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„The establishment of paternity" Hugo Steinhaus

In early 50's when DNA has not been discovered yet, scientists had a problem to establish paternity. Genetics gave some posibilities to serological exclusion of paternity by comparing blood types. Hugo Steinhaus tried to solve that problem giving indicators to court in those cases where the court calls an expert and obtains from him a report based on a serological test of the bood of the mother, the child and the man indicated by the mother. He showed a point of view of propabilty calculus in material offered by serological evedience, which schould help with judge's verdicts.

1) <u>Legal problem</u>

Steinhaus's papers were prepered in addiction to Polish Family Law of 1950. Nawdays some very important principles, e.g. mater *semper certa est*(conotation of *in vitro* method's) are not valid and other quot are changed like: „It is surmised that the child's father is the man who had sexual intercourse with the child's mother in the period from 300th to 180th day before its birth"(the assertion of this proof was called **E(X)**).
Steinhaus shall called more: **F(X)** – the fact, that man **X** sued by the mother **M** of the child **D** is the child's father.

2) <u>Blood Groups</u>

The blood group theory arose at begining of XX's century, so Steinhaus had data of three systems of blood groups {(A,B,AB,0),(Rh+,Rh-),(M,N)}.Moreover Rh classification was discover during The Second World War and the presented three methods were independent of one another. One of creators of (A,B,AB,0) is Polish, Ludwik Hirszfeld, who worked in Wrocław in the same time as Hugo S.
The application of the theory of blood groups to the establishment of paternity is based on the laws of inheriting blood characteristics.Thus, *for instance, each of the characteristics has the following property: it cannot appear in the child blood if it is found neither in the father's nor in the mother's blood.*

Table 1. Blood group inheritance for (A,B,AB,O) system

Mother/Father	**O**	**A**	**B**	**AB**
O	**O**	**O, A**	**O, B**	**A, B**
A	**O, A**	**O, A**	**O, A, B, AB**	**A, B, AB**
B	**O, B**	**O, A, B, AB**	**O, B**	**A, B, AB**
AB	**A, B**	**A, B, AB**	**A, B, AB**	**A, B, AB**

Generally, *if a serological test ascertains the absence of a characteristic **C** of the type **Z** in the blood of the child **D**'s mother and in the blood of the defendant **X**, and the presence of characteristic in **D**'s blood, it will prove **non-F(X)**.*
Statisctics showed that under 10% cases stated the exclusion of paternity and over 90% do not cathegorally say **F(X)** or **non-F(X)**.

3) <u>Propability of paternity</u>

Let **f** is a fraction of characteristic **C** for a given population. The frequencies of diffrent blood charcteristics were known in times when Steinhaus work wih this paper. He showed errors in people's thinking. Let consider:

(1) **M** has no **C**, **D** has **C**

(**f** = 0,05)

(2) **X** has **C**

„In view of (1) the probability of **F(X)** is 95%", but the answer is wrong.
In Poland in 50's there was 7000000 adults men, so if **X** has been chosen at random, **F(X)** is 1/350000 after (1) (2).

4) <u>The *a priori* probability of the fact **F(X)**</u>

Let **p** denote that fraction of the presumed fathers **X** who are actual fathers. The probability that **X** will be excluded by means of the test for characteristic **C** is **(1-f)**, the expected number of exclusion among **n** cases is equal to **n(1-f)(1-p)**. Steinhaus used the material of 1515 cases consists of 15 classes with its frequency **f**$_i$. The expected number of exclusions in whole polulation is:

(3)
$$g=(1-p)\sum n_i\cdot(1-f_i)$$

There g is experimental value of successful exclusion.

(4)
$$p=1-\frac{g}{\sum n_i\cdot(1-f_i)}$$

The **p** calculated by formula (4) gives a priori value of **F(X)** (71,3% in Poland in 1952).

5) <u>The *a posteriori* probability of the fact **F(X)**</u>

Steinhaus used Bayes' rule to calculate conditional probability **F(X)** (contrete values are given in brackets in situaction (1))

$$P(A/B)=\frac{P(B/A)P(A)}{\sum P(B/A_i)A_i}$$ (to remain)

The a priori probability of **F(X)** (**p**=71,3%)
The a priori probability of **non-F(X)** (**q**=1-**p**=28,7%)
The conditional probability that if **F(X)**, then **X** has **C** (**r**=100%)
The conditional probability that if **non-F(X)**, then **X** has **C** (**f**=5%)
The probability that if (1) and (2), then **F(X)** is:

$$P=\frac{p\cdot r}{p\cdot r+q\cdot f}$$

(5)
$$P=\frac{p}{p+f-p\cdot f}$$

6) <u>Aplication of formula (5) and (4)</u>

Since this paper the judges had had possibility to use mathematics in making decision. For the beginning the judge schould ascribe equal probability to both sides. It is clear that the proof of the assertion **E(X)** increases the probability of **F(X)** from 50% to 71,3%. E.g if (1) and (2) then **P**=98% so the pointer of balance has shifted to establish paternity. But it is still not certain fact. The judge schould use this information only as advise and gives verdict based on this education and experience.
Steinhaus asked philosophical question in addiction to problem of charging **X** with the cost of child's maintenance even there is a small chance that **X** is not father- how compare material true with legal true. He tried not to decrease authoritate of the judge, but only gave him some statistical

tool.
He discused some properties of **p**, which can be understand as a coefiction of women's truthful and depend on blood group's disrtibution of population.

7) <u>My conclusions</u>

I really like those papers, becouse there is very simple maths with great application to law and social science. This method gave people, who had only basic mathematics background (little statistics and logic) tool to use in their work. He learned reader's about some problems, which could appear, becuose of using this method. E.g. *Logic tells us that the statements „If R, then S" and „If non-S, then non-R" are equivalent, but „If R, then S with 5%" and „If non-S, then non-R with 5%" are not equivalent.* This paradox was only one from all Steinhaus's thougths.
Staihaus explained some other cases of using **P** and **p**. He showed how simple it is in situation (1) and (2). *Let us remember, however, that **P** can be calculated in all possible cases, even when both **M** and **D** have the characteristic **C**; the presence of the characterisic **C** in **X** increases **P** above **p**, while its absence lovers **P** below **p**.*
I was little disapointed with lack of solution of other situations, but there would be only addiction to this paper and I could think of them and do it by myself. Unfortunatlly this paper is not recent nowdays, when we can state or exclude paternity in over 99,9% by genetic test and problem of probability of serological estiblishment is not being developed any more. On the other hand I am really impressed observing Steinhaus as a humanist, who cared about people's fate and thougth in cathegory of social justice.

Bibliography:

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