a real scalar field in the Heisenberg picture and $j(x)$ is an arbitrary function of space-time.

Ex. 2. For a real scalar field $\phi(x)$ with a Lagrangian density

$$\mathcal{L} = \frac{\hbar^2 c^2}{2} g^{\mu\nu} \partial_\mu \phi(x) \partial_\nu \phi(x) - \frac{M^2 c^4}{2} \phi^2 - j(x)\phi$$

with a real source $j(x)$ determine a form of the evolution operator $U(t_f, t_i)$ in the interaction picture to terms including $O(j^4)$.

Ex. 3. Consider a complex scalar field interacting with the external real field (not quantum) $W(x)$:

$$H_{int} = g \int d^3x \Phi^\dagger(x) W(x) \Phi(x)$$

Using Wick's theorem determine a form of the evolution operator $U(t_f, t_i)$ for $t_i \to -\infty,\ t_f \to \infty$ to the order g^3 (included).

Ex. 4. Consider a self-interacting real scalar field ϕ. Assume that the Lagrangian density has a form

$$\mathcal{L} = \frac{\hbar^2 c^2}{2} g^{\mu\nu} \partial_\mu \phi(x) \partial_\nu \phi(x) - \frac{M^2 c^4}{2} \phi^2 - \frac{g}{3!} \phi^3.$$

Determine a form of H_{int} in the interaction picture. Using Wick's theorem determine a form of the evolution operator $U(t_f, t_i)$ for $t_i \to -\infty,\ t_f \to \infty$ to the order g^2 (included). What would be these terms if $H_{int} \to :H_{int}:$?