ISMD2003 at Krakow

Analysis of transverse momentum distributions observed at the RHIC by a stochastic model in the hyperbolic space

N. Suzuki[1] and M. Biyajima[2]

[1] Matsumoto University, Matsumoto, Japan

[2] Shinshu University, Matsumoto, Japan

1. Stochstic equation in longitudinal rapidity and transverse momentum variables
2. Diffusion equation in the hyperbolic space H^3
3. Analysise of Transverse momentum distributions observed at the RHIC
4. Summary

1. Stochastic equation in longitudinal rapidity and transverse momentum variables

Ornstein-Uhlenbeck process in 3D

$$\frac{\partial}{\partial t}\phi(\boldsymbol{r},\,t) = \sum_{i=1}^{2}\left\{\mu_i\frac{\partial}{\partial x_i}x_i\phi(\boldsymbol{r},\,t) + D_i\frac{\partial^2}{\partial x_i^2}\phi(\boldsymbol{r},\,t)\right\} + \gamma\frac{\partial}{\partial x_3}x_3\phi(\boldsymbol{r},\,t) + D_3\frac{\partial^2}{\partial x_3^2}\phi(\boldsymbol{r},\,t)$$

initial condition: $\phi(\boldsymbol{r},\,t=t_0) = \delta^3(\boldsymbol{r}-\boldsymbol{r}_0)$

$$\begin{cases}\mu_1=\mu_2=\mu_r\\ D_1=D_2=D_r\end{cases}\qquad \begin{cases}D_3=D\\ x_3=y\\ x_{30}=y_0\end{cases}\qquad \begin{cases}x_1=\rho\cos\theta\\ x_2=\rho\sin\theta\\ x_{10}=\rho_0\cos\theta_0\\ x_{20}=\rho_0\sin\theta_0\end{cases}$$

$$\begin{cases}x_1 = p_{T_1}/m\\ x_2 = p_{T_2}/m\end{cases}\qquad \begin{cases}p_r = 1-e^{-2\mu_r(t-t_0)}\\ p_\gamma = 1-e^{-2\gamma(t-t_0)}\end{cases}$$

$$\begin{aligned}\phi(\boldsymbol{r},\,t) &= (2\pi)^{-3/2}\frac{\mu_r}{D_rp_r}\left(\frac{\gamma}{Dp_\gamma}\right)^{1/2}\exp\left\{-\frac{\gamma\left(y-y_0\sqrt{1-p_\gamma}\right)^2}{2Dp_\gamma}\right.\\ &\qquad\left.-\frac{\mu_r[(x_1-x_{10}\sqrt{1-p_r})^2+(x_2-x_{20}\sqrt{1-p_r})^2]}{2D_rp_r}\right\}\\ &= \phi_\gamma(y,\,t)\phi_r(\rho,\,\theta,\,t)\,,\end{aligned}$$

where $\phi_\gamma(y,\,t)$: rapidity distribution,

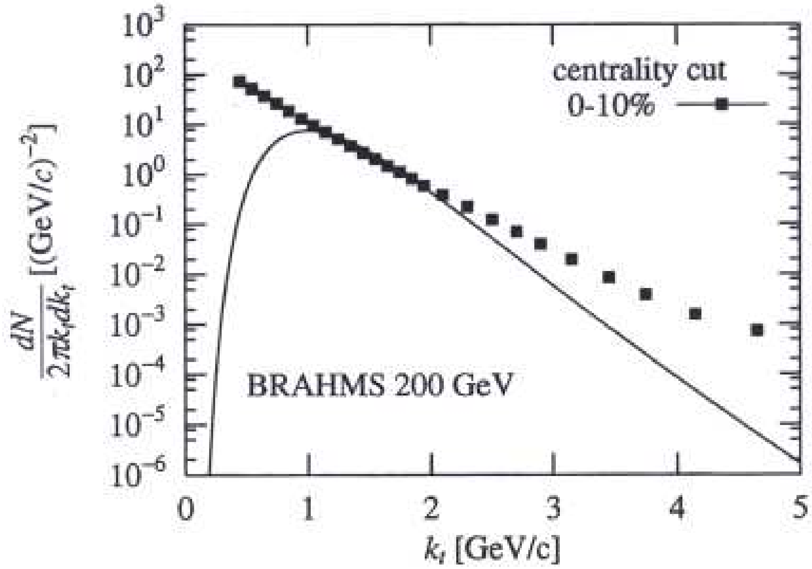
$\rho\to p_T$: $\phi_r(\rho,\,\theta,\,t)$: p_T distribution

After θ integration

$$\int_0^{2\pi} d\theta \phi_r(\boldsymbol{r},\, t) \;=\; \frac{\mu_r}{D_r p_r} \exp\left\{ -\frac{\mu_r}{2D_r p_r}[\rho^2 + \rho_0^2(1-p_r)] \right\}$$
$$\times I_0\left(\frac{\mu_r \rho \rho_0 \sqrt{1-p_r}}{D_r p_r} \right)$$

$$\rho^2 = p_T^2, \quad \rho_m = \sqrt{m_h^2 + \langle p_T \rangle^2}, \quad D_r/\mu_r = A$$
$$= \frac{1}{Ap_r} \exp\left\{ -\frac{p_T^2 + \rho_m^2(1-p_r)}{2Ap_r} \right\} \times I_0\left(\frac{p_T \rho_m \sqrt{1-p_r}}{Ap_r} \right)$$

This Equation is equivalent to the expression obtain by Voloschin, Phys. Rev. C55 (1997) 1630,



if $\quad \rho = |p_T|/m$,

$\rho_0 \cdot \sqrt{1-p_r} = |p_0|/m$

and $\dfrac{\mu}{D_r p_r} = \dfrac{m}{T}$

PHYSICAL REVIEW C VOLUME 55, NUMBER 4 APRIL 1997

Transverse radial expansion and directed flow

Sergei A. Voloshin*
Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany
(Received 19 November 1996)

The effects of an interplay of radial expansion of the thermalized system created in a heavy ion collision and directed flow are discussed. It is shown that the study of azimuthal anisotropy of particle distribution as a function of rapidity *and* transverse momentum could reveal important information on both radial and directed flow. [S0556-2813(97)50304-9]

PACS number(s): 25.75.Ld, 24.10.Jv

The transverse isotropic expansion of the source can be described as a superposition of different sources moving radially with an expansion velocity β_0. Then the (nonrelativistic) transverse momentum distribution of protons from a radially expanding thermal source can be written as

$$\frac{1}{N}\frac{d^2N}{d\mathbf{p_t}}=\frac{1}{(2\pi)^2(2mT)}\int d\psi \exp \times\left(-\frac{[p_x-p_0\cos(\psi)]^2+[p_y-p_0\sin(\psi)]^2}{2mT}\right), \quad (4)$$

where $p_0=m\beta_0$. The integration over ψ (the orientation of the expansion velocity) results in the distribution

$$\frac{1}{N}\frac{d^2N}{d\mathbf{p_t}}=\frac{1}{2\pi(2mT)}\exp\left(-\frac{p_t^2+p_0^2}{2mT}\right)I_0(\xi), \quad (5)$$

where I_0 is the modified Bessel function, and $\xi=\beta_0 p_t/T$.

Using the formula (3) one gets an expression for v_1:

$$V_1 = \frac{\langle p_x \rangle}{\langle p_t \rangle}$$

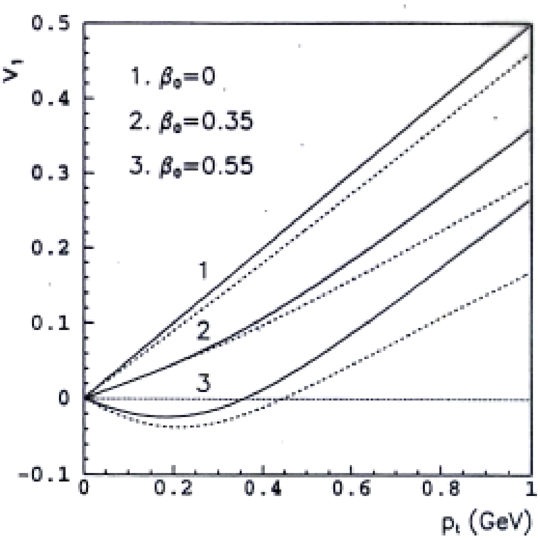


FIG. 1. $v_1(p_t)$: solid lines—Eq. (6), dashed lines—Eq. (8). T=0.1 GeV, β_a=0.1.

$$v_1(p_t)=\frac{p_t\beta_a}{2T}\left(1-\frac{m\beta_0}{p_t}\frac{I_1(\xi)}{I_0(\xi)}\right). \quad (6)$$

In our analysis we work in the frame moving longitudinally with the effective source rapidity. In this frame the longitudinal particle momenta usually can be neglected, which was done in the derivation above. If one considers the particle production with rapidities far from the rapidity of the effective source, when particle longitudinal momenta are large, one should make a substitution $m\rightarrow\sqrt{m^2+p_l^2}$.

2. Diffusion equation in the Hyperbolic Space H^3

The geodesic polar coordinate

$$
\begin{aligned}
v_1 &= \frac{p_1}{E} = \tanh\rho\cos\theta \\
v_2 &= \frac{p_2}{E} = \tanh\rho\sin\theta\cos\phi \\
v_3 &= \frac{p_3}{E} = \tanh\rho\sin\theta\sin\phi
\end{aligned}
$$

$$
\begin{aligned}
E &= m\cosh\rho \\
p &= \sqrt{p_1^2+p_2^2+p_3^2} = m\sinh\rho \\
\rho &= \ln\frac{E+p}{m}
\end{aligned}
$$

ρ : radial rapidity

The metric is written as

$$ds^2 = d\rho^2 + \sinh^2\rho\,(d\theta^2 + \sin^2\theta d\phi^2)$$

Diffusion equation

$$\frac{\partial f}{\partial t} = \frac{D}{\sinh^2\rho}\left[\frac{\partial}{\partial\rho}\left(\sinh^2\rho\frac{\partial f}{\partial\rho}\right)\right.$$
$$\left.+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}\right]$$

initial condition

$$f(\rho,\theta,\phi,t=0) = \frac{\delta(\rho)\cancel{\delta(\theta)\delta(\phi)}}{\sinh^2\rho\sin\theta}$$

Approximate solution for $Dt << 1$

$$f = (4\pi Dt)^{-3/2}\mathrm{e}^{-Dt}\boxed{\frac{\rho}{\sinh\rho}\exp\left[-\frac{\rho^2}{4Dt}\right]}$$

Molchanov, Rus. Math. Surveys, 30 (1975) 1-63

This is the solution of the equation with radial symmetry;

$$\frac{\partial f}{\partial t} = \frac{D}{\sinh^2\rho}\frac{\partial}{\partial\rho}\left(\sinh^2\rho\frac{\partial f}{\partial\rho}\right)$$

The geodesic cylindrical coordinate

$$
\begin{aligned}
v_1 &= \frac{p_1}{E} = \tanh\eta \\
v_2 &= \frac{p_2}{E} = \frac{1}{\cosh\eta}\tanh\xi\cos\phi \\
v_3 &= \frac{p_3}{E} = \frac{1}{\cosh\eta}\tanh\xi\sin\phi
\end{aligned}
$$

$$
\begin{aligned}
E &= m\cosh\eta\cosh\xi = m_{\mathrm{T}}\cosh\eta \\
p_1 &= m\sinh\eta\cosh\xi = m_{\mathrm{T}}\sinh\eta \\
p_{\mathrm{T}} &= \sqrt{p_2^2 + p_3^2} = m\sinh\xi \\
m_{\mathrm{T}} &= \sqrt{p_{\mathrm{T}}^2 + m^2}
\end{aligned}
$$

longitudinal rapidity $\quad \eta = \ln\dfrac{E + p_1}{m_T}$

transverse rapidity $\quad \xi = \ln\dfrac{m_T + p_T}{m}$

Identity $\quad E/m = \boxed{\cosh\rho = \cosh\eta\cosh\xi}$

$\rho = \xi$, when $\eta = 0$ $\quad \rho = \ln\dfrac{E + |p|}{m}$

$(p_1 = 0)$

We can analyse transvers momentum (rapidity) distributions at fixed longitudinal rapidity ($\eta = 0, 2.2, \cdots$), using the equation,

$$
f(\rho, t) = C\frac{\rho}{\sinh\rho}\exp\left[-\frac{\rho^2}{2\sigma(t)^2}\right]
$$

Parameters $\quad C, \quad \sigma(t)^2 = 2Dt$

3. Analysis of P_t distributions observed at the RHIC

centrality	m	C	$\sigma(t)^2$	χ^2_{min}
00-05%	0.51	380.5 $\pm$11.5	0.356$\pm$ 0.001	149.08
05-10%	0.50	316.0 $\pm$ 9.4	0.362$\pm$ 0.001	119.96
10-20%	0.49	235.4 $\pm$ 7.1	0.372$\pm$ 0.001	115.20
20-30%	0.48	166.6 $\pm$ 5.0	0.378$\pm$ 0.001	97.94
30-40%	0.45	111.1 $\pm$ 3.3	0.396$\pm$ 0.002	67.8
40-60%	0.43	59.0 $\pm$ 1.8	0.409$\pm$ 0.002	67.8
60-80%	0.36	26.8 $\pm$ 0.8	0.451$\pm$ 0.002	25.9

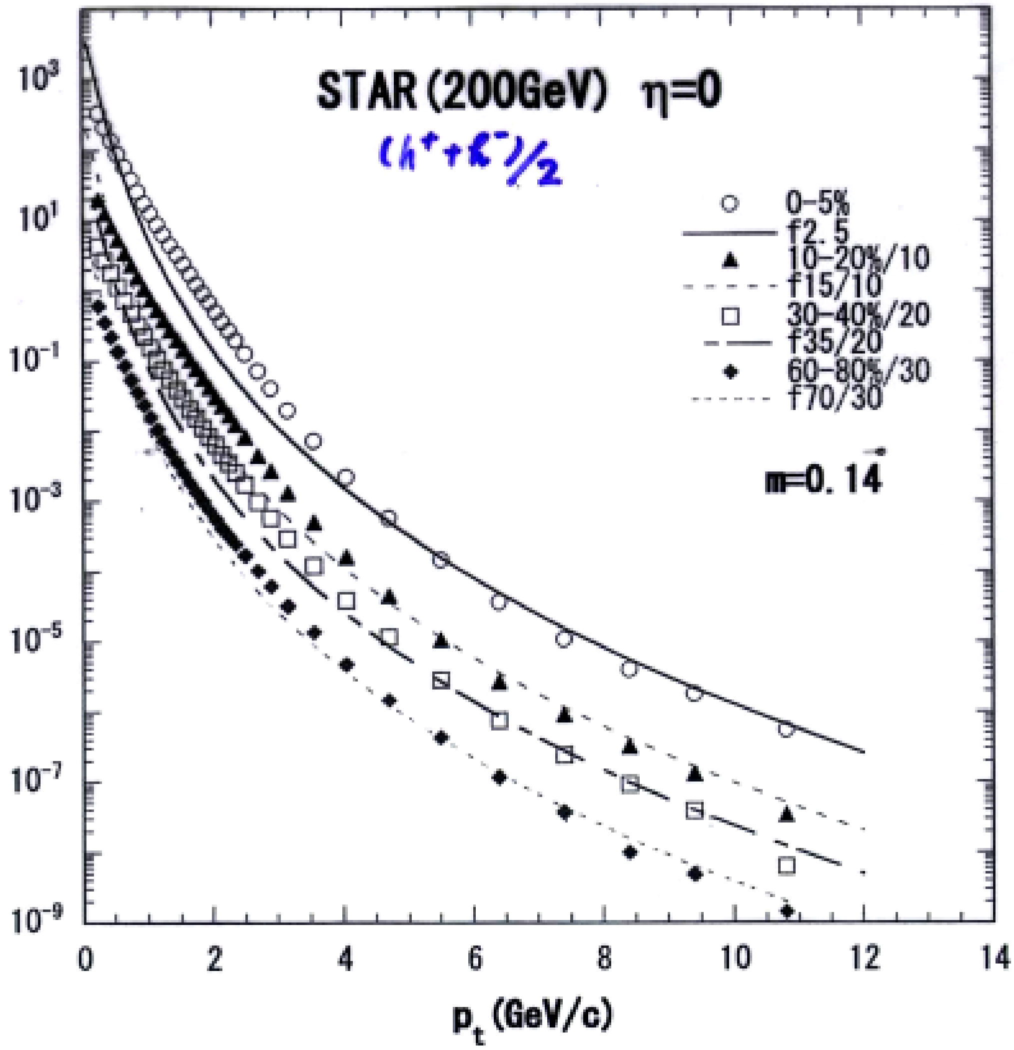
STAR($\eta = 0$) error:10% n.d.f. = 35 - 3

centrality	m	C	$\sigma(t)^2$	χ^2_{min}
00-10%	0.71	121.0 $\pm$ 0.7	0.307$\pm$ 0.001	351.2
10-20%	0.67	88.7 $\pm$ 0.6	0.325$\pm$ 0.001	233.5
20-40%	0.63	56.9 $\pm$ 0.7	0.340$\pm$ 0.001	97.8
40-60%	0.57	27.5 $\pm$ 0.5	0.358$\pm$ 0.002	28.7

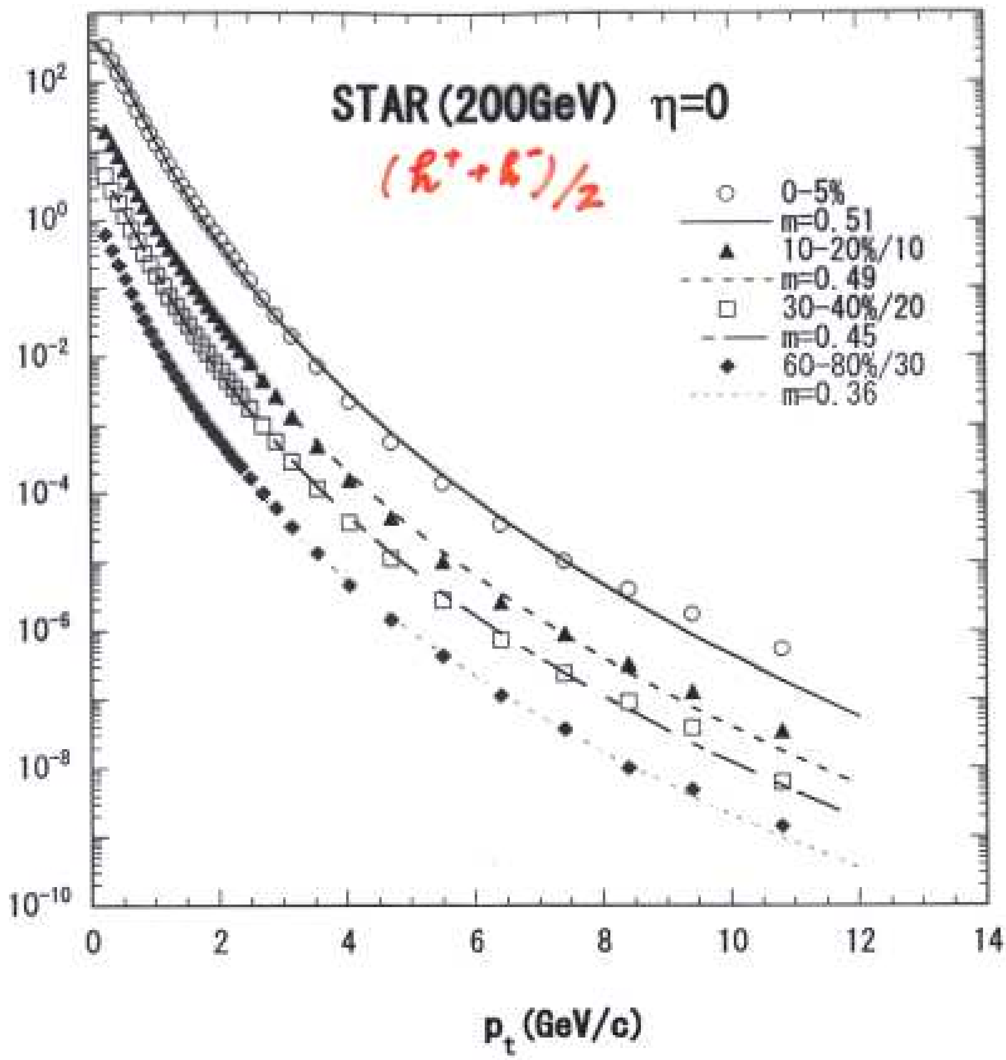
BRAHMS($\eta = 0$) : statistical error only n.d.f. = 26-3

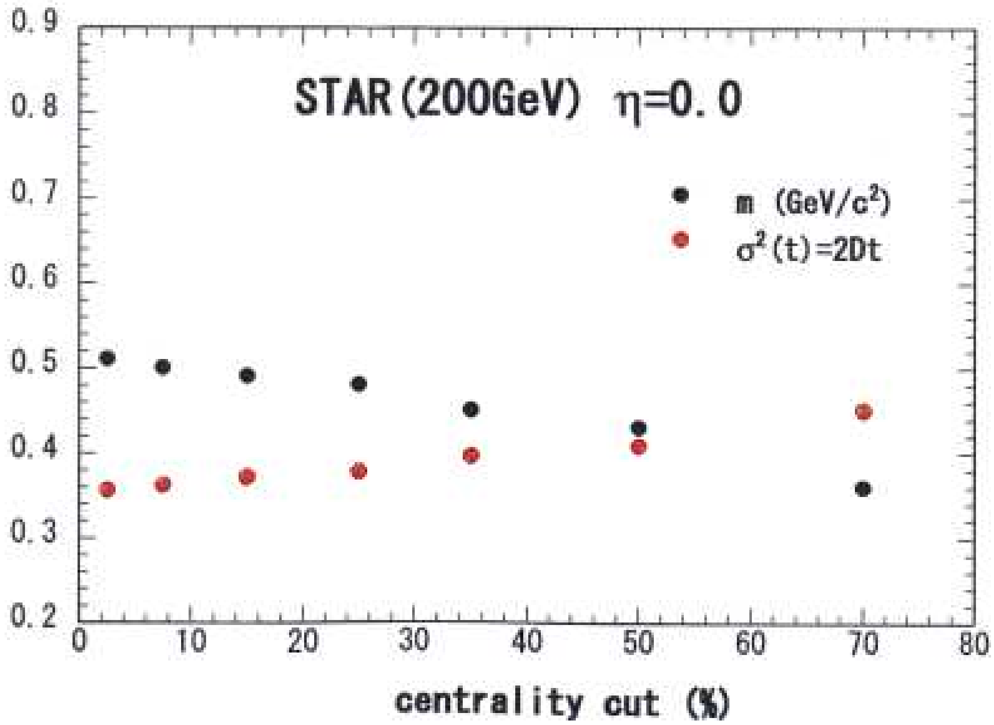
centrality	m	C	$\sigma(t)^2$	χ^2_{min}
00-10%	0.80	52286 $\pm$ 1575	0.470$\pm$ 0.002	93.7
10-20%	0.84	41669 $\pm$ 1561	0.452$\pm$ 0.002	30.5
20-40%	0.81	19364 $\pm$ 862	0.469$\pm$ 0.003	68.9
40-60%	0.70	6070 $\pm$ 418	0.524$\pm$ 0.005	3.1

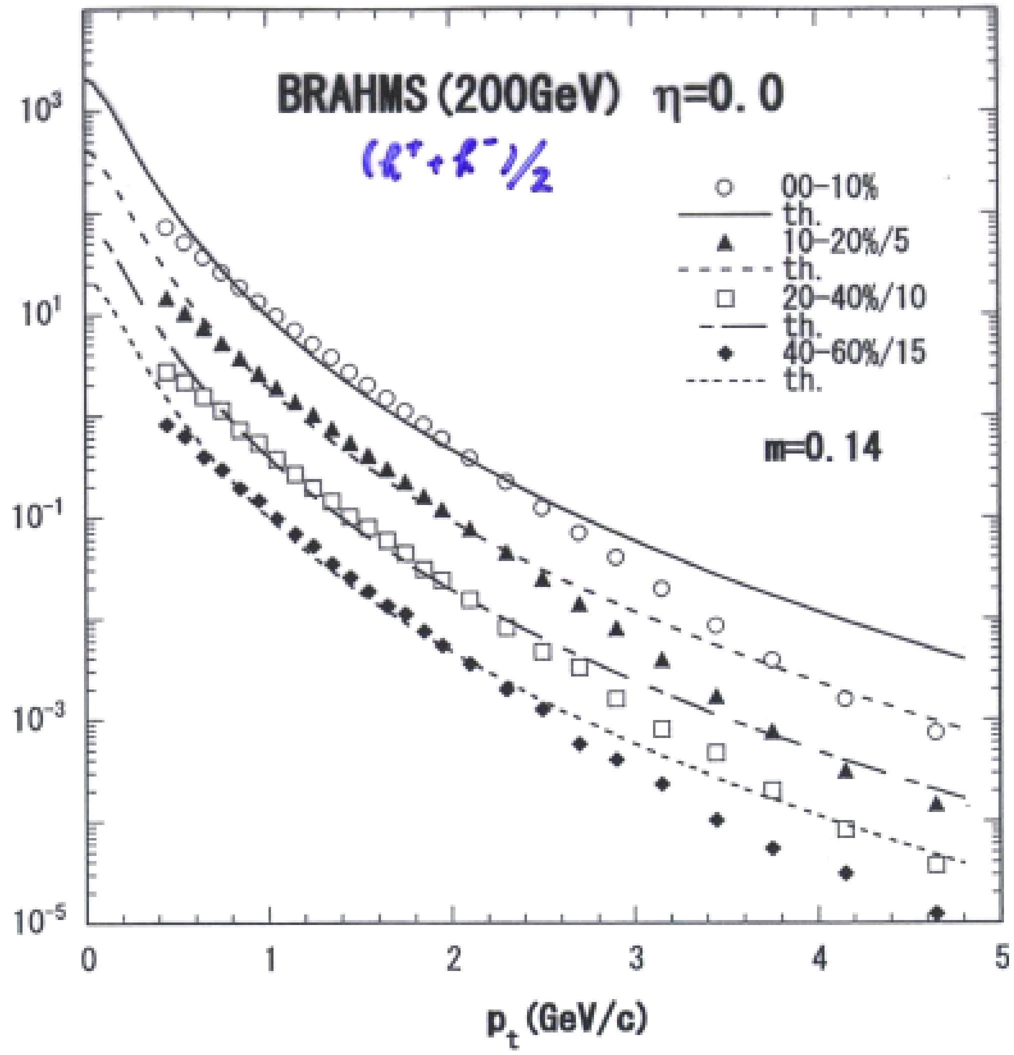
BRAHMS($\eta = 2.2$) : statistical error only n.d.f. = 11- 3



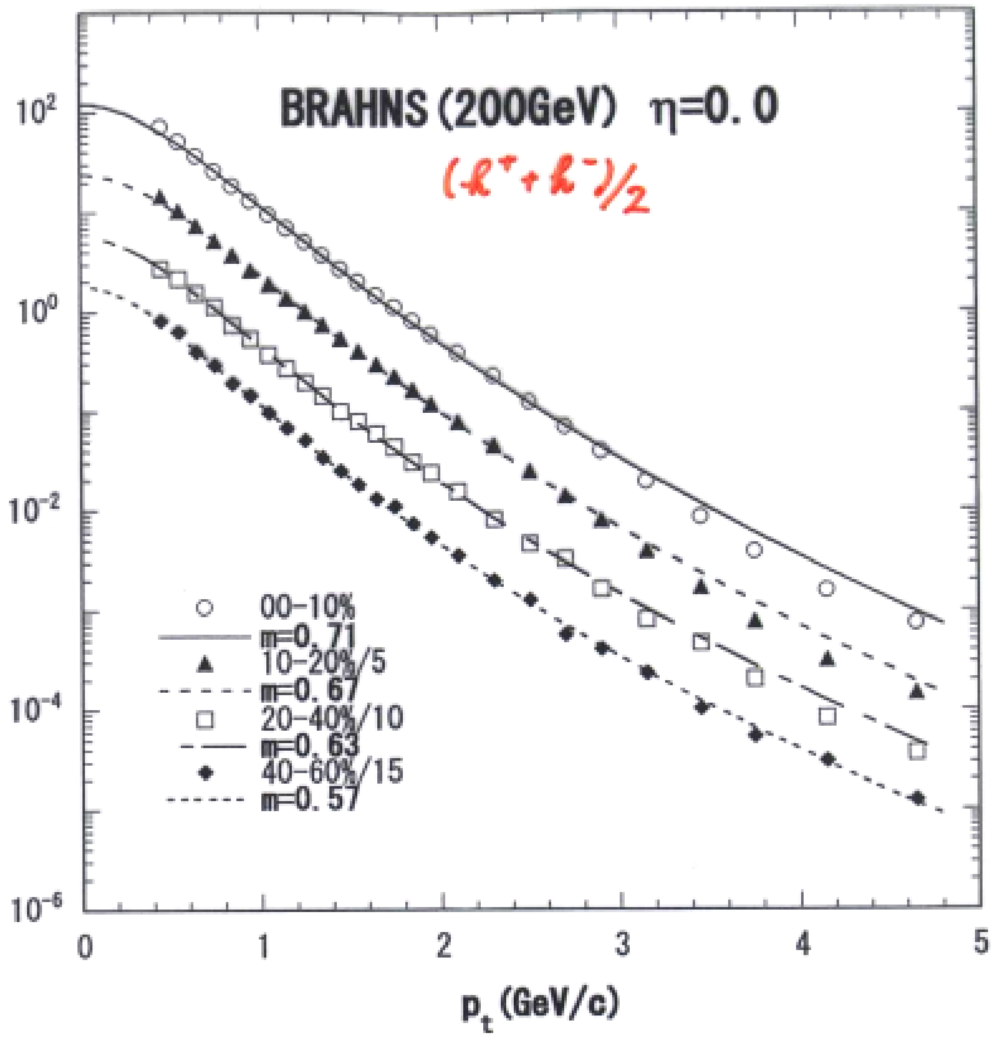
STAR (200GeV) η=0
(h⁺+h⁻)/2
0-5%
f2.5
10-20%/10
f15/10
30-40%/20
f35/20
60-80%/30
f70/30
m=0.14
10³
10¹
10⁻¹
10⁻³
10⁻⁵
10⁻⁷
10⁻⁹
0
2
4
6
8
10
12
14
pt (GeV/c)


STAR(200GeV) η=0
(h⁺+h⁻)/2
0-5%
m=0.51
10-20%/10
m=0.49
30-40%/20
m=0.45
60-80%/30
m=0.36
10^2
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
0 2 4 6 8 10 12 14
p_t (GeV/c)

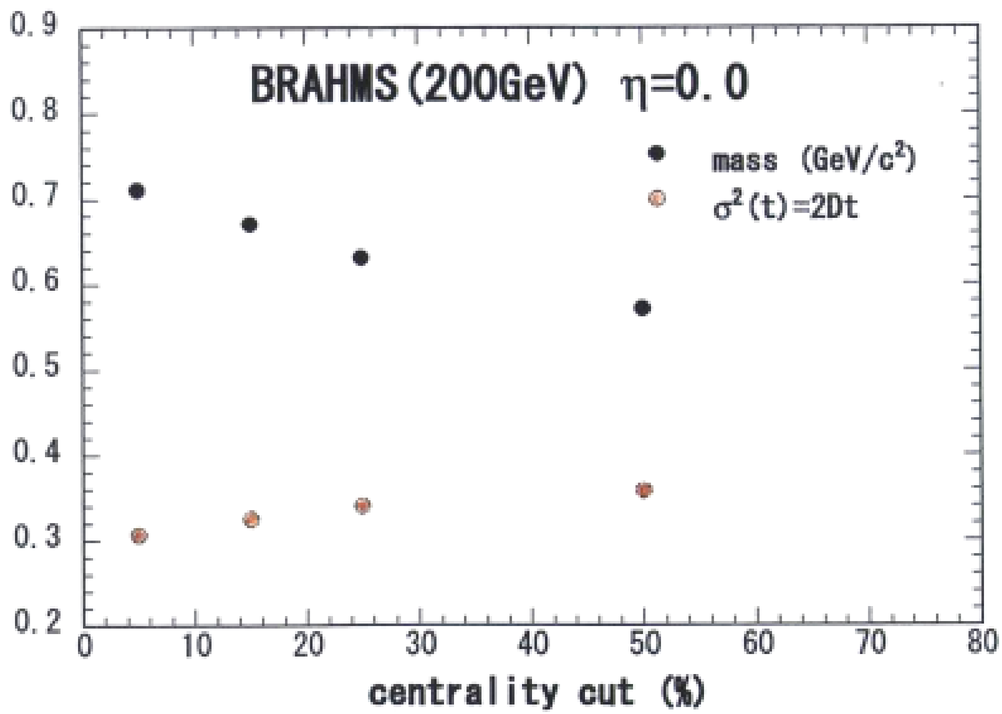

STAR(200GeV) η=0.0
m (GeV/c²)
σ²(t)=2Dt
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0 10 20 30 40 50 60 70 80
centrality cut (%)



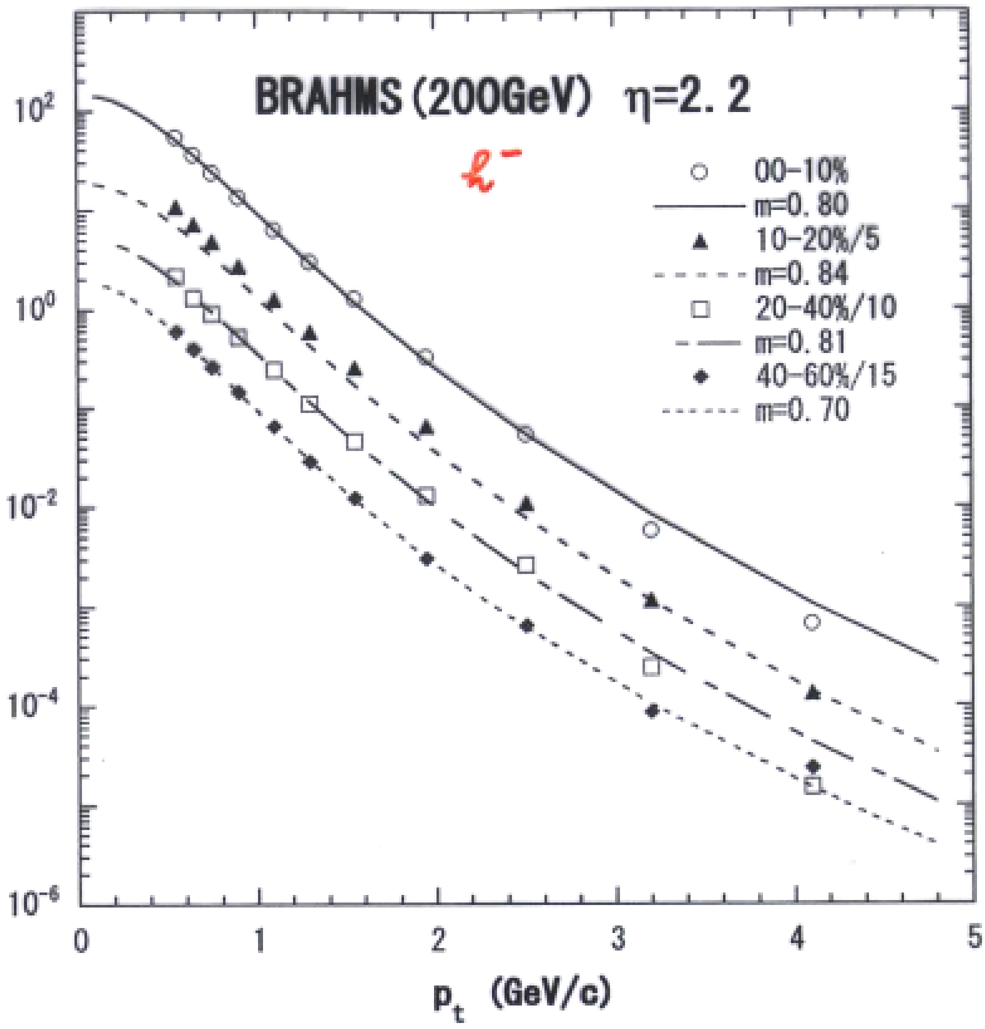
BRAHMS (200GeV) η=0.0
(π⁺+π⁻)/2
00-10%
th.
10-20%/5
th.
20-40%/10
th.
40-60%/15
th.
m=0.14
10³
10¹
10⁻¹
10⁻³
10⁻⁵
0
1
2
3
4
5
p_t (GeV/c)

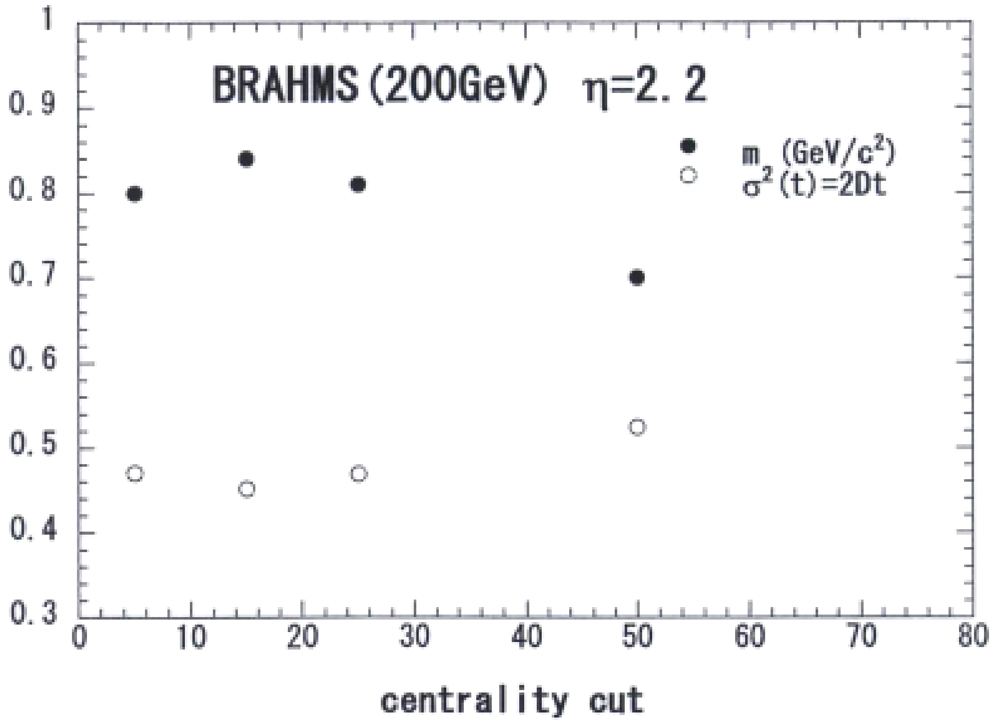


BRAHNS (200GeV) η=0.0
(h⁺+h⁻)/2
00-10%
m=0.71
10-20%/5
m=0.67
20-40%/10
m=0.63
40-60%/15
m=0.57
p_t (GeV/c)



BRAHMS (200GeV) η=0.0
mass (GeV/c²)
σ²(t)=2Dt
centrality cut (%)


BRAHMS(200GeV) η=2.2
k⁻
00-10%
m=0.80
10-20%/5
m=0.84
20-40%/10
m=0.81
40-60%/15
m=0.70
p_t (GeV/c)


BRAHMS(200GeV) η=2.2
m_2 (GeV/c²)
σ²(t)=2Dt
centrality cut

4. Summary

A formula proposed by Voloshin is derived from the stochastic model (Ornstein-Uhlenbeck process). However, it is gaussian in p_t, and observed p_t distributions at the RHIC cannnot be described by the formula.

A stochastic process in the three dimensional hyperbolic space H^3 is taken as a model of particle production processes. The approximate solution is gaussian-like in radial rapidity.

P_t distributios at $\eta = 0$ and $\eta = 2.2$ observed by the BRAHMS collaboration and those at $\eta = 0$ ~~and 2.2~~ observed by the STAR collaboration are analysed.

Observed p_t distributions are well described by the formula (gaussian in rapidity variable), if value of mass contained in rapidity is used as parameter.

Variance (Width) $\sigma^2(t)$ increases as the centrality cut increases.
Mass m decreases as the centrality cut increases.

Estimated values of $\sigma^2(t)$ and m^{-1} from the STAR collab. are larger than those from the BRAHMS collab. at the same centrality cut.