Quantum Mechanics III, set 11.

Ex. 1. Find the following anticommutators of Dirac field operators in the Schrödinger representation:

$$\{\Psi_\alpha^{(S)}(\vec{x}), \Psi_\beta^{(S)}(\vec{y})\}, \quad \{\Psi_\alpha^{(S)}(\vec{x}), \bar{\Psi}_\beta^{(S)}(\vec{y})\}.$$

Similar excersise in the Heisenberg representation:

$$\{\Psi_\alpha^{(H)}(x), \Psi_\beta^{(H)}(y)\}, \quad \{\Psi_\alpha^{(H)}(x), \bar{\Psi}_\beta^{(H)}(y)\}.$$

Ex. 2. Find a form of the propagator of the Dirac fermion field as a four-dimensional Fourier transform

$$iS_F(x,y) = \langle 0|T\left(\Psi_\alpha^{(H)}(x)\bar{\Psi}_\beta^{(H)}(y)\right)|0\rangle.$$

Show that

$$iS_F(x,y) = T\left(\Psi_\alpha^{(H)}(x)\bar{\Psi}_\beta^{(H)}(y)\right) - :\Psi_\alpha^{(H)}(x)\bar{\Psi}_\beta^{(H)}(y):$$

Ex. 3. Consider a Dirac field interacting with the external field (not quantum) with the interaction

$$\mathcal{H}_{int} = \bar{\Psi}(x)\Gamma(x)\Psi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find a form of the generating functional $\mathcal{Z}(\xi,\bar{\xi}) = \exp W(\xi,\bar{\xi})$ using the analogy with the case of bosonic field. $\Gamma(x)$ is an arbitrary external matrix function.

Ex. 4. Assume that Dirac field with a mass m interacts with the quantum real scalar field with a mass M and the interaction has a form

$$\mathcal{H}_{int} = g\bar{\Psi}(x)\phi(x)\Psi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}.$$

Find all connected diagrams to order g^2. What are their signs as compared to similar diagrams from Set 10.