Magnetoconductance of the Corbino disk in bilayer graphene

Grzegorz Rut and Adam Rycerz

Jagiellonian University, Reymonta 4, PL-30059 Krakow, Poland

grzegorz.rut@uj.edu.pl and rycerz@th.if.uj.edu.pl

Foundation for Polish Science

INNOVATIVE ECONOMY NATIONAL COHESION STRATEGY

EUROPEAN UNION EUROPEAN REGIONAL DEVELOPMENT FUND

Quantum transport in an impurity-free Corbino disk in bilayer graphene (BLG) pierced by magnetic flux is analytically investigated by mode-matching method. As in the monolayer, conductance oscillations are present at pseudodiffusive regions. At the Dirac point the oscillation aplitude highly depends on the interaction between the layers as well as the ratio of outer and inner radii, yet the period remains the same as in the monolayer. At higher Landau levels or at large potential difference between the layers oscillations per valley behave the same way as in a monolayer. A comparison with a standard 2DEG Corbino disk is provided.

Model

The Corbino disk can be characterized by inner R_i and outer R_o radii. In the discussed system the ring area is made of weakly-doped bilayer graphene whereas metallic contacts are modelled with heavily-doped BLG. We assume that the magnetic field pierces only the ring. Since the analyzed system posseses a polar symmetry, the corresponding Hamiltonian commutes with the angular momentum operator $J_z = -i\hbar\partial_\varphi + \hbar\sigma_z/2$, thus we can take wavefunctions as products of angular and radial parts $\psi(r,\varphi) = e^{ij\varphi}\left(\phi_1(r), ie^{-i\varphi}\phi_2(r), \phi_3(r), ie^{i\varphi}\phi_4(r)\right)^T$, where $j = 0, \pm 1, \pm 2, \ldots$ We choose a symmetrical vector potential $\vec{A} = [-\sin(\varphi), \cos(\varphi)]^T rB/2$ ($B = 0$ outside $0 < x < L$), thus the gauge invariant momentum equals $\vec{\pi}/v_F = \left(-i\hbar\partial_{\vec{r}} + e\vec{A}\right)$.

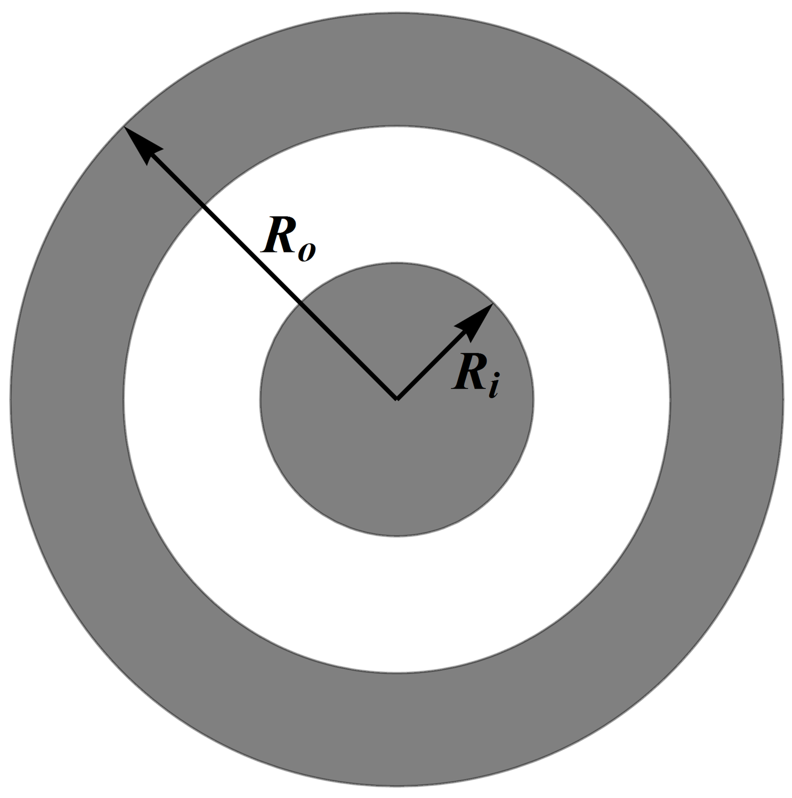

Figure 1: *Weakly doped bilayer graphene ring characterized with inner R_i and outer radii R_o between metallic contacts.*

For the K valley the 4-band Hamiltonian of such a system reads

$$H = \begin{pmatrix} U_1(x) & \pi_x + i\pi_y & t_\perp & 0 \\ \pi_x - i\pi_y & U_1(x) & 0 & 0 \\ t_\perp & 0 & U_2(x) & \pi_x - i\pi_y \\ 0 & 0 & \pi_x + i\pi_y & U_2(x) \end{pmatrix}, \qquad (1)$$

where $t_\perp \approx 0.38\,\text{eV}$ is the interlayer nearest-neighbour hopping energy, and $v_F \approx c/300$ is the Fermi velocity in a monolayer. $U_j(x)$ (with $j = 1,2$ the layer index) is an electrostatic potential energy

$$U_j(x) = \begin{cases} U_\infty & \text{if } x < R_i \text{ or } x > R_o, \\ \gamma_j V/2 & \text{if } R_i < x < R_o, \end{cases} \qquad (2)$$

where V is the potentials difference between the layers and $\gamma_j = (-1)^j$.

The conductance is derived by the Landauer-Büttiker formula

$$G = G_0 \mathrm{Tr}\, T, \qquad (3)$$

where $G_0 \equiv g_0 = e^2/h$, $T = t^\dagger t$ and t is a block-diagonal matrix with each block corresponding to different transmission mode (we presume that these modes do not mix). In order to derive a possibly general solution we keep a bias potential between the layers. Since the spin degree of freedom does not play an important role in the QRCE, we neglect the Zeeman effect.

Quantum relativistic Corbino effect

Transmission probability through the analysed system can be retrieved from the solution of Eq. (4) and mode matching analysis. Not surprisingly, at the Dirac point as in a case when there is no magnetic field, two transmission peaks emerge

$$T = \cosh^{-2}\left[\mathcal{L}(j \pm \mathcal{A} + \phi_D/\phi_0)\right], \qquad (4)$$

where $\phi_D = \pi\left(R_o^2 - R_i^2\right)B$ is the flux piercing the ring, $\phi_0 = 2(h/e)\mathcal{L}$, $\mathcal{L} = \ln(R_o/R_i)$, $\mathcal{A} = (2\mathcal{L})^{-1}\ln\left[\Upsilon/2 - \sqrt{(\Upsilon/2)^2 - 1}\right]$ with $\Upsilon = 2\cosh(\mathcal{L}) + \tau^2\sinh(\mathcal{L})/2$, $\tau = \sqrt{R_o^2 - R_i^2}\, l_\perp^{-1}$ and $l_\perp = \hbar v_F/t_\perp$. Since the Landauer-Büttiker formula requires the summation over all modes j, conductance G exhibits oscillatory behavior in ϕ_D/ϕ_0 with period equal one, just as in a monolayer (see Fig. 2). It is also important to note that since the distance between the transmission peaks is $2\mathcal{A}$, the oscillations resulting from these resonances may interfere with each other.

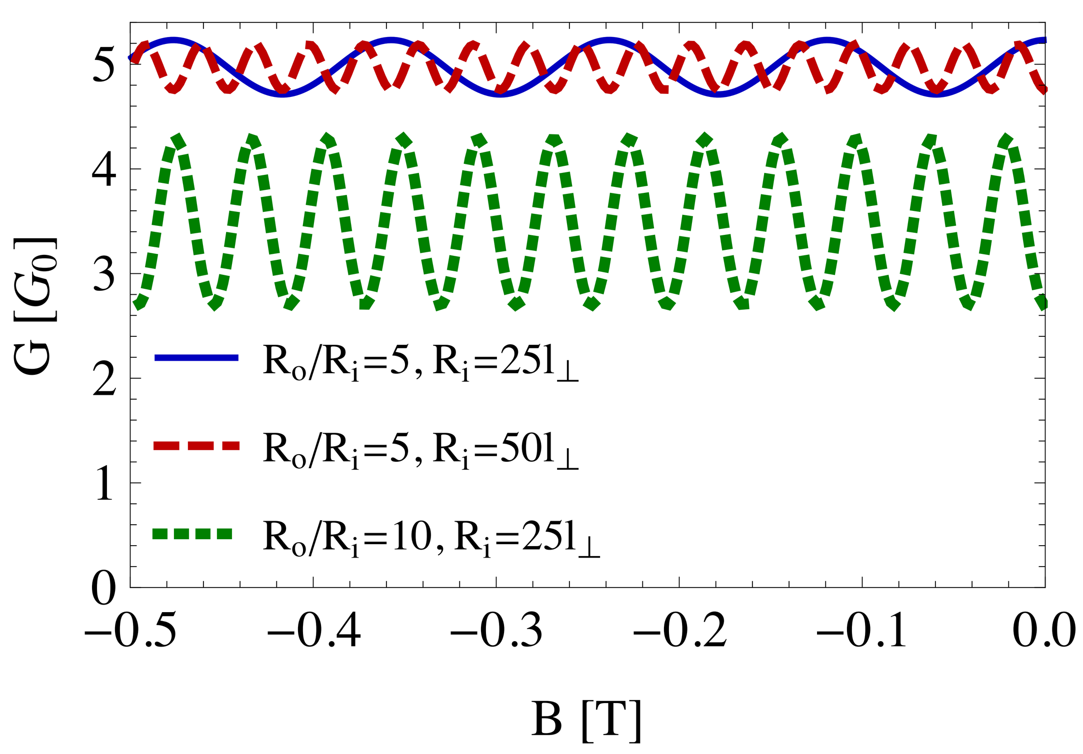

Figure 2: *Oscillatory behavior of conductance at the Dirac point for three different setups. Note that the amplitudes as well as average conductance depend only on the ratio whereas the period depends on the size of the sample as well.*

A clear view on this effect can be obtained by presenting G in the Fourier series

$$G = \frac{4g_0}{\mathcal{L}} + \sum_{m=1}^{\infty} G_m \cos\left[2\pi m \phi_D/\phi_0\right], \qquad (5)$$

with $G_m = 2g_0(2\pi/\mathcal{L})^2 m\,\mathrm{csch}\left(\pi^2 m/\mathcal{L}\right)\cos\left[2\pi m\mathcal{A}\right]$. The first term, $4g_0/\mathcal{L}$, gives the mean value of conductance which is simply twice as large as in a monolayer. It is possible to estimate the condition for extreme values of amplitude oscillations, since $G_1/G_2 \ll 1$ and provided the system is large enough ($R_i \gtrsim 10 l_\perp$, $R_o/R_i \gtrsim 3$; see: Fig. 3)

$$\mathcal{L} \approx 4\ln\left(R_i/2l_\perp\right)/p, \qquad (6)$$

where p is an odd (even) number for vanishing (maximal) oscillations.

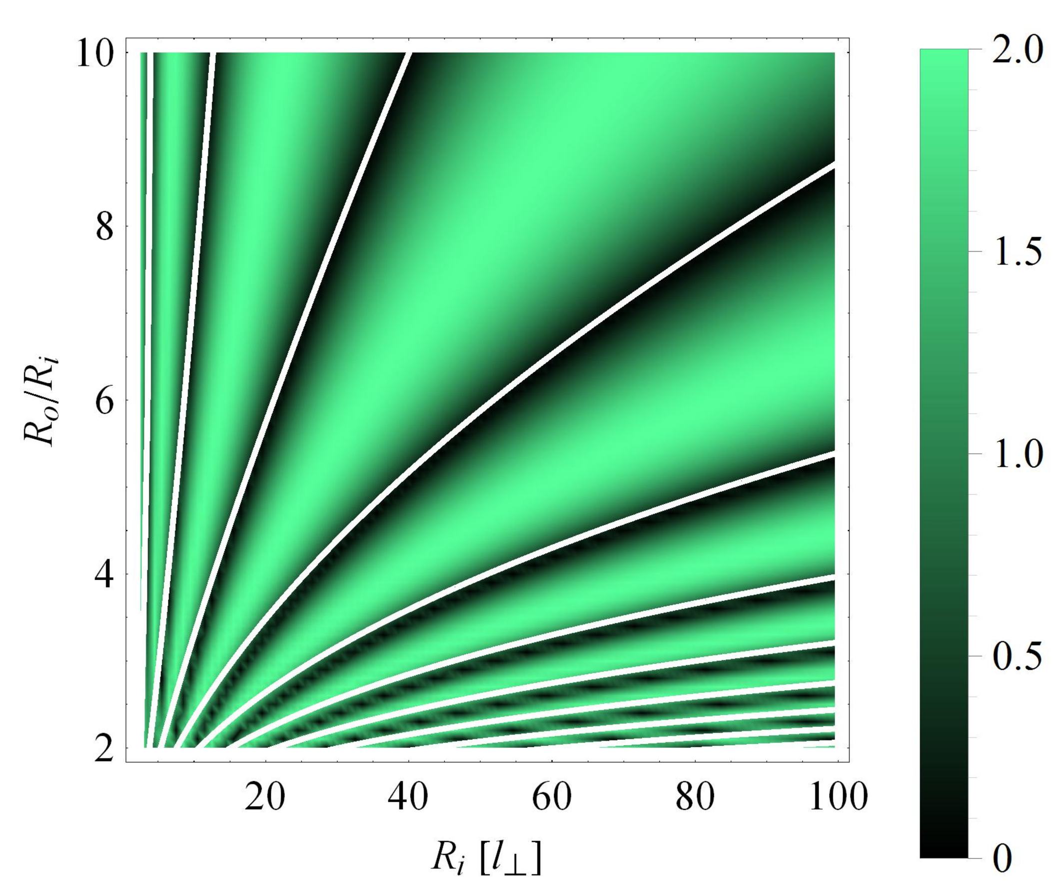

Figure 3: *Relation between oscillation amplitudes in bilayer and monolayer graphene. (a) White lines follow Eq. (6) for vanishing oscillations in bilayer graphene. (b) Ratio between oscillations in bilayer and monolayer graphene for two selected inner radii.*

For the sake of completeness of the discussion, it is worthy to check how QRCE emerges in the Andreev-Corbino (AC) setup, where the conductance can be written as

$$G^{NS} = 2g_0 \sum_j \frac{T_j^2}{(2 - T_j)^2}. \qquad (7)$$

In case of the Dirac point we get

$$G^{NS} = g_0 \sum_j \frac{32\left(1 + \cosh[2\mathcal{A}]\cosh[2\Phi_D/\Phi_0]\right)^2}{\left(\cosh[4j + 4\Phi_D/\Phi_0] + \cosh[4\mathcal{A}] - 2\right)^2}. \qquad (8)$$

Unfortunately, it is not possible to retrieve a compact Fourier series form (5) unless conditions $R_i \gtrsim 10 l_\perp$, $R_o/R_i \gtrsim 3$ are met. In that case, with a very good approximation we can write

$$G^{NS} \approx \frac{4g_0}{\mathcal{L}} + \sum_{m=1}^{\infty} G_m^{NS}\cos\left[2\pi m\phi_D/\phi_0\right], \qquad (9)$$

Comparison with 2DEG

One might put forward a question whether conductance oscillations in graphene are obtainable also in non-relativistic systems. Although there is no analog of the Dirac point in non-relativistic 2D systems, one might expect a similar behavior in the vicinity of Landau levels. Thus, we complement our investigation with an analysis of the Corbino disk in a Schrödinger system.

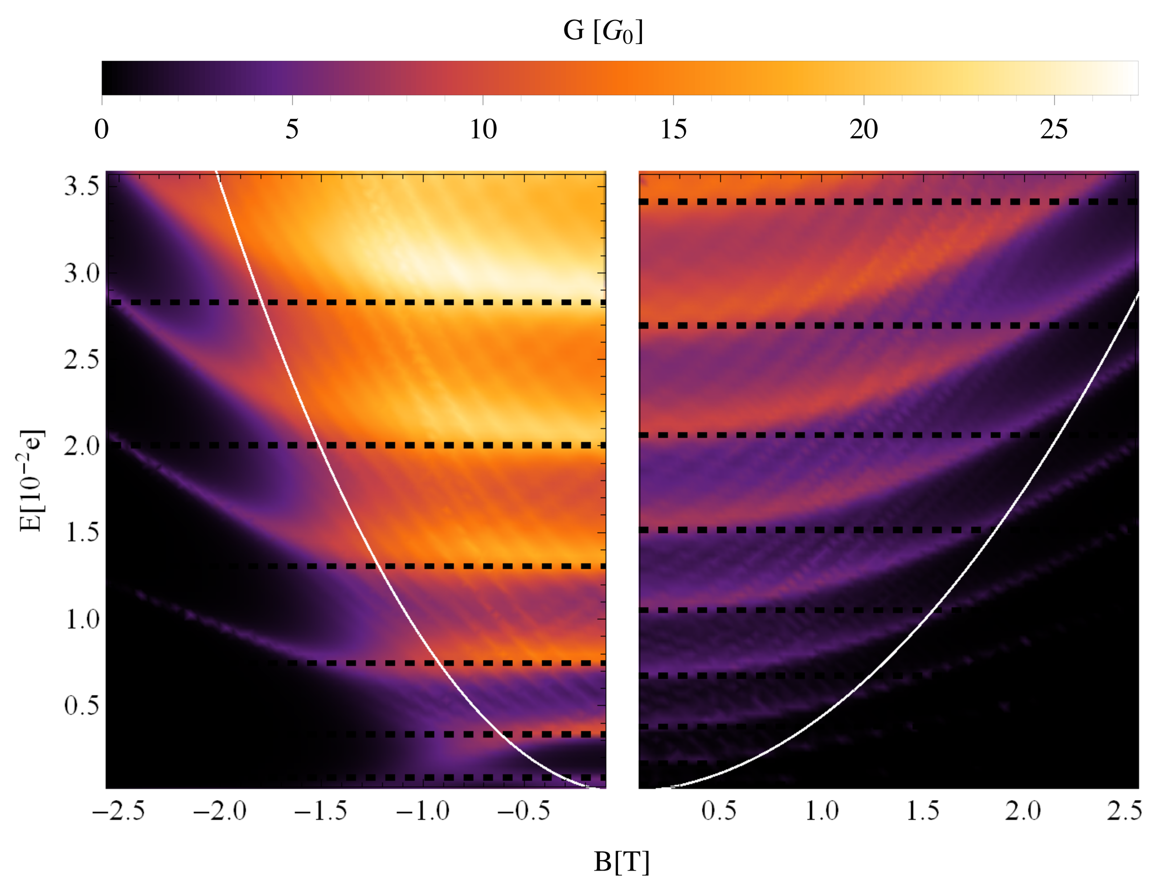

Figure 4: *Comparison of conductance as a function of doping and magnetic field of bilayer graphene (left) and 2DEG (right) disks with the same inner and outer radii $R_i = 25 l_\perp$ and $R_o = 100 l_\perp$. Dotted lines indicate quantum-well-like resonances, white lines depict the ballistic regime limit marked by the cyclotrone radii.*

In our calculations we have chosen a sample with an effective mass as in GaAs $m_\star = 0.067 m_e$, inner radius $R_i = 25 l_\perp$ and the doping on the leads $E = 0.4$eV. As one can see on Fig. 4, just as in graphene, at low magnetic fields one can observe conductance peaks corresponding to potential well energies $E \approx \hbar^2 n^2\left[8m_\star(R_o - R_i)\right]^{-1}$ which, along with increasing magnetic field, turn into Landau level resonances. The ballistic transport regime estimated by the cyclotron radius

$$r_c = \frac{\sqrt{2m_\star E}}{\hbar e B} \gtrsim (R_o - R_i)/2. \qquad (10)$$

Outside this region, the conductance is strongly suppresed by the magnetic field even at the Landau levels. It turns out that conductance oscillations emerge in 2DEG as well yet they vanish at high magnetic fields.

References

[1] A. Rycerz, Phys. Rev. B 81, 121404(R) (2010).
[2] G. Kirczenow, J. Phys.: Condens. Matter 6, L583 (1994).
[3] G. Rut and A. Rycerz, arXiv:1307.7093 (2013).
[4] A.R. Akhmerov and C.W.J. Beenakker, Phys. Rev. B 75, 045426 (2007).