Comment on "Holographic Thermalization, Stability of Anti–de Sitter Space, and the Fermi-Pasta-Ulam Paradox"

A recent interesting Letter [1] revisits the problem of stability of anti–de Sitter spacetime (AdS) under massless scalar perturbations, first studied in Ref. [2], and claims to "uncover a large class of spherically symmetric initial conditions that are close to AdS but whose numerical evolution does not result in black hole formation." This assertion is supported by two independent numerical computations of the evolution of these initial data (which comprise the small amplitude two-mode data with energy equally distributed among the modes and the specially constructed small "quasiperiodic" data): (i) the full dynamical evolution and (ii) the newly proposed two-time scale approximation. The aim of this Comment is to scrutinize these results in the case of two-mode data. Hereafter, we use the notation of and references to the equations of Ref. [1].

To verify computation (i), using our code (see Ref. [3] for the detailed description), we solved the system of equations (2)–(4) for the two-mode initial data (20) with $\kappa = 3/5$ and $\varepsilon = 0.09$ used in Fig. 3 in Ref. [1]. The comparison of our result with the one of Ref. [1] is shown in Fig. 1, which depicts the upper envelope of the quantity $\Pi^2(t, 0)$ (related linearly to the Ricci scalar at the origin).

Until the first local minimum at $t \approx 500$ the two curves stay together (although small discrepancies are seen to develop earlier). However, for later times the two curves begin to diverge. In particular, after the second local minimum we observe a rapid growth of the Ricci scalar at the origin and the formation of an apparent horizon at $t \approx 1080$, whereas the numerical solution of Ref. [1] remains bounded and enjoys a long (possibly infinite) lifetime.

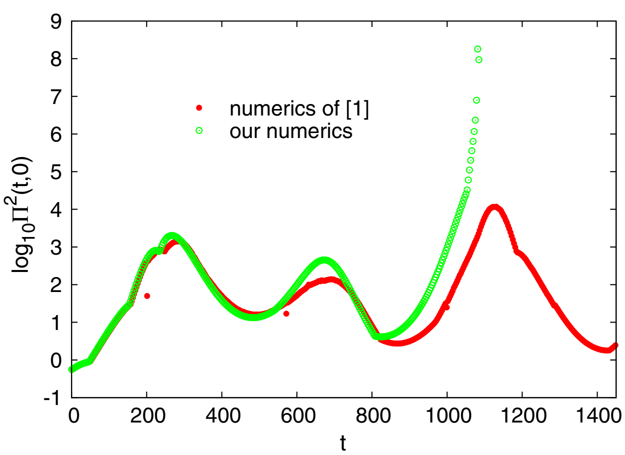

FIG. 1 (color online). The upper envelope of $\Pi^2(t, 0)$ for solutions starting from the two-mode initial data (20) with $\kappa = 3/5$ and $\varepsilon = 0.09$. Superimposed (red curve) is the numerical result of Ref. [1].

To feel confident that our computation is correct, we have validated it by convergence tests. The evidence for the expected fourth-order convergence is given in Fig. 2 in Ref. [4]. We stress that in numerical simulations of turbulent phenomena the convergence test is an indispensable tool of verifying whether small spatial scales are properly resolved. We suspect that the numerical solution depicted by the red curve in Fig. 1 suffered from the gradual loss of spatial resolution (presumably due to a too coarse grid or/and ineffective adaptive mesh refinement) and, consequently, the simulation stepped over the collapse and went off track. Unfortunately, the "visual" convergence test shown in Fig. 3 of the Supplemental Material in Ref. [1] was stopped much too early to spot the loss of resolution.

Concerning the two-time scale computation (ii) (which is a valuable result of Ref. [1] on its own) we wish to point out that it provides a good approximation to the true solution only for times $\lesssim \varepsilon^{-2}$ so it cannot be used to infer stability on longer time scales. Moreover, for the concrete implementation used in Ref. [1], for most (but not all) initial data this approximation cannot be even used to infer stability for times of order ε^{-2} because its time of validity is much shorter due to errors caused by the truncation of the number of modes at a relatively low cutoff $j_{\max}$. For quasiperiodic solutions the truncation error is negligible because their energy spectrum is very steep; however, for the solution shown in Fig. 1 over 500 modes are activated at the first maximum, hence the cutoff $j_{\max} = 47$ used in Ref. [1] does not suffice to capture the dynamics of the turbulent cascade (which *nota bene* is evident from Fig. 3 in Ref. [1]).

In conclusion, the claims about stability islands around AdS made in Ref. [1] seem premature.

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