Sub-Sharvin conductance in doped graphene nanosystems

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NARODOWE CENTRUM NAUKI

Where resistivity comes from in PERFECT nanosystems?

Time-energy uncertainty principle:

For given energy interval $\Delta E = e\,|\,V\,|$, time-of-flight cannot be shorter than $\Delta t \geqslant \hbar/(2\Delta E)$. Therefore, the current (per *quantum channel*) is limited by $I = e/\Delta t \leqslant 2(e^2/\hbar)\,|\,V\,|$, and the conductance $G = I/V \leqslant 2e^2/\hbar = (4\pi)e^2/h$ [— *not far from the conductance quantum* e^2/h …]

The above — requires a presence of **charge carriers** (and so ***propagating modes***)

In undoped graphene, conductance appears in the absence of charge carriers, due to *evanescent modes*
[$\sigma_0 = GL/W = 4e^2/(\pi h)$, $\mathscr{F} = 1/3$]

Takeaway message —

Usual mesoscopic contacts (*2DEG, breakable junctions etc.*) show so–called *Sharvin conductance*:

$$\frac{G}{g_0} \approx \frac{W}{\lambda_F/2} = \frac{k_F W}{\pi} \equiv G_{\text{Sharvin}} / \, g_0,$$

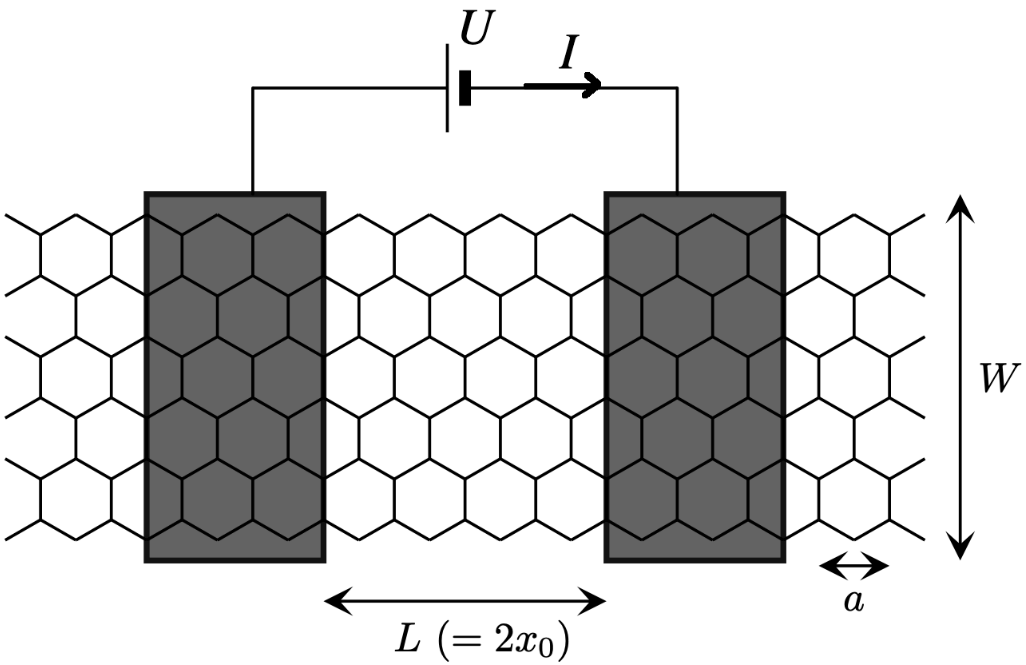
where g_0 is the conductance quantum [$g_0 = 2e^2/h$ in 2DEG or $4e^2/h$ in graphene], W is the constriction width, and λ_F (k_F) if the wavelength (wavenumber) for an electron at the Fermi level.

In *doped* graphene, we have:

$$G \approx (\pi/4) \, G_{\text{Sharvin}}.$$

Additionally, the **Fano factor** $\mathscr{F} \approx 1/8 > 0$.

Basic definitions (1)



The conductance:

$$G = \frac{I}{U} = \frac{\langle Q \rangle}{U \Delta t},$$

with $\langle Q \rangle$ being the average charge transferred during the time interval Δt upon a voltage difference $U = \left(\mu_L - \mu_R\right)/e$, and μ_L (μ_R) denotes the chemical potential in the left (right) reservoir.

[*Notation after*: Nazarov and Blanter, *Quantum transport: Introduction to Nanoscience.* Cambridge University Press, Cambridge, UK, 2009.]

Basic definitions (2)

Fano factor:

$$\mathscr{F} = \frac{\langle (Q - \langle Q \rangle)^2 \rangle}{\langle (Q - \langle Q \rangle)^2 \rangle_{\text{Poisson}}},$$

where the variance of charge transferred for a Poissonian process is

$$\langle (Q - \langle Q \rangle)^2 \rangle_{\text{Poisson}} = e \langle Q \rangle = eI\,\Delta t.$$

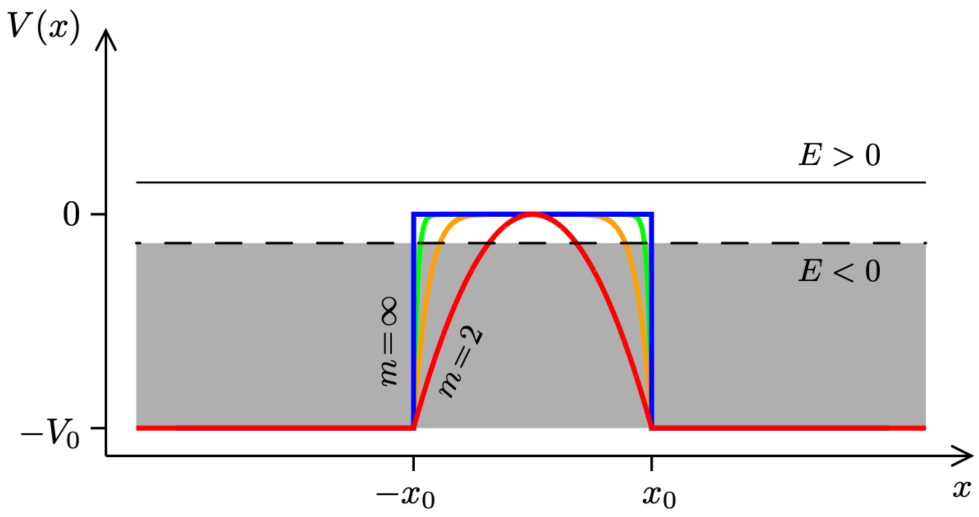
[*See*: Nazarov and Blanter, *Quantum transport: Introduction to Nanoscience.* Cambridge University Press, Cambridge, UK, 2009.]

Landauer–Büttiker approach

In the linear–response regime:

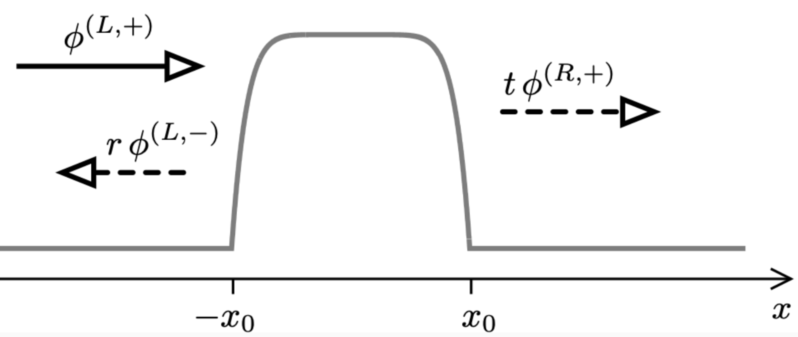
$$G = g_0 \sum_{n=0}^{N-1} T_n,$$

$$\mathscr{F} = \frac{\sum_{n=0}^{N-1} T_n(1-T_n)}{\sum_{n=0}^{N-1} T_n},$$

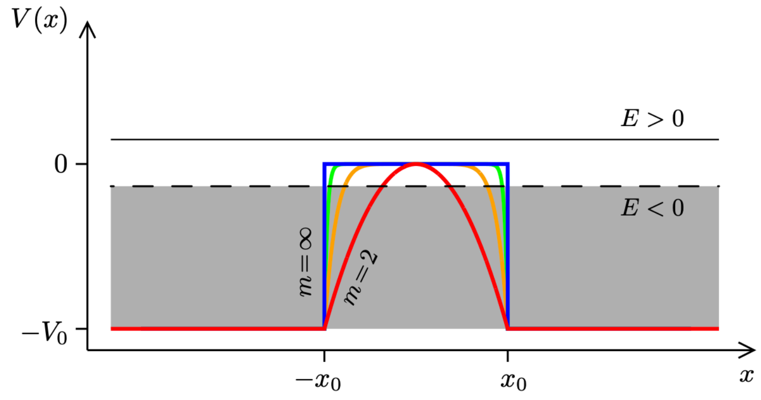


where T_n – transmission probability for n–th normal mode,

N – no. of normal modes in a selected lead.



The model



The electrostatic potential is choosen as

$$V(x) = -V_0 \times \begin{cases} \left| x/x_0 \right|^m & \text{if } |x| \leqslant x_0 \\ 1 & \text{if } |x| > x_0, \end{cases}$$

with the limit $m \to \infty, V_0 \to \infty$ corresponding to the rectangular barrier studied in earlier works [*Katsnelson, 2006; Tworzydło, 2006*].

The Dirac equation, $\left[v_F\, \mathbf{p} \cdot \sigma + V(x)\right] \Psi = E\Psi,$ with $\Psi = \phi(x) e^{i k_y y}$, $\phi(x) = (\phi_a, \phi_b)^T$, [*mass confinement:* $k_y = \pi(n+\frac{1}{2})/W$], brought us to:

$$\phi_a' = k_y \phi_a + i \frac{E - V(x)}{\hbar v_F} \phi_b, \qquad \phi_b' = i \frac{E - V(x)}{\hbar v_F} \phi_a - k_y \phi_b \,.$$

The rectangular barrier case (1)

In the $m \to \infty, V_0 \to \infty$ limit, analytical considerations lead to:

$$T_{k_y}(E) = \left[1 + \left(\frac{k_y}{\varkappa} \right)^2 \sin^2(\varkappa L) \right]^{-1},$$

where $$\varkappa = \begin{cases} \sqrt{k_F^2 - k_y^2}, & \text{for } |k_y| \leqslant k_F, \\ i\sqrt{k_y^2 - k_F^2}, & \text{for } |k_y| > k_F, \end{cases}$$

and the Fermi wavenumber $k_F = |E|/(\hbar v_F)$.

The rectangular barrier case (2)

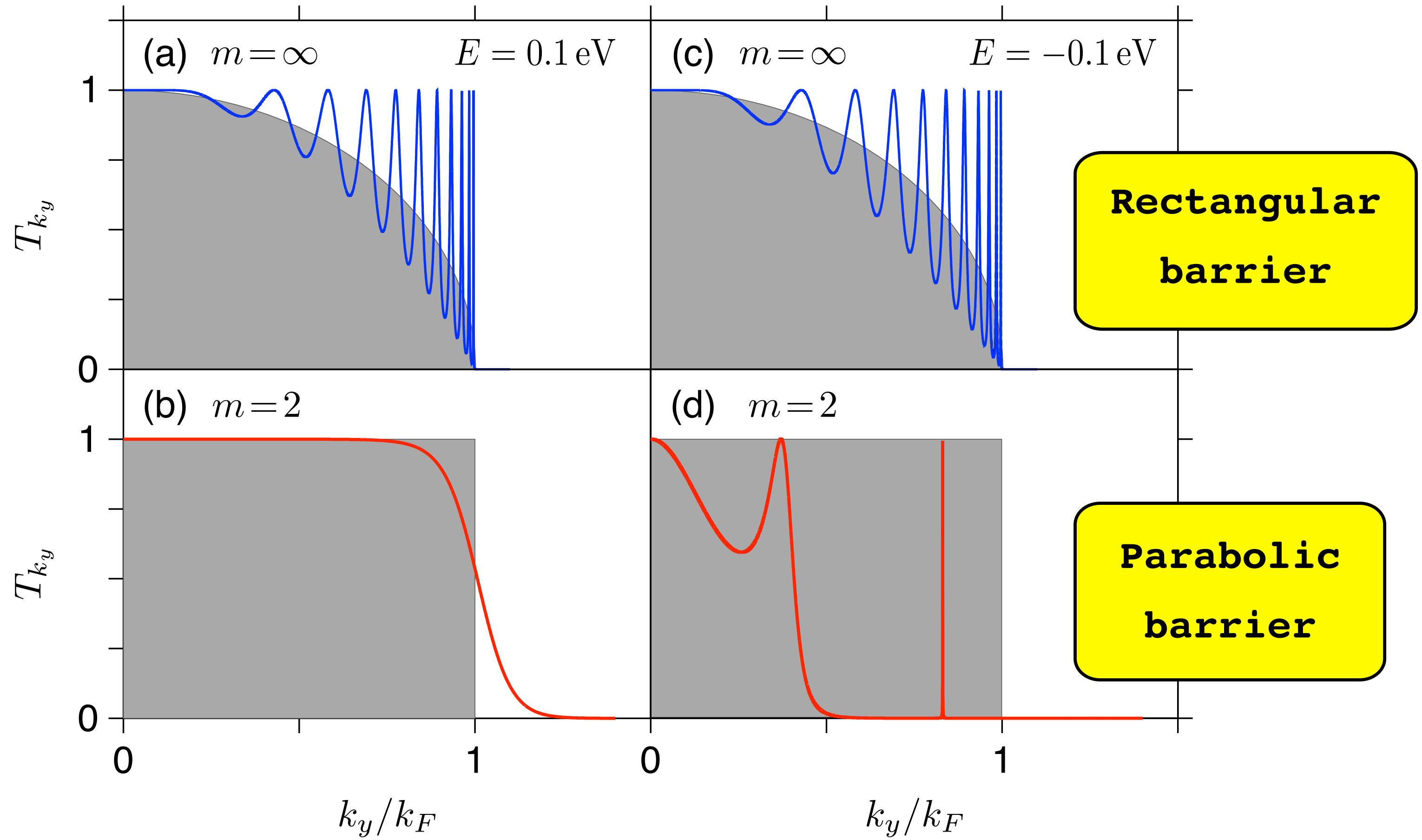
For a doped sample $k_F L \gg 1$, and – for $|k_y| \leqslant k_F$ – fast oscillations in $T_{k_y}(E)$ can be approximated by an average, namely

$$\left(T_{k_y}\right)_{\text{approx}} = \frac{1}{\pi}\int_0^{\pi} \frac{d\varphi}{1 + \left(k_y^2/\varkappa^2\right)\sin^2\varphi} = \sqrt{1 - \left(k_y/k_F\right)^2},$$

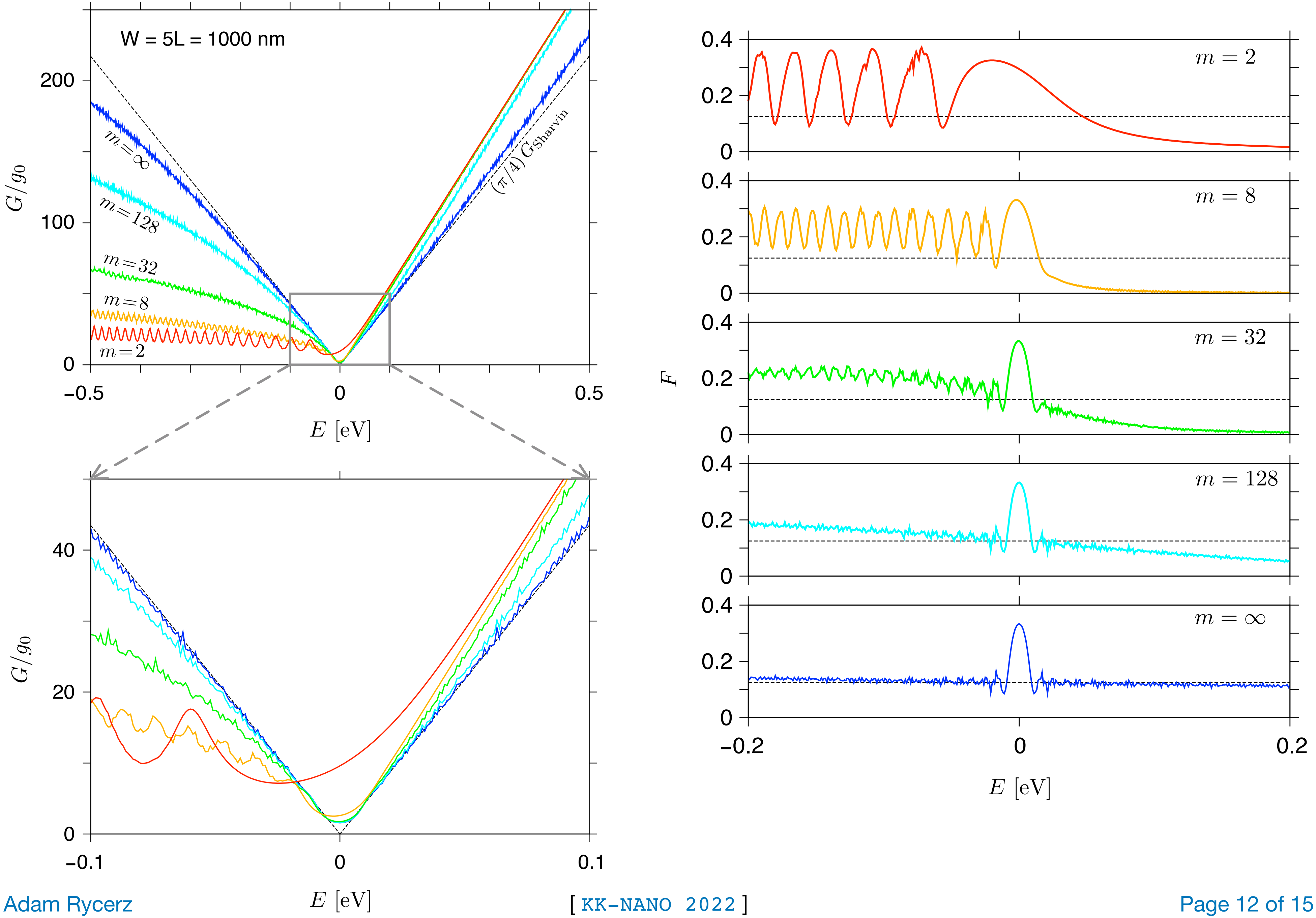
without affecting $G = g_0 \sum_n T_n \approx \frac{g_0 W}{\pi}\int dk_y T_{k_y}(E)$ [*Approx. of a sum by integral reffers to* $W \gg L$]. For $|k_y| > k_F$, $\left(T_{k_y}\right)_{\text{approx}} = 0$.

In turn, $G \approx \frac{g_0 W}{\pi}\int_0^{\infty} dk_y \left(T_{k_y}\right)_{\text{approx}} = \frac{\pi}{4} G_{\text{Sharvin}}\,.$

The rectangular barrier *vs* Sharvin contact

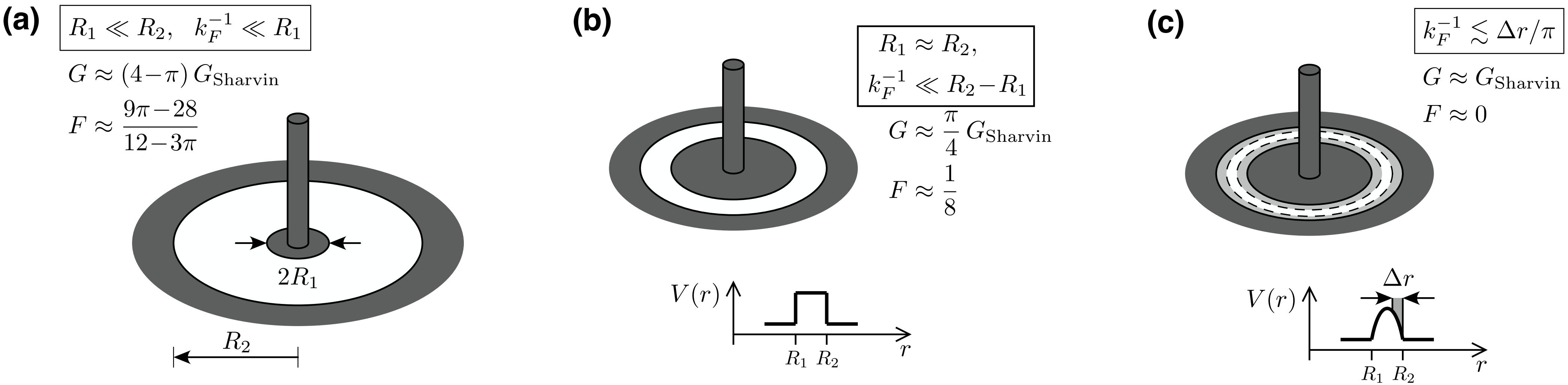


Numerical integration was performed for $V_0 = 1.35\,\mathrm{eV}$, $L = 200\,\mathrm{nm}$.



W = 5L = 1000 nm
G/g₀
200
100
0
−0.5
0
0.5
E [eV]
m=∞
m=128
m=32
m=8
m=2
(π/4) G_Sharvin
40
20
−0.1
0.1
F
0.4
0.2
m = 2
m = 8
m = 32
m = 128
m = ∞
−0.2

The Corbino geometry



Summary

For $E > 0$ (*electron* doping, no *p–n* junctions) parabolic potential reproduces *Sharvin* transport; increasing *m* leads to crossover to the **sub–Sharvin regime** [$G \approx (\pi/4)\, G_{\rm Sharvin}$, $\mathscr{F} \approx 1/8$].

For $E < 0$ (*hole doping*, two *p–n* interfaces in series) the conductance is strongly suppressed; sub–Sharvin regime (and the spectrum symmetry upon $E \leftrightarrow -E$) is gradually restored with increasing *m*.

In the Corbino geometry, one interface is effectively removed for $R_2 \gg R_1$, leading to *intermediate* values of $G \approx (4-\pi)\, G_{\rm Sharvin}$, $\mathscr{F} \approx (9\pi - 28)/(12 - 3\pi) \approx 0.1065 < 1/8$ even for $m \rightarrow \infty$.

Related works:

Paraoanu, *New J. Phys.* **23**, 043027 (2021).
Silvestrov and Efetov, *Phys. Rev. Lett.* **98**, 016802 (2007).
Cayssol, Huard, Goldhaber–Gordon, *Phys. Rev. B* **79**, 075428 (2009).

Acknowledgment

The work was supported by the National Science Centre of Poland (NCN) via Grant No. 2014/14/E/ST3/00256.

THANK YOU!