Quantum Mechanics III, set 10.

Ex. 1. Consider a complex scalar field interacting with the external real field (not quantum) $V(x)$

$$\mathcal{H}_{int} = \Phi^\dagger(x)V(x)\Phi(x), \quad H_{int} = \int d^3x \mathcal{H}_{int}$$

where

$$V(x) = A\exp(-\vec{x}^2/2\sigma)$$

(no time dependence).

Find the scattering amplitude of a one-particle state (particle or anti-particle) to order A^2. Try to push the calculations as far as possible. The in- and out-going particles have respectively momenta $\vec{p}$ and $\vec{q}$.

Ex. 2. Consider a system containing a complex scalar field $\Psi(x)$, $\Psi^\dagger(x)$ with a mass M interacting with a real scalar field $\phi(x)$ with a mass m through the interaction

$$\mathcal{H}_I = g\Psi^\dagger(x)\phi(x)\Psi(x).$$

Find all connected diagrams to order g^2. Find all possible amplitudes, which have two-particle states as the in- and out-states. Calculate the differential cross sections for these processes.