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Altruism, reputation, and collective collapse of cooperation in a simple model

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Agenda

- Motivation
- Introduction to model
- Set of cases with constant altruism
- Set of cases with changing altruism
- Conclusion and sociological analyse

Motivation

- The Prisoner's Dilemma (PD) is a canonical example of game, where mutual cooperation is not profitable for an individual player and simultaneously it is profitable for a society

- Two archetypes of players:

  Homo Economicus - a creature who is rational and purely self-regarding

  Homo Sociologicus - a creature who follows prevailing social norms

Motivation

Conditions for a Social Norm to Exist:

Let R be a *behavioral rule* for situations of type S, where S can be represented as a mixed-motive game. We say that R is a social norm in a population P if there exists a sufficiently large subset P_{cf} P such that, for each individual $i \in P_{cf}$:

•*Contingency*: i knows that a rule R exists and applies to situations of type S;

•*Conditional preference*: i prefers to conform to R in situations of type S on the condition that:

(a) *Empirical expectations*: i believes that a sufficiently large subset of P conforms to R in situations of type S;

(b) *Normative expectations*: i believes that a sufficiently large subset of P expects i to conform to R in situations of type S;

(b') *Normative expectations with sanctions*: i believes that a sufficiently large subset of P expects i to conform to R in situations of type S, prefers i to conform, and may sanction behavior.

C. Bicchieri, *The Grammar of Society,* Cambridge UP 2006

Motivation

- Social change and norm's change
- Interest in dynamics of social process rather than final stage
- Toy model with only few assumtions: society characterized with reputation and altruism

Model

Each of two identical players has two different strategies: to cooperate (C) with the other or to defect (D) from cooperation. The probability that *i* cooperates with *j* depends on level of:

- Reputation *W* of co-player
- Altruism $\boldsymbol{\varepsilon}$ of player *i*, as a measure of her/his willingness to cooperate with others or to defect.

$$P(i,j) = W_j(i) + \varepsilon_i$$

If $P(i,j)>1$ is set 1, otherwise if $P(i,j)<0$ then 0.
Reputation *W*- is in range of [0,1]
Altruism ε- is in range of [-1/2,1/2]

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Model

Main rule:

- Reputation of player increase (decrease) if he cooperates (defects);
- Altruism ofplayer increase (decrease) if co-player cooperates (defects);

Speed of change is defined by x_W/x_ε as a procentage change of reputatation / altruism.

(C) $\boldsymbol{\varepsilon := (0.5-\varepsilon)x_\varepsilon + \varepsilon}$

(D) $\boldsymbol{\varepsilon := \varepsilon + (-0.5-\varepsilon)x_\varepsilon}$

(C) $\boldsymbol{W := (1-W)x_W + W}$

(D) $\boldsymbol{W := W - Wx_W}$

Model

There is 100 (sometimes 1000) players in game with some initial conditions *W* and *ε*. Network is implemented as a flully connected graph, square lattice or Erdős–Rényi graph.

W and *ε* are random unitary distributed in range of:

$$\boldsymbol{[\langle W\rangle - d;\ \langle W\rangle + d]}$$

$$\boldsymbol{[\langle \varepsilon\rangle - d;\ \langle \varepsilon\rangle + d]}$$

where *d* is a half of whole range

For example:

- if *d*=0 *W* (*ε*) is exaclly the same for all players
- if *d*=0,5 distribution is established on whole range and mean value is exaclly in the middle

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Model

Parameters	Model descriptions	Observations
x_W=0,5 and ε=*const*	Reputation changes in „bisection” way and altruism is constant	Mutual choices exist and create symmetric coexisting frequencies curves
x_W=W' and ε=*const*	*W'* is reputation of co-player, so player's reputation change as quickly as *W'*	Symmetry broken – cooperation is more common, unstabile final state
x_W=0,5 and x_ε=0,5	Reputation and altruism changes in „bisection” way the same time	All players choose only one strategy,and the initial state is divided into two attractors
x_W=0,5 and x_ε={0,5;0}	Altrusim changes only in some cases: CC (goes up) and CD (goes down)	Symmetry broken – cooperation is more common, non-full negative state
$\boldsymbol{W_j(i)}$	Individual vision of agent's *W*	System slows down
E-R or lattice	Agents on special networks	Spatial correlations appear

Model

Possible mutual choices:

R- both cooperate

S- co-player defects when a player has cooperated

T- player defects when the co-player has cooperated

P- both defect

x_w=0,5 and ε=const

x_w=0,5 and ε=*const*	Reputation changes in „bisection" way and altruism is constant	Mutual choices exist and create symmetric coexisting frequencies curves

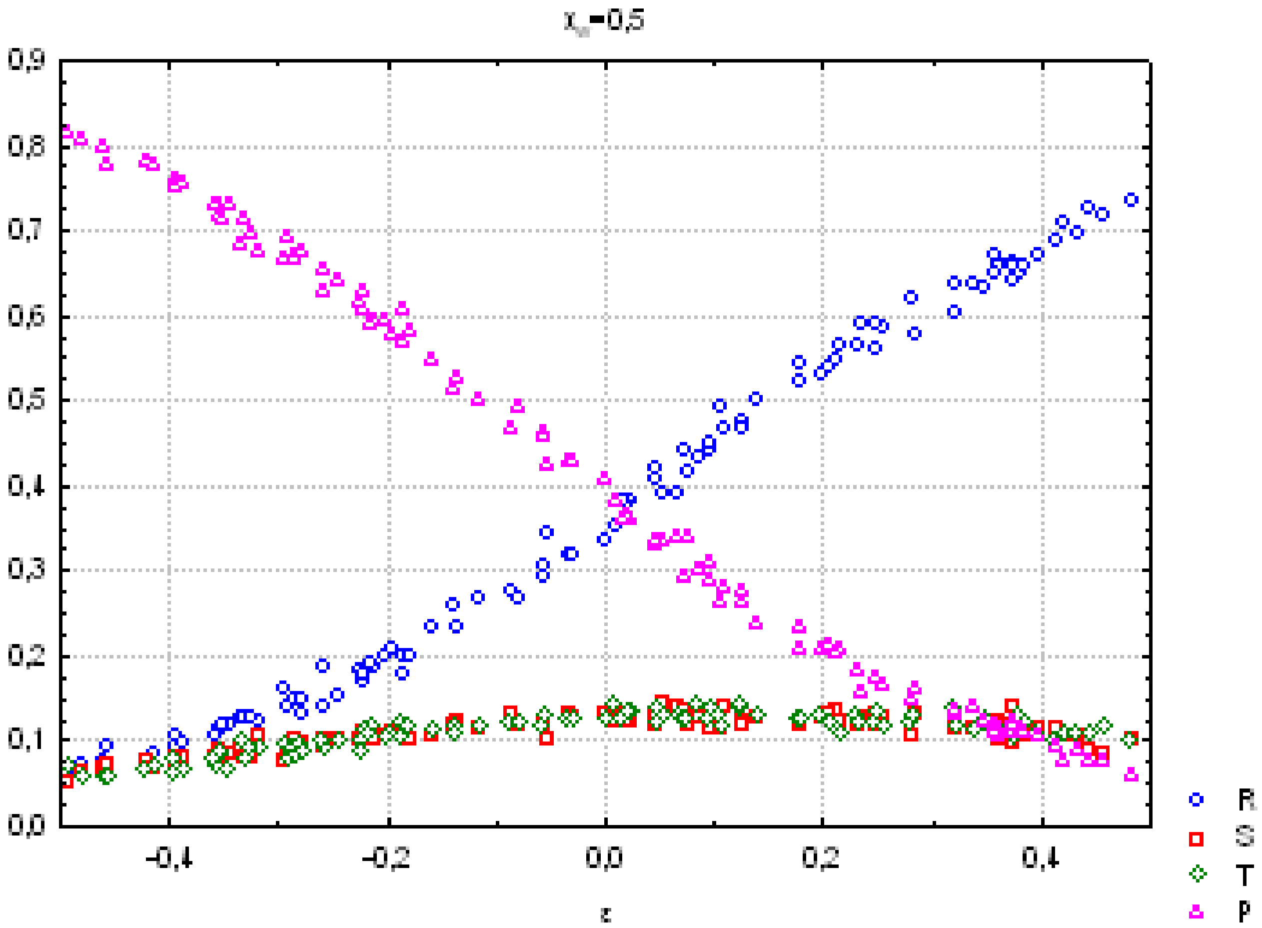
x_w=0,5 and ε=const

Evolution simplify to:

(C) $W := (1-W)x_w + W$

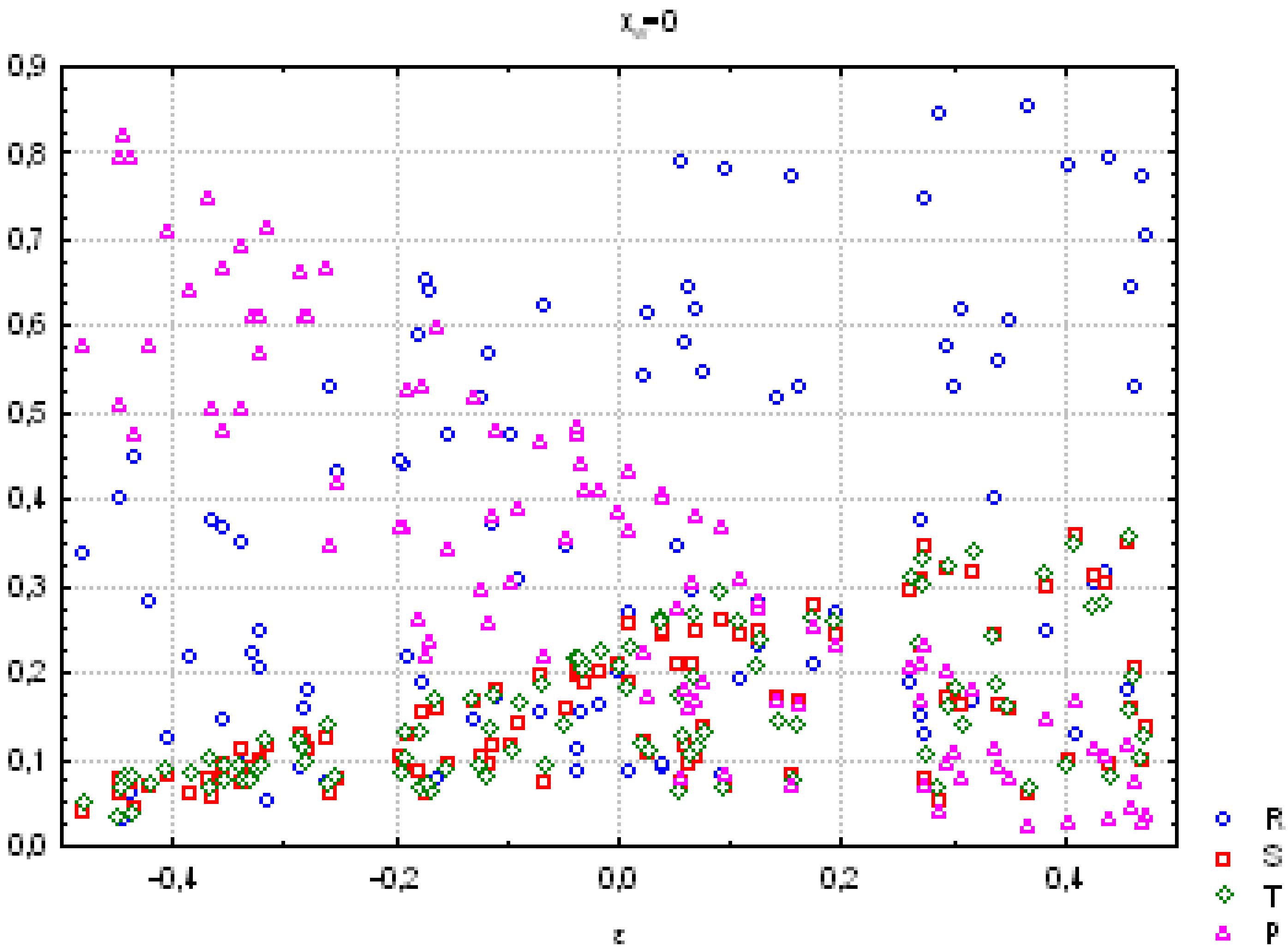
(D) $W := W - Wx_w$

Frequencies of mutual choices for 10^5 MC steps



ε=const, W=const

Absolutly simplified

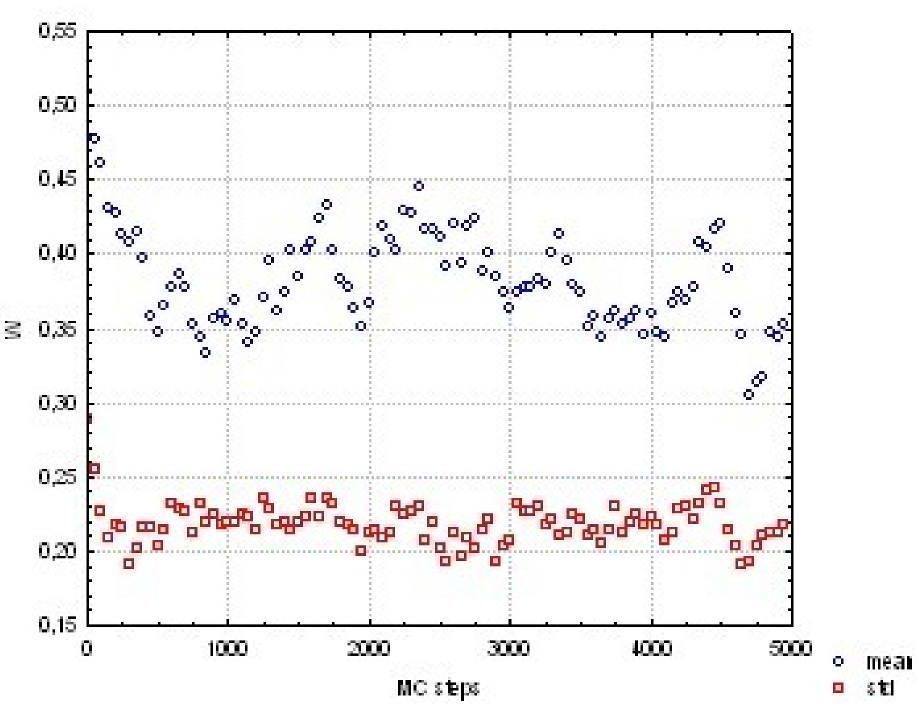


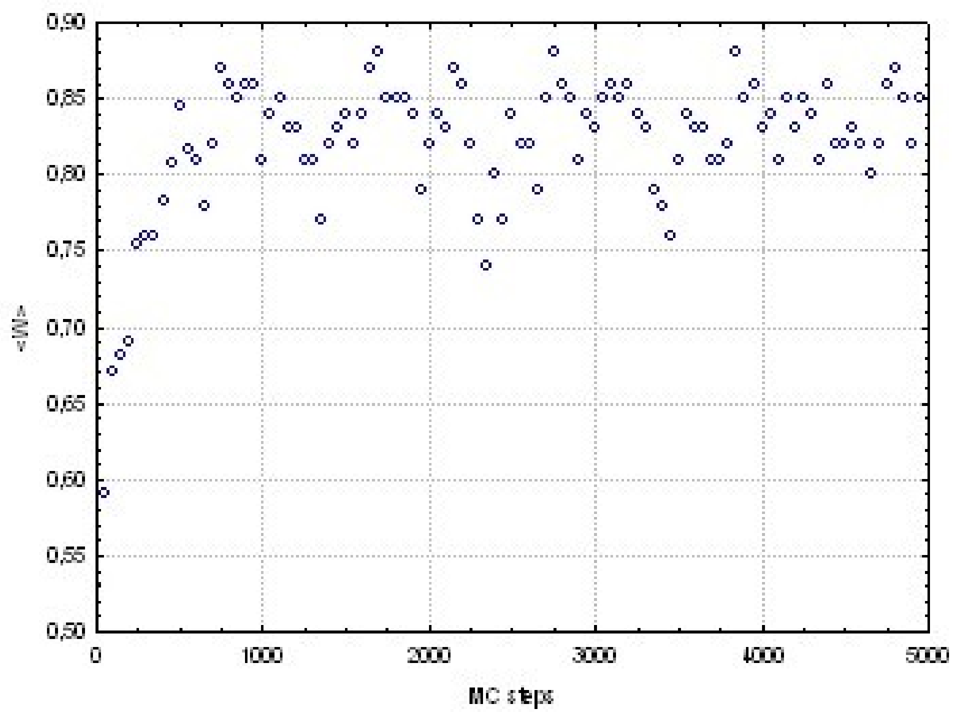
ε=const, $x_w = z \cdot W'$

x_W=W' and ε=*const*	*W'* is reputation of co-player, so player's reputation change as quickly as *W'*	Symmetry broken – cooperation is more common, unstabile final state

ε=const, $x_w = z \cdot W'$

$x_w = z^* W'$ where *W'* is reputation of co-player, and *z* is additional parameter of speed of change

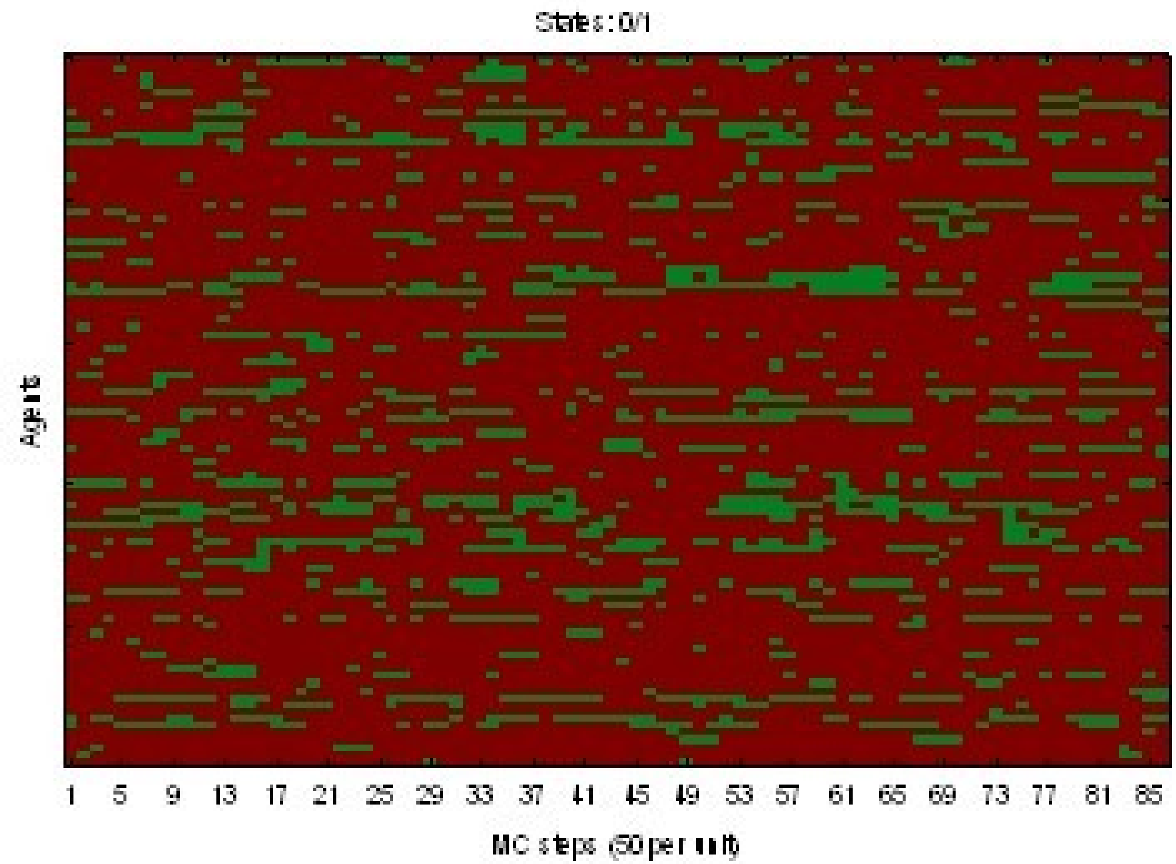




Oscilation of mean reputation in time for *z*=0,5 (left) and *z*=1(right)

ε=const, $x_w = z \cdot W'$

Spontaneous transitions between reputation's states (0/1) for z=1. „Positive” state 1 domitates.



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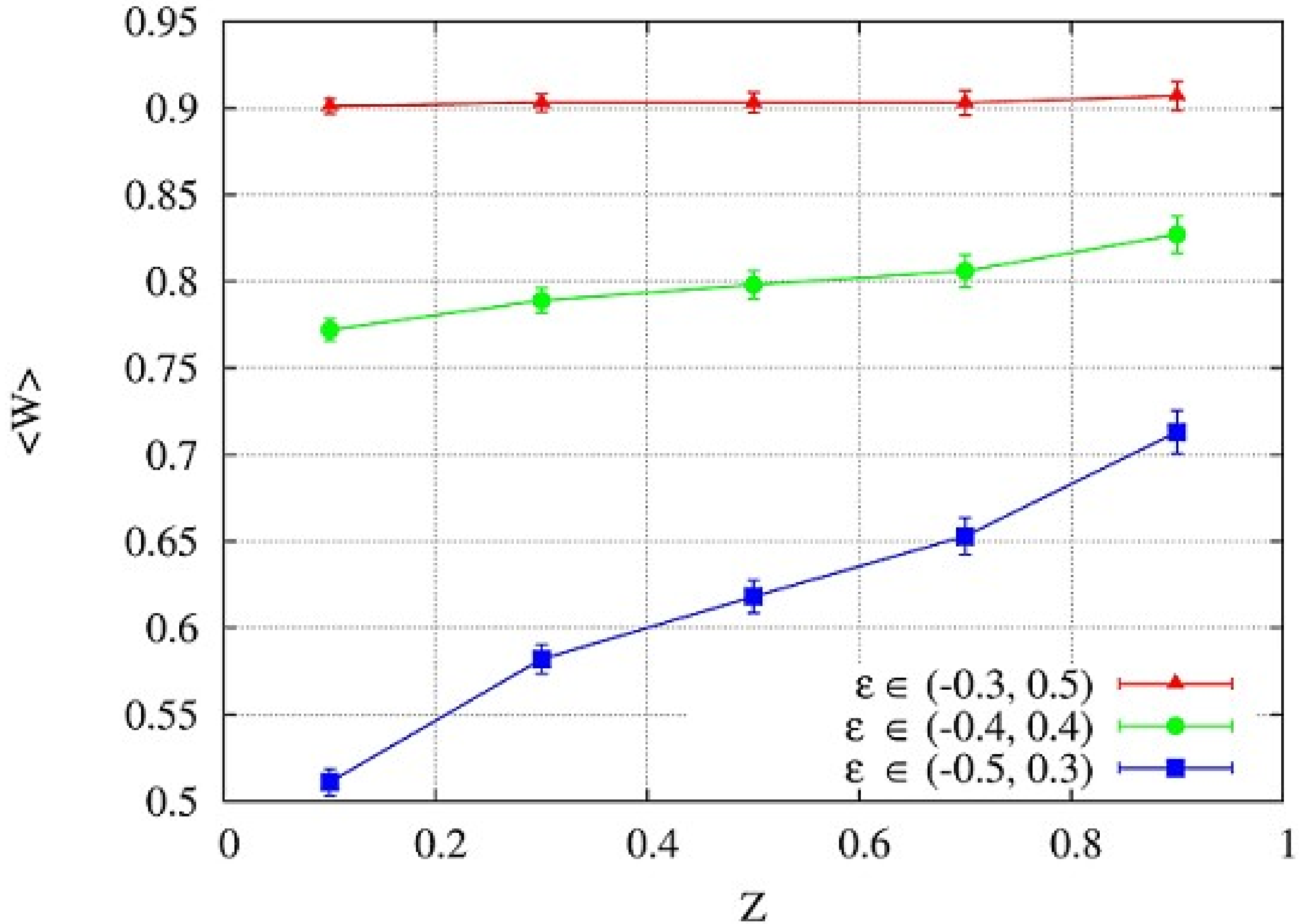
ε=const, x_w=z·W'



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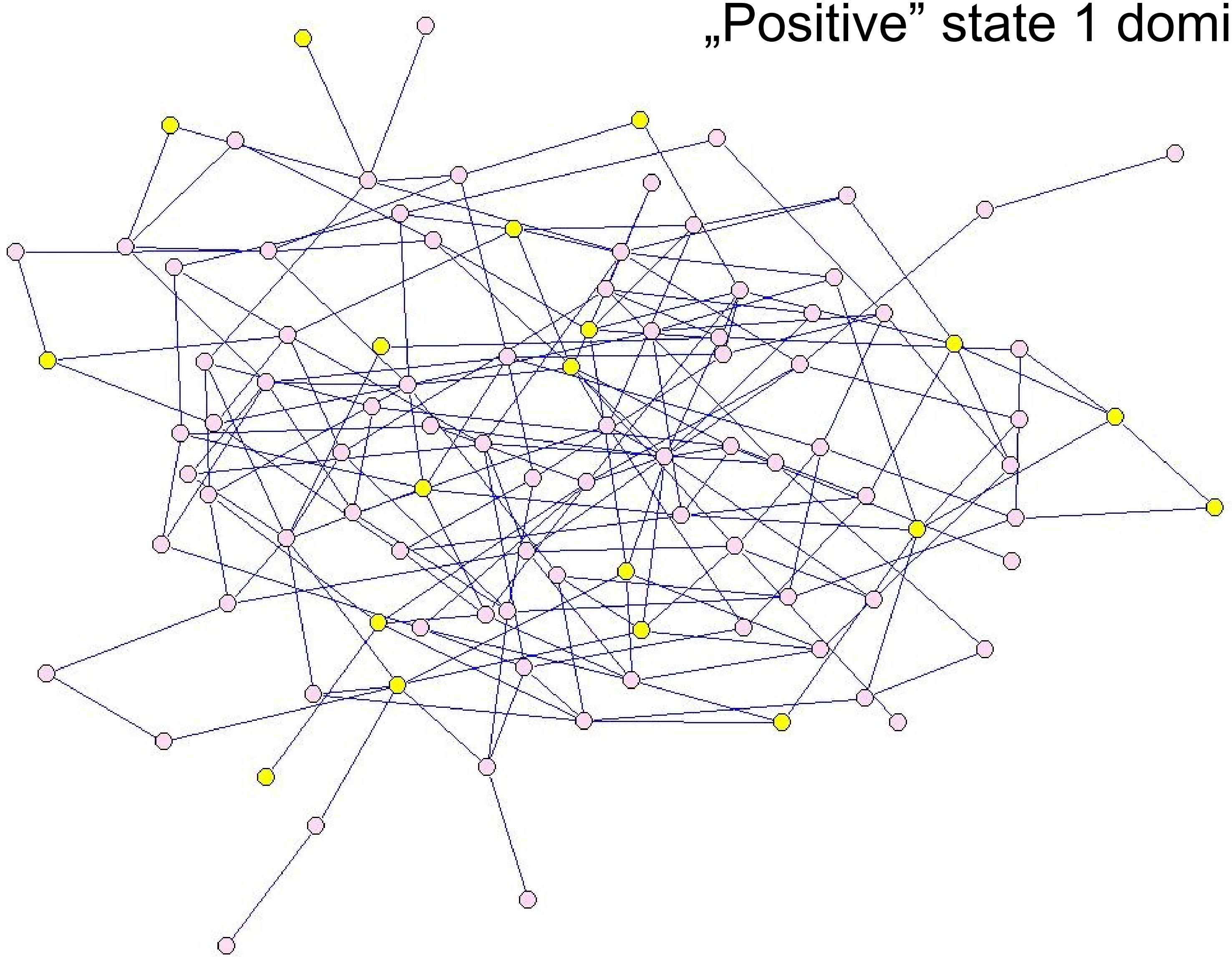
ε=const, $x_w = z \cdot W'$, E-R net

x_W=W' and ε=*const*	*W'* is reputation of co-player, so player's reputation change as quickly as *W'*	Symmetry broken – cooperation is more common, unstabile final state
E-R or lattice	Agents on special networks	Spatial correlations appear

ε=const, x_w=$z\cdot W'$, E-R net

„Positive" state 1 domitates

(by pajek)



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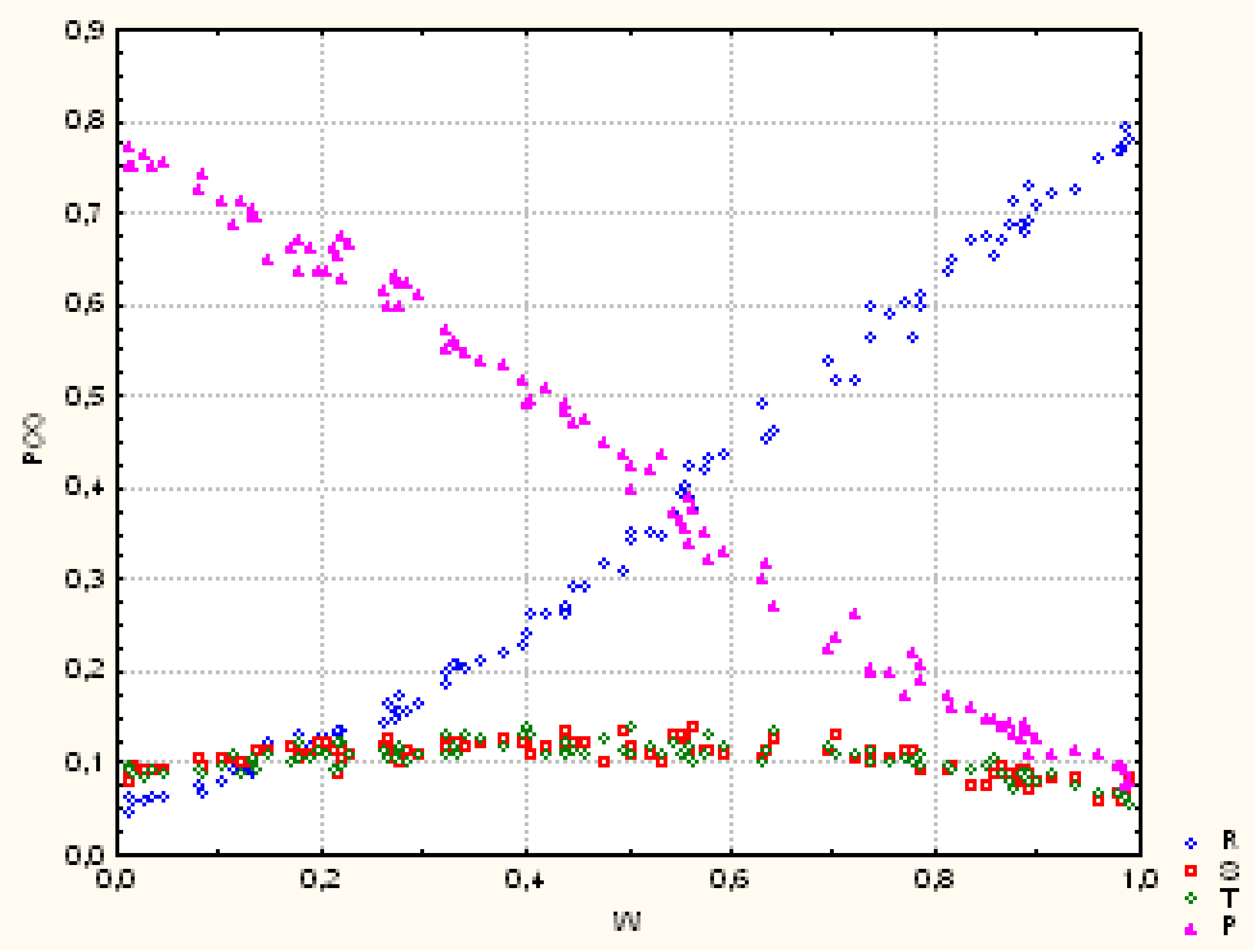
$x_\varepsilon \neq 0$ and $x_W = 0$

Group of cases with changing altruism starts with the simplest case x_W=0

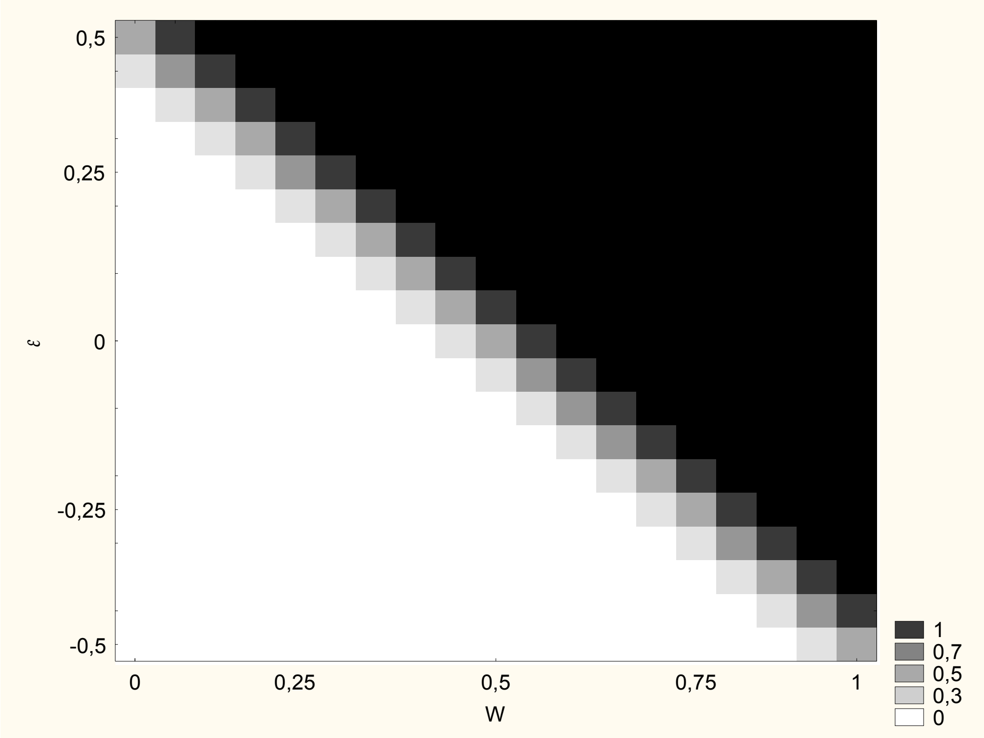


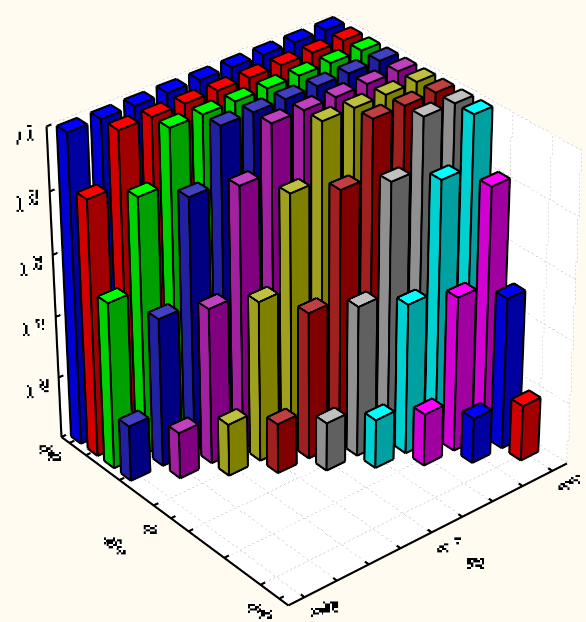
$x_\varepsilon \neq 0$ and $x_W = 0,5$

$x_W=0,5$ and $x_\varepsilon=0,5$	Reputation and altruism changes in „bisection” way the same time	All players choose only one strategy,and the initial state is divided into two attractors

$x_\varepsilon \neq 0$ and $x_W = 0,5$

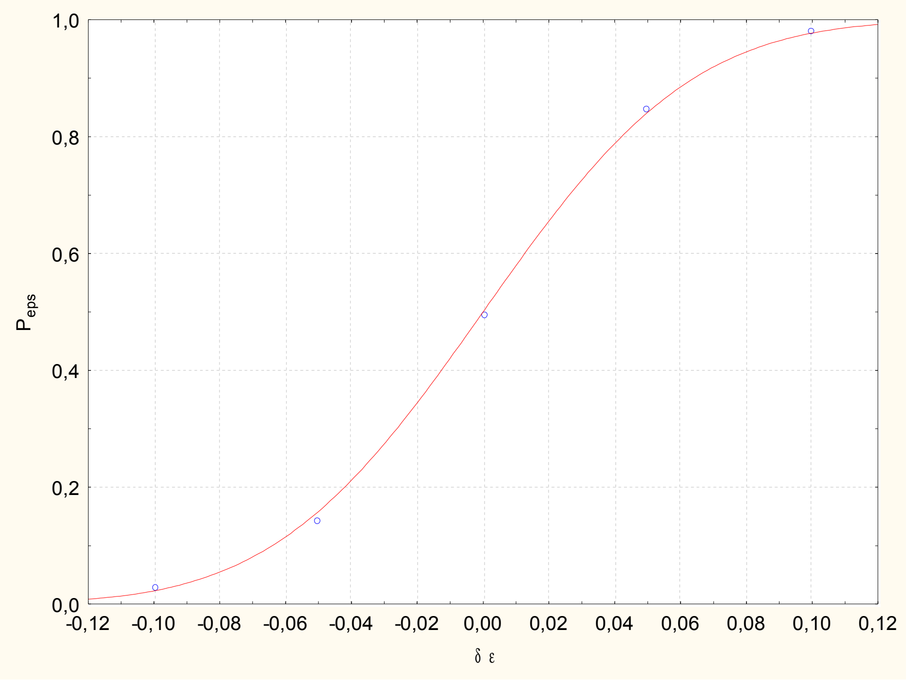
Let consider case where both x_W and x_ε are non-zero. For simplicity, let assume that both are equal to ½. Because of both parameters change we cannot present characteristics graphs from previous paragraphs where we could draw strategies versus constant vector of initial states *W* and ε.

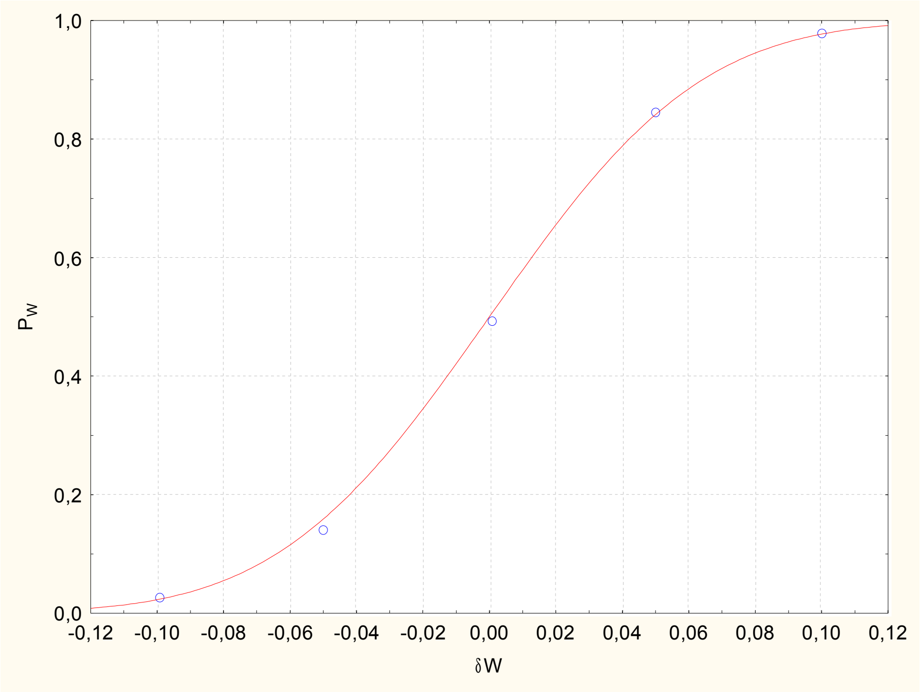




$x_\varepsilon \neq 0$ and $x_W = 0{,}5$

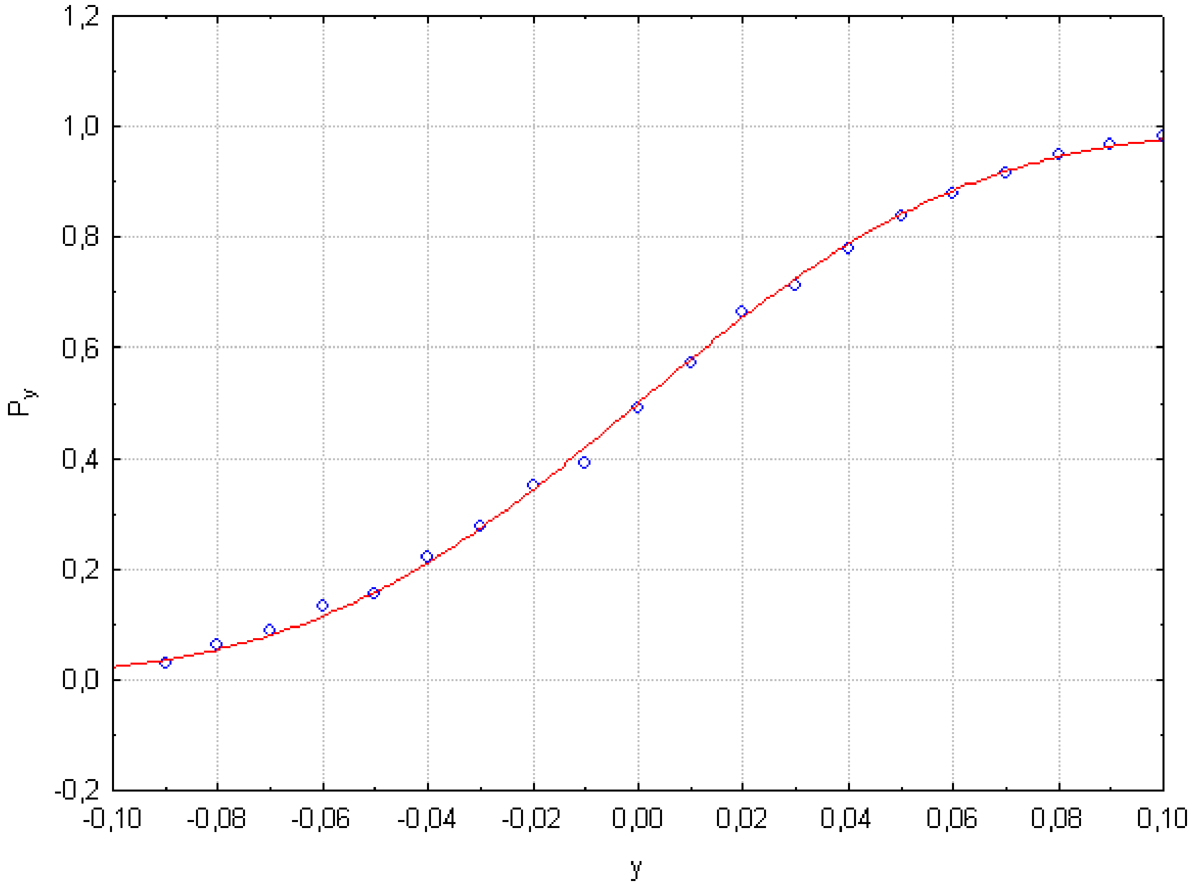
Gaussian cumulative distribution fit to probability of all cooperating





$x_\varepsilon \neq 0$ and $x_W = 0,5$

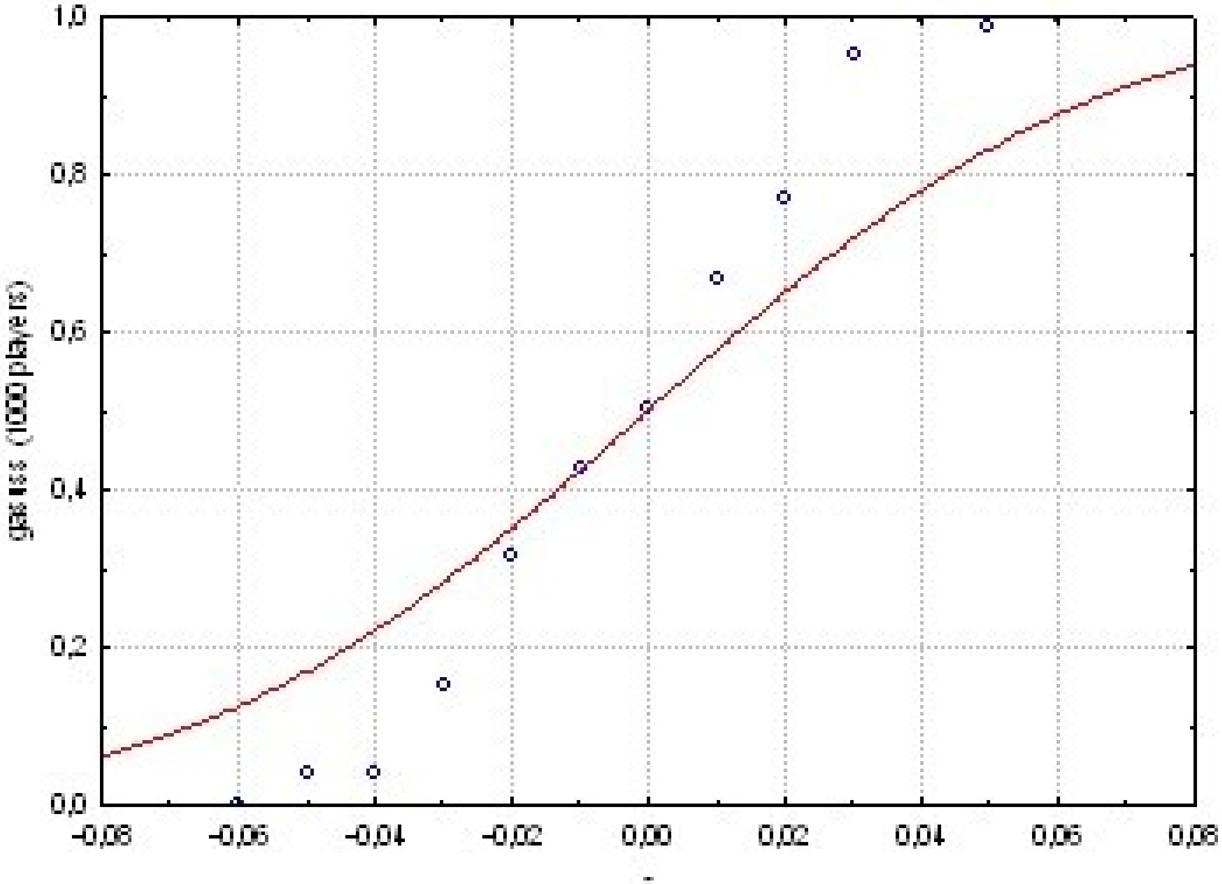
Stochastic condition: $1/2-3\sigma<(W+\varepsilon)<1/2+3\sigma$, where σ is a standard deviation of fitted CDF.



$$y = W + \varepsilon - 1/2$$

$x_\varepsilon \neq 0$ and $x_W = 0,5$

Size effect



$y = W + \varepsilon - 1/2$

$x_{\varepsilon} \neq 0$ and $x_W = 0,5$

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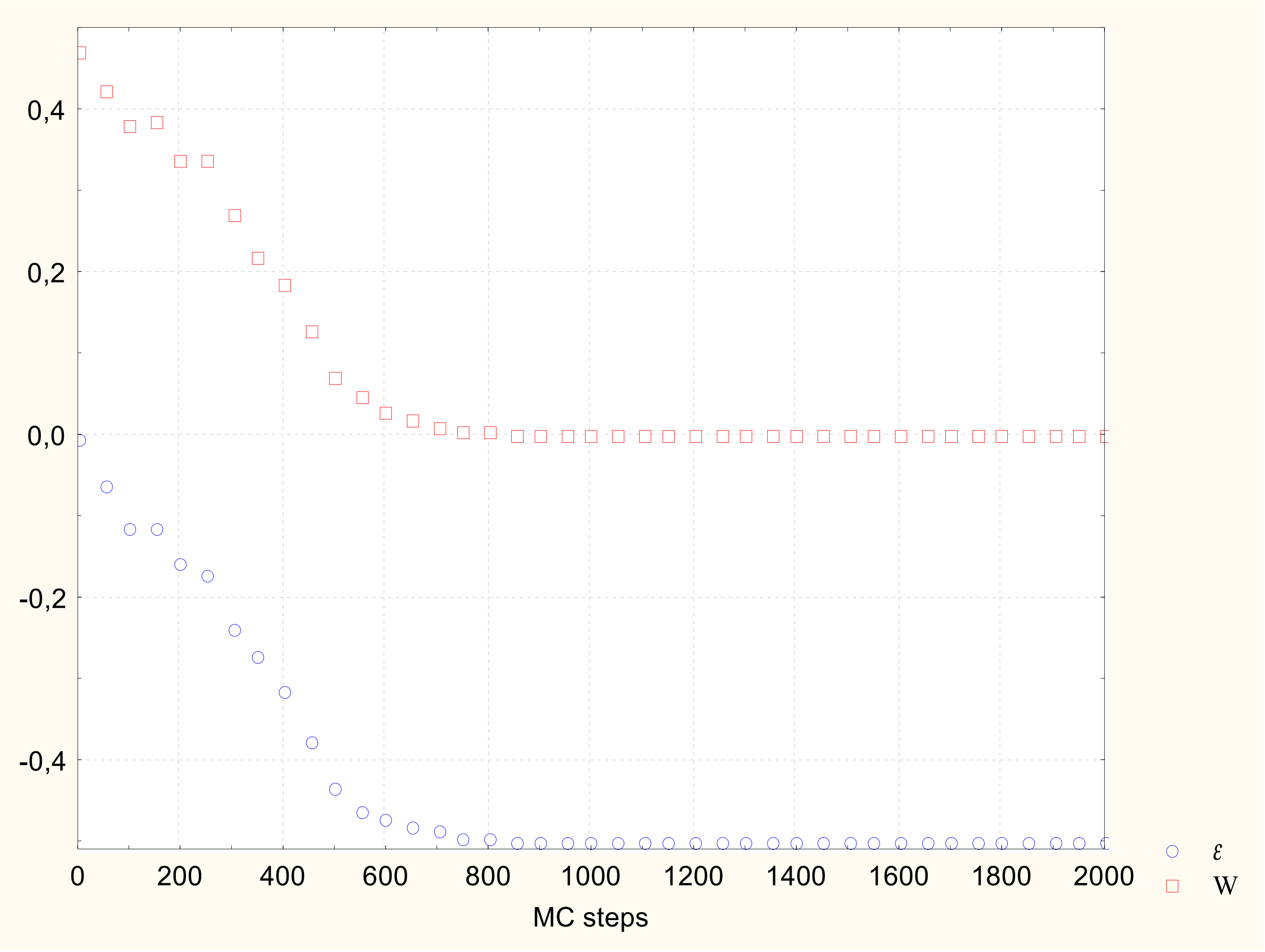
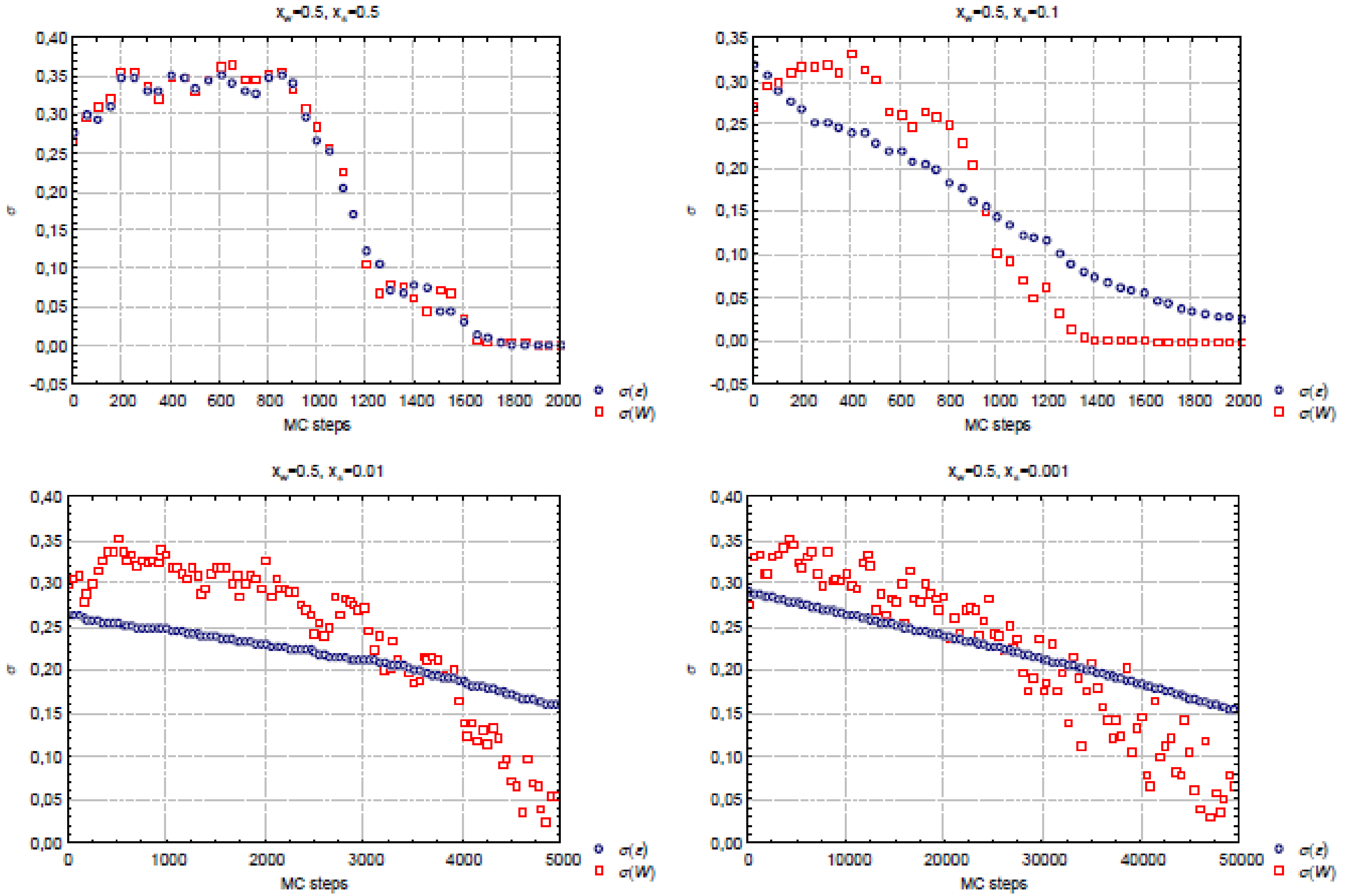
Convergence to final state



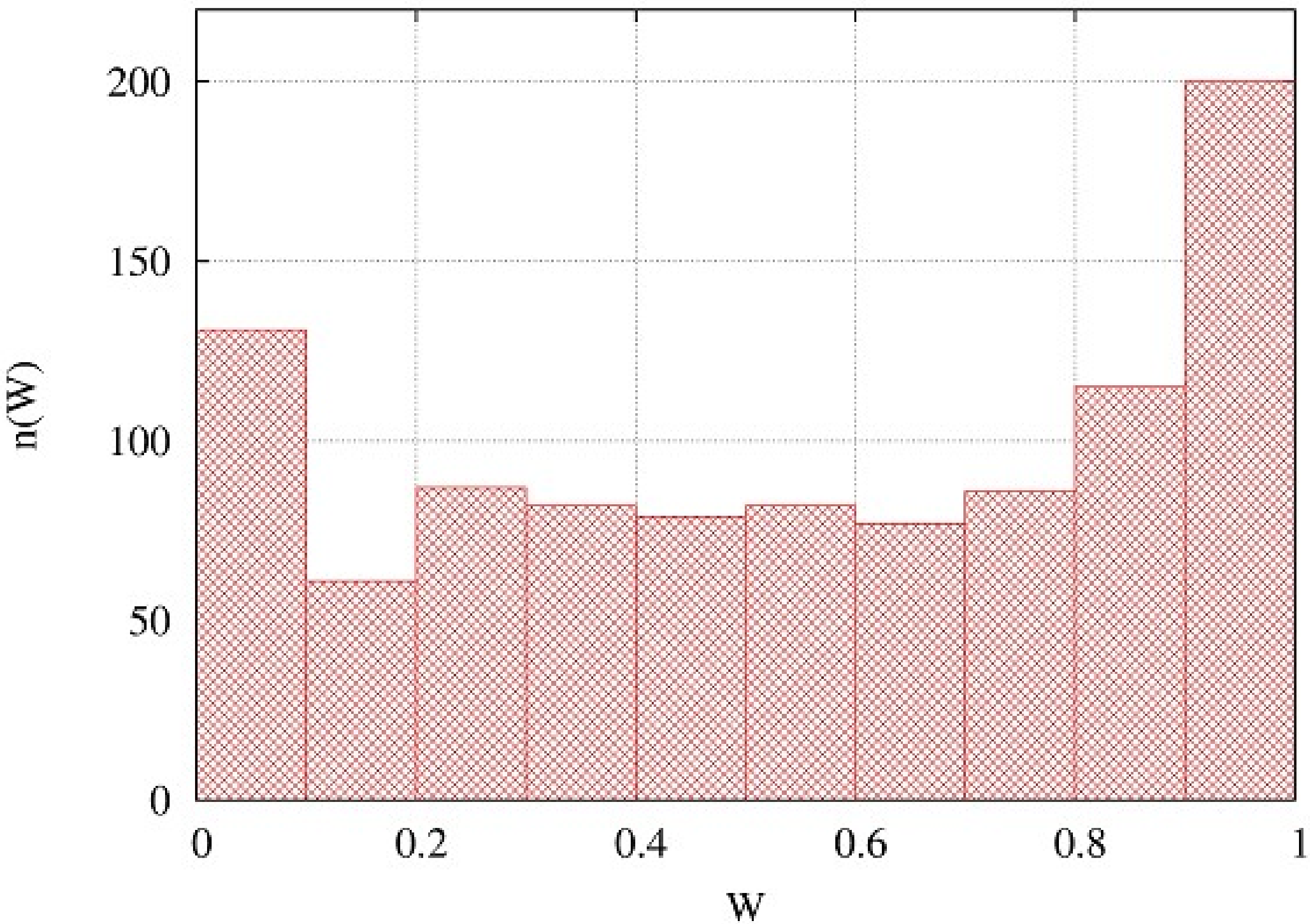
x_W=0,5 and x_ε={0,5;…;0,001}

Convergence to final state: different velocities



$x_{\varepsilon} \neq 0$

Initial dynamics



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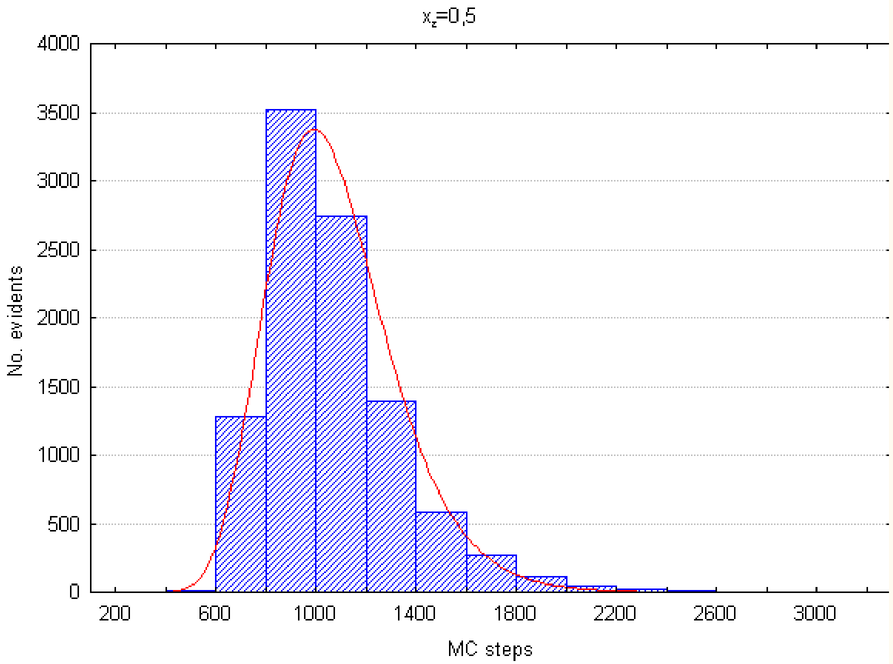
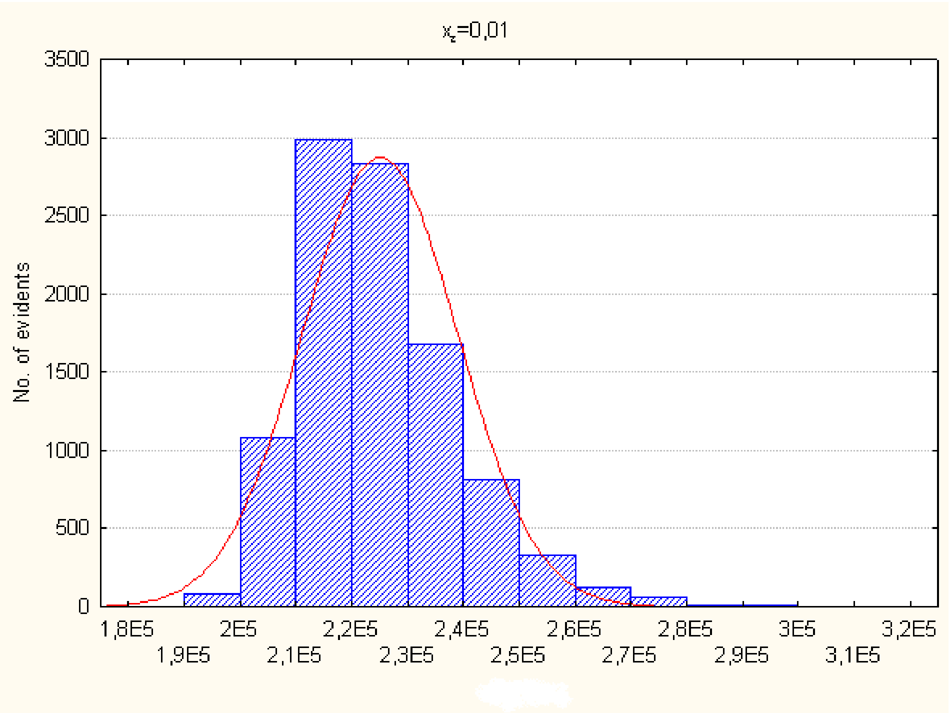
x_ε=0,5 or 0,01

Relaxation time



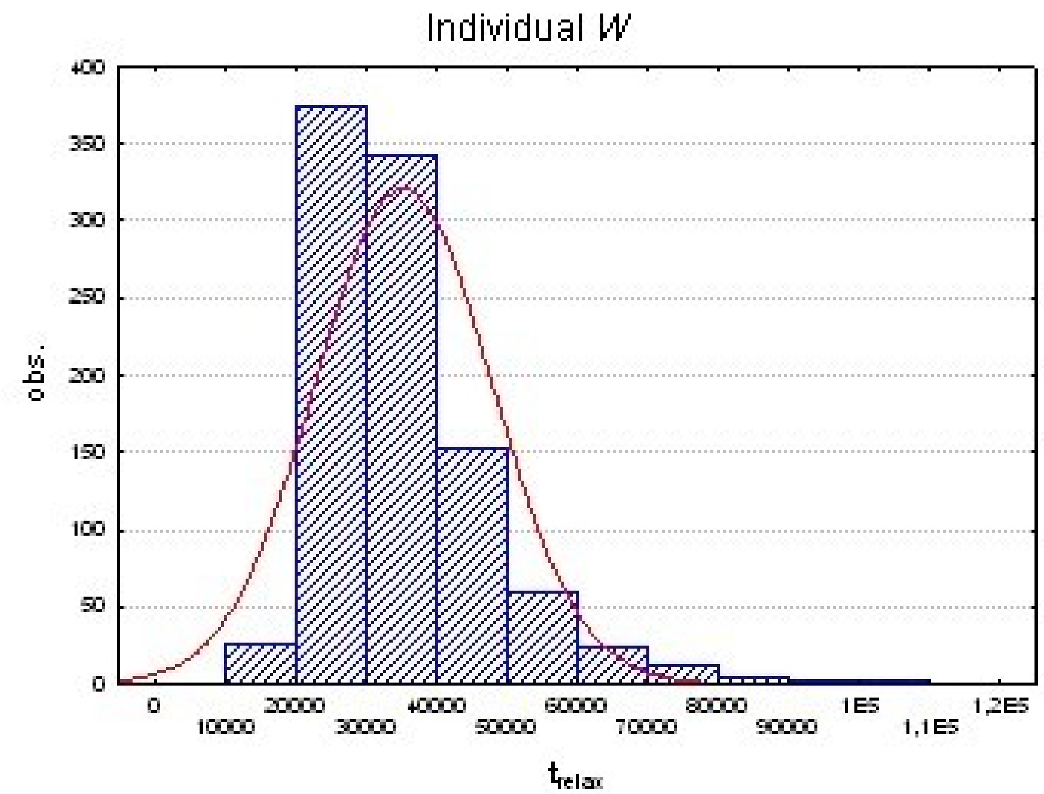


$x_\varepsilon \neq 0$, individual W

x_W=0,5 and x_ε=0,5	Reputation and altruism changes in „bisection” way the same time	All players choose only one strategy,and the initial state is divided into two attractors
$W_j(i)$	Individual vision of agent's W	System slows down

$x_\varepsilon \neq 0$, individual *W*

Let repeat, that reputation can be individual (every player i has his own vision of all *N*-1 other players)



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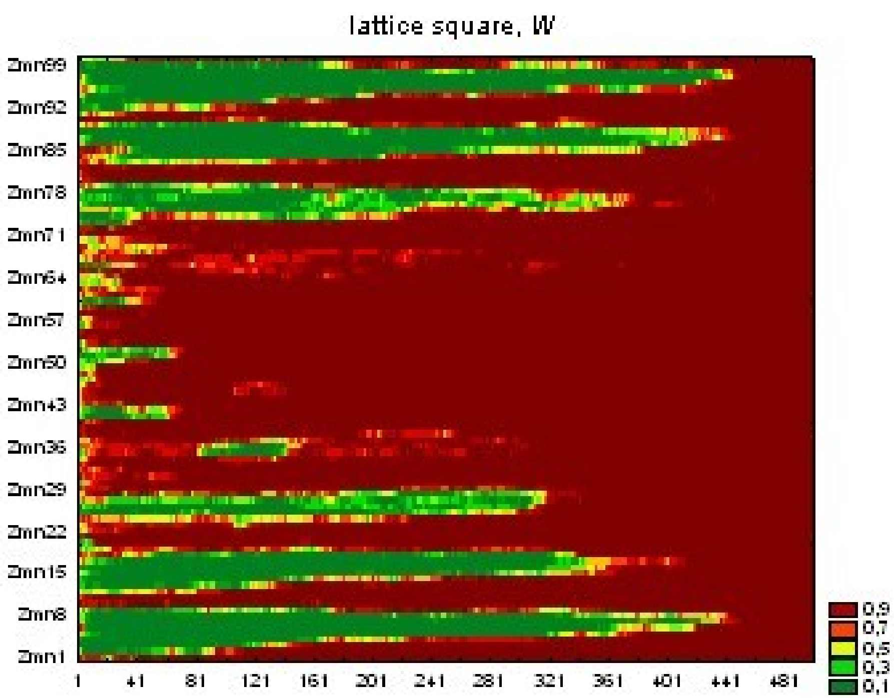
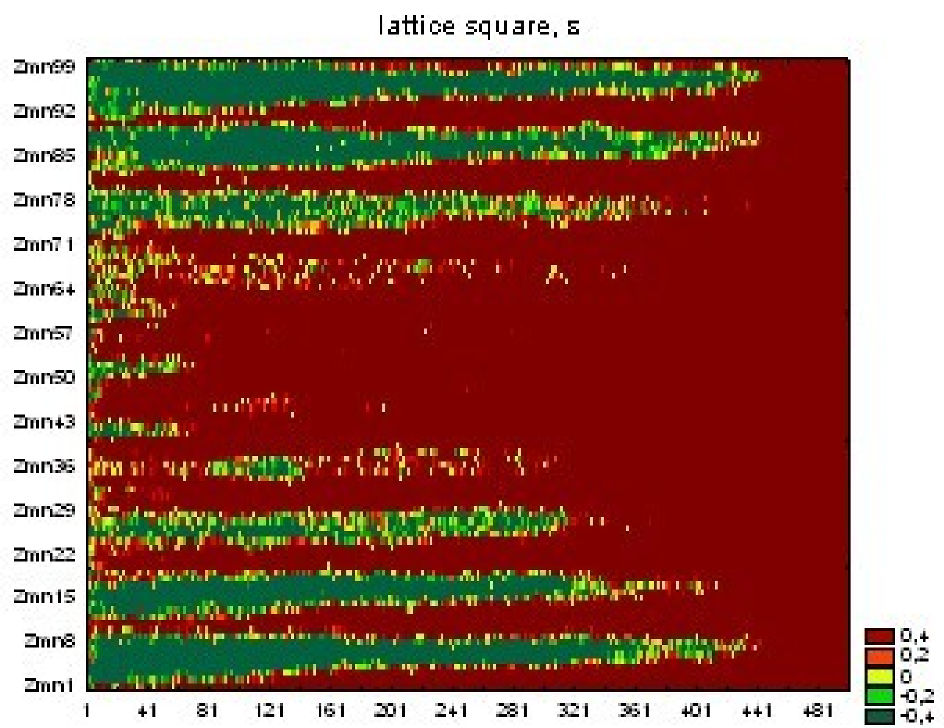
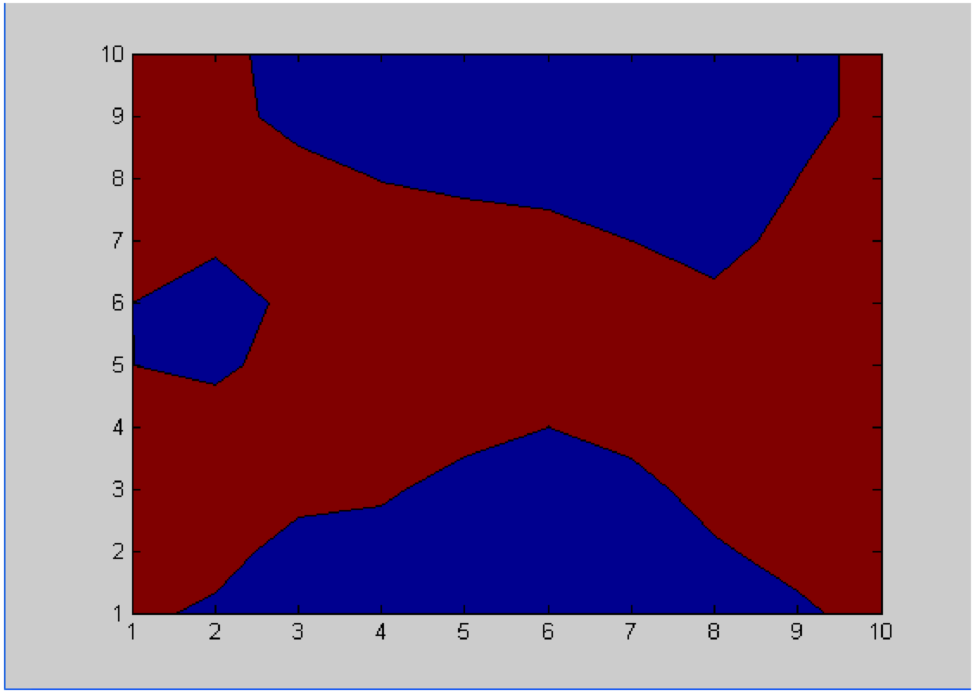
$x_\varepsilon \neq 0$, square lattice



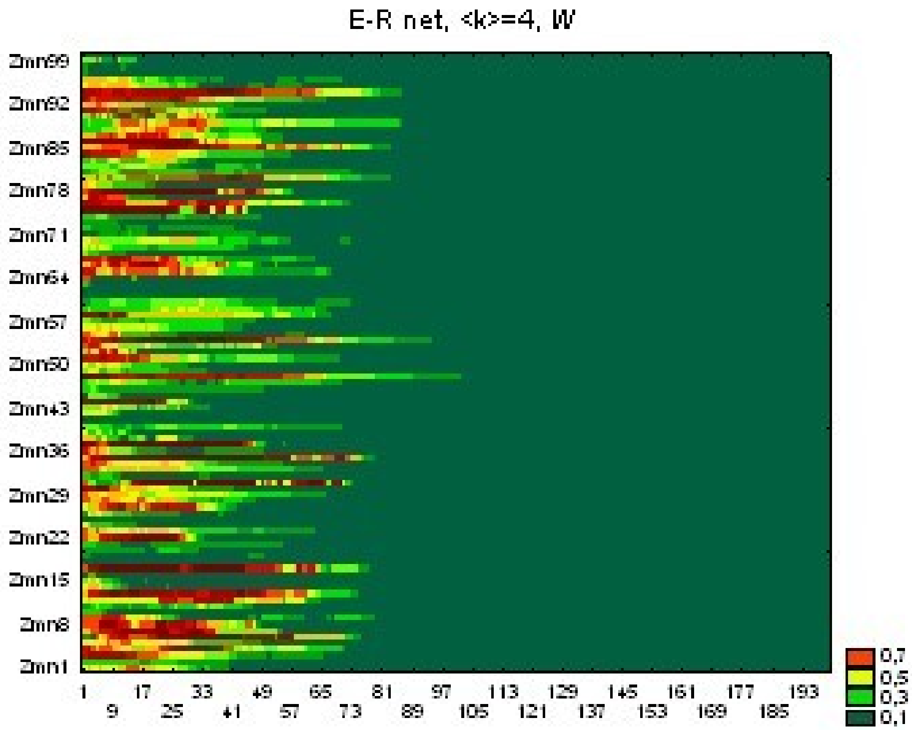


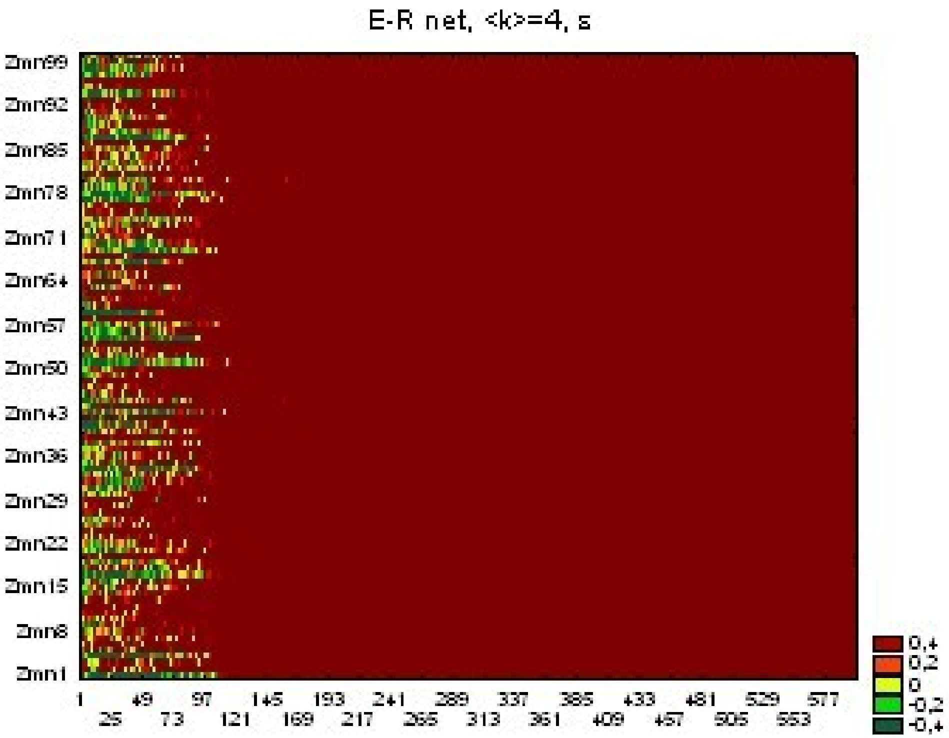
$x_\varepsilon \neq 0$, square lattice



$x_\varepsilon \neq 0$, E-R network

Erdős–Rényi network model with <k>=4 (proportional to lattice)





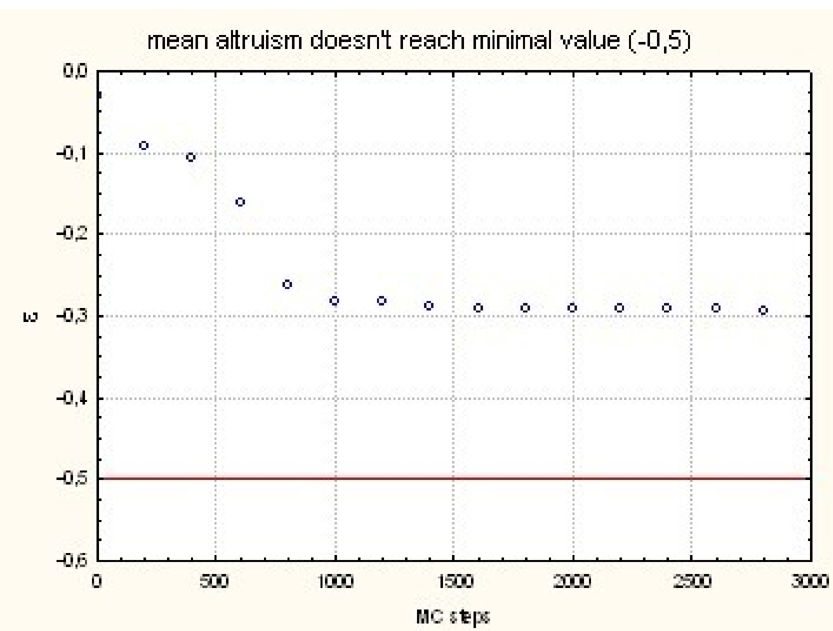
$x_\varepsilon \neq 0$, conditional change

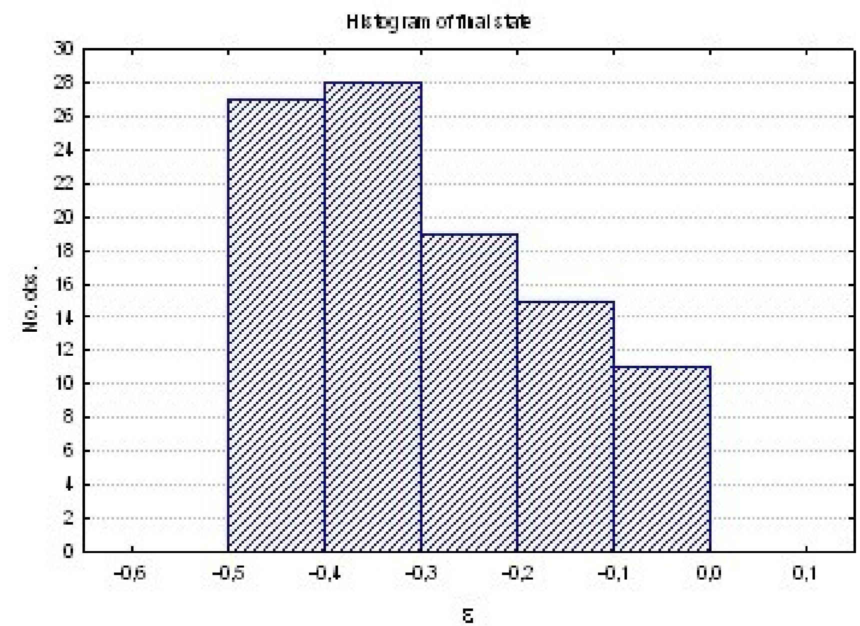
x_W=0,5 and x_ε={0,5;0}	Altrusim changes only in some cases: CC (goes up) and CD (goes down)	Symmetry broken – cooperation is more common, non-full negative state

$x_\varepsilon \neq 0$, conditional change

Let altrusim change only in some cases (not after every game):

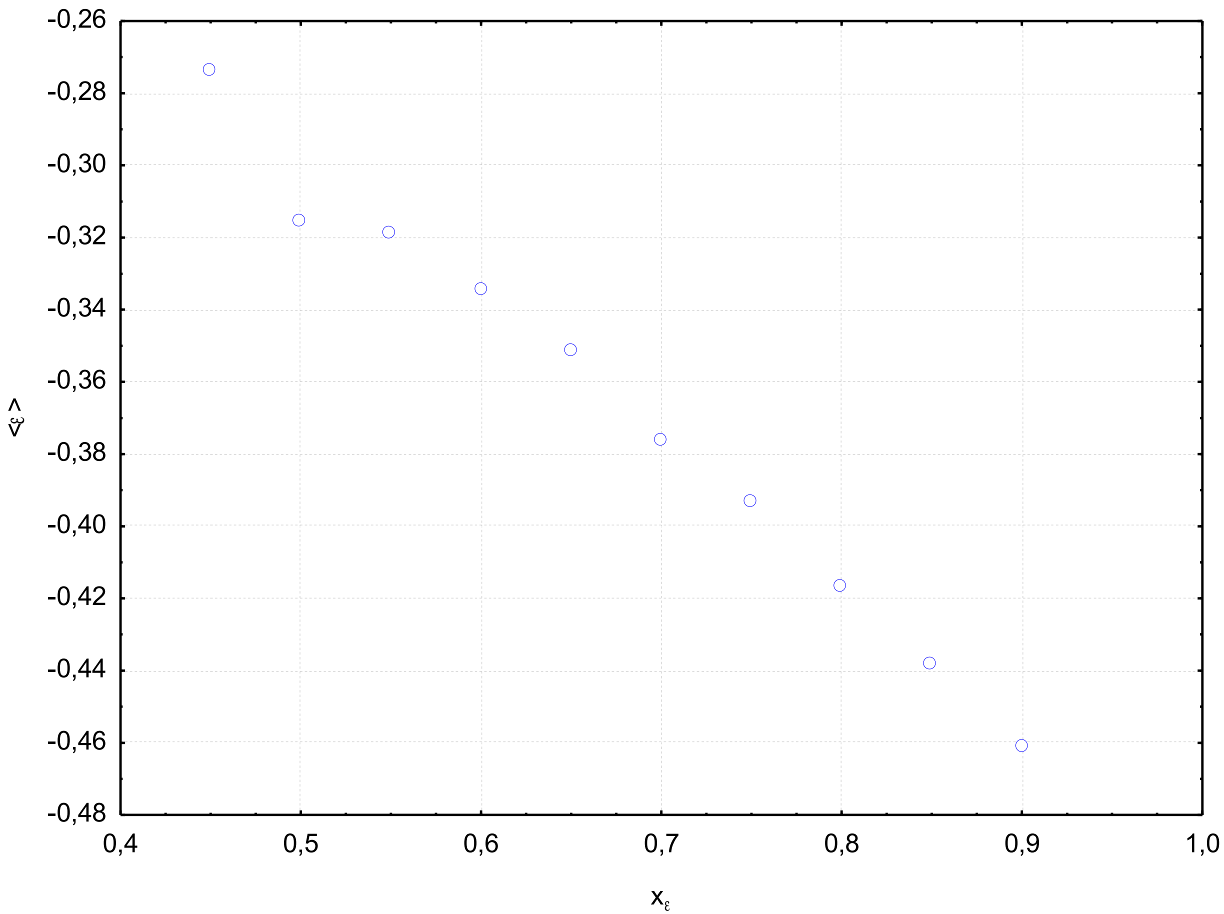
- mutual respect of two agents expressed by their cooperation enhances their selfevaluation, what in turn reinforces their willingness to cooperate;
- a cooperating agent is humiliated when mets a defection, what reduces his willingness to cooperate.





$x_\varepsilon \neq 0$

Final „negative" state depends on x_ε



However, probability of ending in „positive" state is around 0,66

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Conclusions

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Conclusions

- Once the altruism is allowed to evolve, in long time limit the simulated players adopt one strategy, the same for the whole population.
- Direction of the process is closer to real situatio rather than the stationary state in the long time limit

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- Sociotechnics and system controlling

- If symmetry of final state is broken – cooperation state is promoted! (social norm works)

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Conclusions

• Once the altruism is allowed to evolve, in long time limit the simulated players adopt one strategy, the same for the whole population.

• Dynamics of the process is closer to real situation rather than the stationary state in the long time limit

• Sociotechnics and system controlling

• If symmetry of final state is broken – cooperation state is promoted! (social norm works)

Thank You for Your attention!