Pseudodiffusive conductance and Landau level hierarchy in biased graphene bilayer

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We demostrate, by means of mode-matching analysis for the Dirac equation, that splittings of the Landau-level (LL) degeneracies associated with spin, valley, and layer degrees of freedom, directly affect the ballistic conductance of graphene bilayer. For wide samples ($W \gg L$), the Landauer-Büttiker conductance reaches the maximum $G \simeq se^2/(\pi h) \times W/L$ at the resonance via each LL, with the prefactor varying from $s = 8$ if all three degeneracies are preserved, to $s = 1$ if all the degeneracies are lifted. Our results show that the charge transfer at each LL has pseudodiffusive character, with the second and third cumulant quantified by $\mathcal{F} = 1/3$ and $\mathcal{R} = 1/15$ (respectively). Moreover, we show that if the electrochemical potential is not sharply defined but slowly fluctuates in a finite vicinity of LL, the resulting charge transfer characteristics are still quantum-limited, approaching $\mathcal{F} \simeq 0.7$ and $\mathcal{R} \simeq 0.5$ in the limit of large fluctuations.

Model

We consider a rectangular, weakly doped bilayer flake between two heavily-doped strips modelling contacts (see Fig. 1). It is assumed that the magnetic field is applied only to the central region. For the K valley the 4-band Hamiltonian of such a system reads

$$H = \begin{pmatrix} U_1(x) & \pi_x + i\pi_y & t_\perp & 0 \\ \pi_x - i\pi_y & U_1(x) & 0 & 0 \\ t_\perp & 0 & U_2(x) & \pi_x - i\pi_y \\ 0 & 0 & \pi_x + i\pi_y & U_2(x) \end{pmatrix}, \quad (1)$$

where $t_\perp \approx 0.38\,\text{eV}$ is the interlayer nearest-neighbour hopping energy, $\vec{\pi}/v_F = \left(-i\hbar\partial_{\vec{r}} + e\vec{A}\right)$ is the gauge-invariant momentum with $\vec{A} = (0, -Bx)$ ($B = 0$ outside $0 < x < L$) and $v_F \approx c/300$ is the Fermi velocity in a monolayer. $U_j(x)$ (with $j = 1, 2$ the layer index) is an electrostatic potential energy

$$U_j(x) = \begin{cases} U_\infty & \text{if } x < 0 \text{ or } x > L, \\ \gamma_j V/2 - g\mu_B B m_s & \text{if } 0 < x < L, \end{cases} \quad (2)$$

where V is the potentials difference between the layers, $\gamma_j = (-1)^j$ and $g\mu_B B m_s$ is the Zeeman term (with $m_s = \pm 1/2$ the z-component of spin, for the numerical discussion $g = 2$).

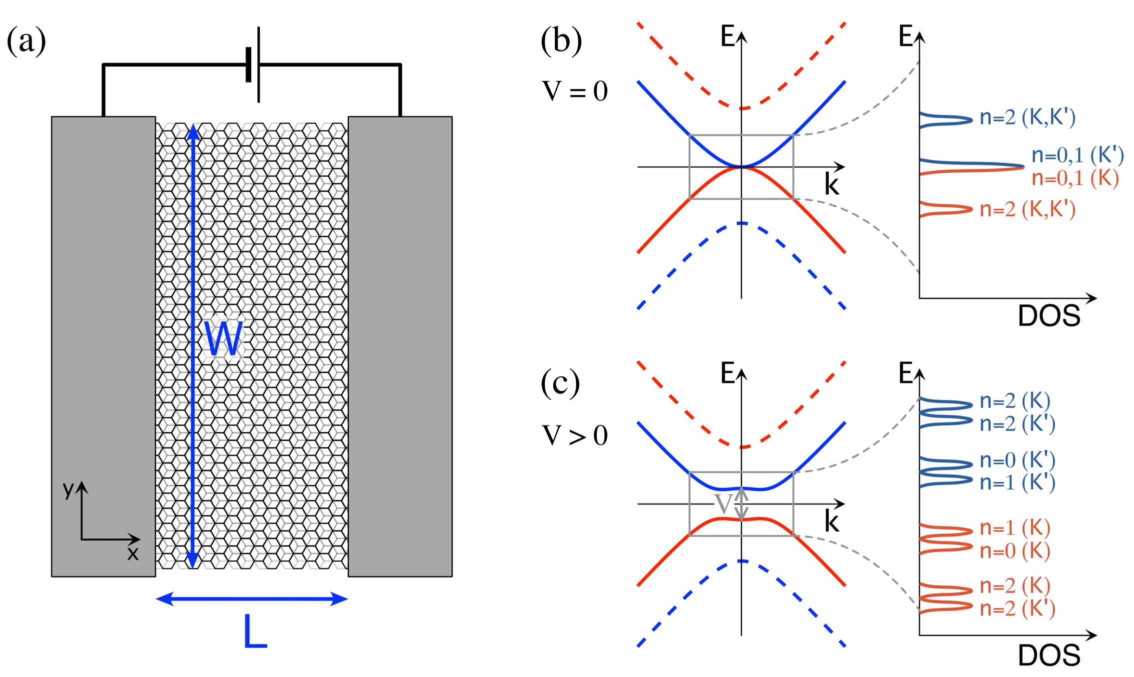

Figure 1: *Schematics of system studied analytically in the paper and energy band structure in the quantum Hall regime. (a) A strip of graphene bilayer of width W attached to two electrodes (shaded rectangles) at a distance L. A voltage source drives a current through the sample area. (b,c) The formation of Landau levels in bilayer graphene with and without a band gap. Landau levels are indexed with the orbital index n and the valley pseudospin (K or K'); the twofold spin degeneracy of each level is assumed for clarity. Both layer and valley degeneracies are splited in the presence of a band gap ($V > 0$).*

The conductance G, the Fano factor $\mathcal{F}$ and the factor $\mathcal{R}$, quantifying the third cumulant of the charge transfer, are derived by using the Landauer-Büttiker formula

$$G = \frac{e^2}{h}\text{Tr}T, \quad (3)$$

$$\mathcal{F} = \frac{\text{Tr}\left[T\,(1-T)\right]}{\text{Tr}T}, \quad (4)$$

$$\mathcal{R} = \frac{\text{Tr}\left[T\,(1-T)\,(1-2T)\right]}{\text{Tr}T}, \quad (5)$$

where $T = t^\dagger t$ and t is a and t is the transmission matrix.

Conductance

At zero doping and zero bias potential the transmission probability reads

$$T\,(k_y) = \cosh^{-2}\left[\left(k_y - l_B^{-2}L/2 \pm k_c\right)L\right], \quad (6)$$

where $k_c = \frac{1}{L}\ln\left[\frac{Lt_\perp}{2v_F} + \sqrt{1 + \left(\frac{Lt_\perp}{2v_F}\right)^2}\right]$, and k_y is the transverse momentum. Compared to the case of bilayer graphene at zero magnetic field, the wave vector is shifted by $-l_B^{-2}L/2$ (with $l_B = \sqrt{\hbar/|eB|}$ the magnetic length). In case we neglect the Zeeman splitting ($g \simeq 0$) the conductance appears to be field independent and twice as large as for the monolayer

$$G = \frac{e^2}{h}\frac{8}{\pi}\frac{W}{L}. \quad (7)$$

An even more interesting effect appears in the presence of a bias field. From the normalization condition of the wave functions one can obtain the following equation for Landau's energy levels

$$\varepsilon^2 + \delta^2 \pm \sqrt{(1-2\delta\varepsilon)^2 + t^2\,(\varepsilon^2 - \delta^2)} = 2n - 1, \quad (8)$$

with $n = 0, 1, \ldots$, $\varepsilon = E/(\hbar v_F l_B)$, $\varepsilon = V/(2\hbar v_F l_B)$ and $t = t_\perp/(\hbar v_F l_B)$.

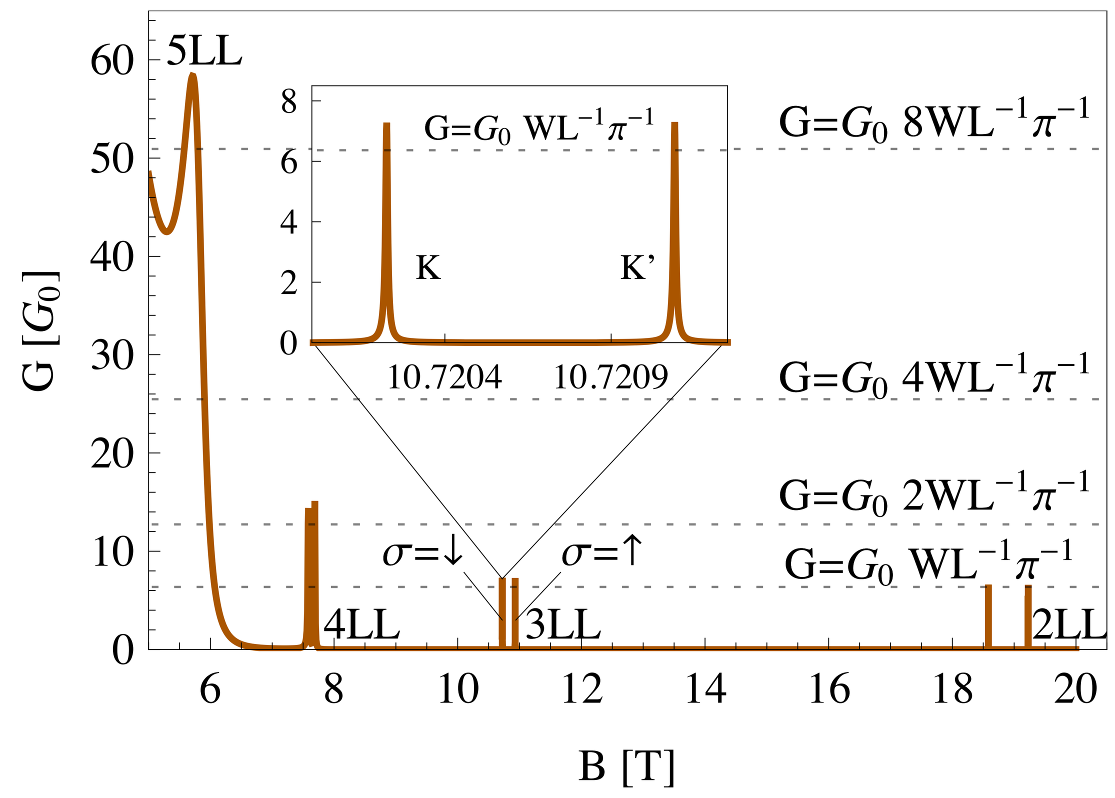

Figure 2: *Pseudodiffusive conductance at the doping $E = 0.2t_\perp$ and the bias $V = 2 \cdot 10^{-4} t_\perp$. For the second (2LL) and the third (3LL) Landau levels high magnetic field separates the resonances corresponding to K' and K valleys.*

In the presence of bias field valley degeneracy is lifted along with electron-hole symmetry. Because of valley splitting the conductance per spin at LLs is two times smaller than in the monolayer, $G = G_0 W/(\pi L)$. The two lowest LLs ($n = 0, n = 1$) exist for electrons (holes) only for the K' (K) valley.

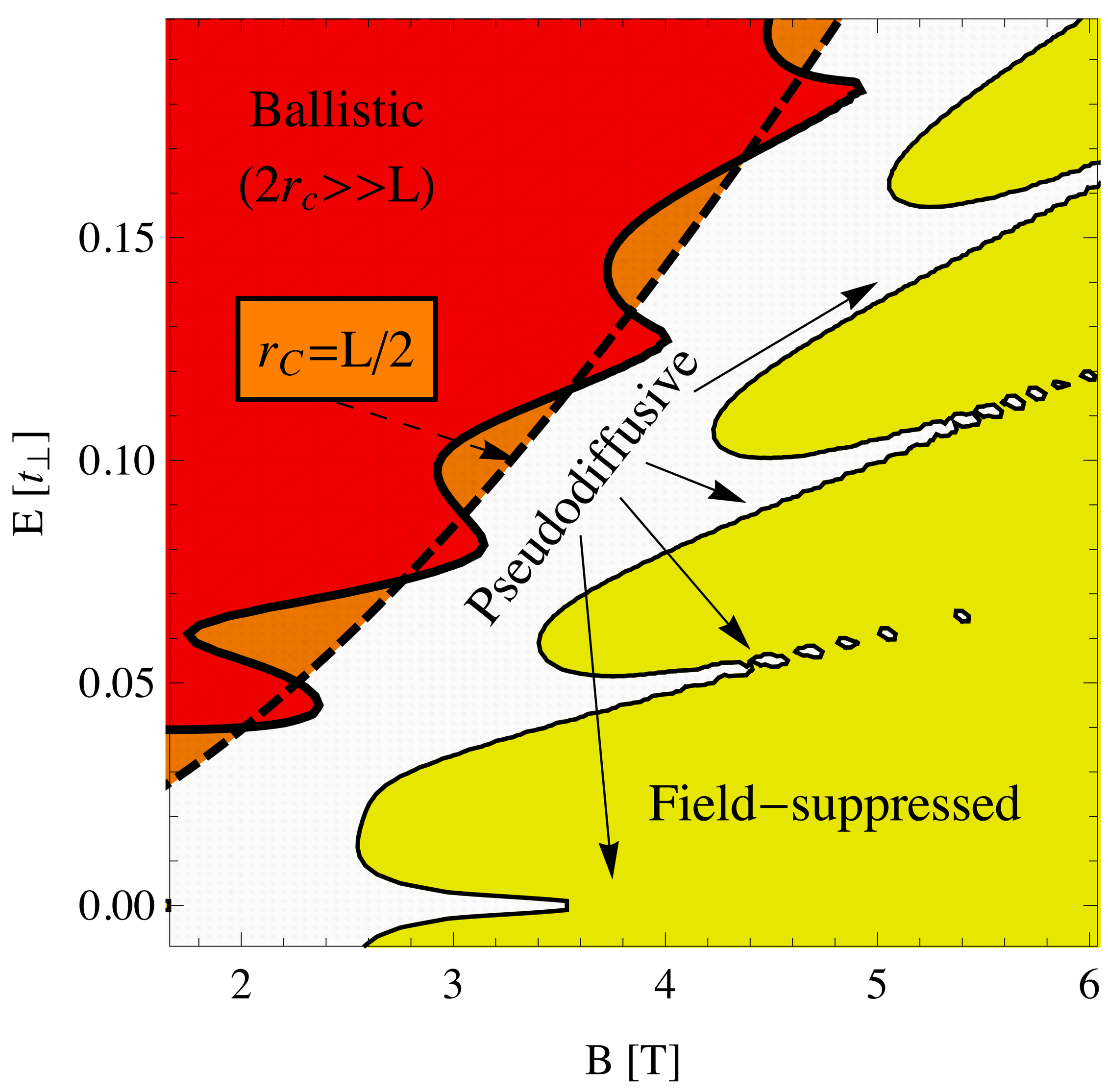

Figure 3: *Transport regimes in unbiased bilayer graphene (Zeeman splitting is not taken into account). Two contour lines mark the areas with $G/G_0 > 8W/L$ (red) and $G/G_0 < 2.4W/L$ (yellow) with $W/L = 20$. The dashed line corresponds to the ballistic regime limit estimated with cyclotron radius r_c.*

The effects of doping fluctuations

During long measurements doping control with sufficient precision might prove challenging, thus measured values of transport-related quantities may turn out to be different form those expected from basic theoretical models. We assume that transmission probability can be with a good approximation written as

$$T\,(k_y, \kappa) \approx \cosh^{-2}\left[\mathcal{L}k_y\right]/\left(1 + \kappa^2\right), \quad (9)$$

where $\mathcal{L}$ is a constant, $\kappa = 2(E - E_0)/\mathcal{W}$ with E_0 being the resonant doping, $\mathcal{W}$ is the full width at half maximum (FWHM) of the transmission resonance).

Placing (9) into the Landauer formula and integrating it over all k_y and fluctutions in the doping range $E \in [E_0 - \Delta\mathcal{W}_0/2, E_0 + \Delta\mathcal{W}_0/2]$ (Δ is a scaling factor) we obtain

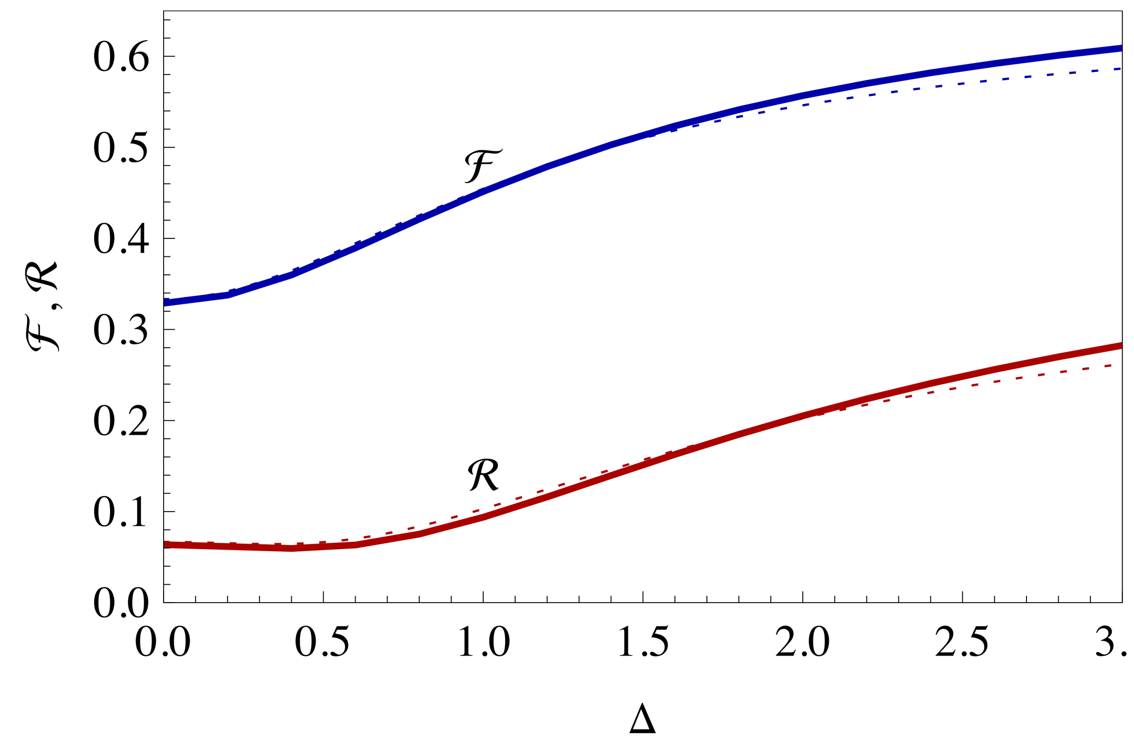

Figure 4: *Fluctuation dependence of $\mathcal{F}$ and $\mathcal{R}$ factors in the vicinity of 2LL at $B = 5T$. Dashed line corresponds to eq. (10). At $\Delta = 0$ $\mathcal{F}$ factor reaches its minimum, $\mathcal{F}(0) = 1/3$ and $\mathcal{R}(0) = 1/15$. $\mathcal{R}$ reaches its minimum value at non-zero fluctuations, namely $\mathcal{R}(\Delta \approx 0.34) \approx 0.064 < 1/15$.*

$$\mathcal{F}\,(\Delta) = \frac{2}{3} - \frac{\Delta}{3(1+\Delta^2)\text{atan}(\Delta)},$$
$$\mathcal{R}\,(\Delta) = \frac{2}{5} - \frac{\Delta}{5(1+\Delta^2)\text{atan}(\Delta)}\left(3 - \frac{4}{3(1+\Delta^2)}\right). \quad (10)$$

Formulas (10) are a good estimation for relatively small fluctuations ($\Delta < 2$). In general $\mathcal{L}$ and $\mathcal{W}$ are not constant although it is relatively easy to increase the accuracy of the estimation. The values of $\mathcal{F}$ and $\mathcal{R}$ at large fluctuations which reach 0.7 and 0.5 respectively. What is noteworthy, (10) are valid for the monolayer as well.

Conclusions

- The pseudodiffusive conductance of unbiased graphene bilayer is twice as large as for the monolayer at the Dirac point for arbitrary filed (0LL).
- At finite dopings (higher LLs) the bilayer conductance becomes equal to that of the monolayer.
- At finite bias potential, splittings of LLs corresponding to K and K' valleys lead to the pseudodiffisive conductance reduced by the factor of 2.
- Both for the monolayer and bilayer graphene, doping fluctuations may strongly affect transport characteristics. However, even for large fluctuations, the values of F and R are predicted to be quantum-limited, i.e. $\mathcal{F} = 0.7$ and $\mathcal{R} = 0.5$.

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