Low x Scattering As A Critical Phenomenon

H. J. Pirner
Universität Heidelberg

March 2002

- Near Light Cone Coordinates and Zero Mode Hamiltonian
- Fundamental domain and $Z(3)$ symmetry for SU3
- Wilson line correlation length
- Matching near light cone Hamiltonian with low x-scattering
- Effective Photon wave function and proton structure function F_2

H.J.P. Phys. Lett.B 521(2001)279,
H.J.P. and Yuan Feng $hep-ph/0203184$

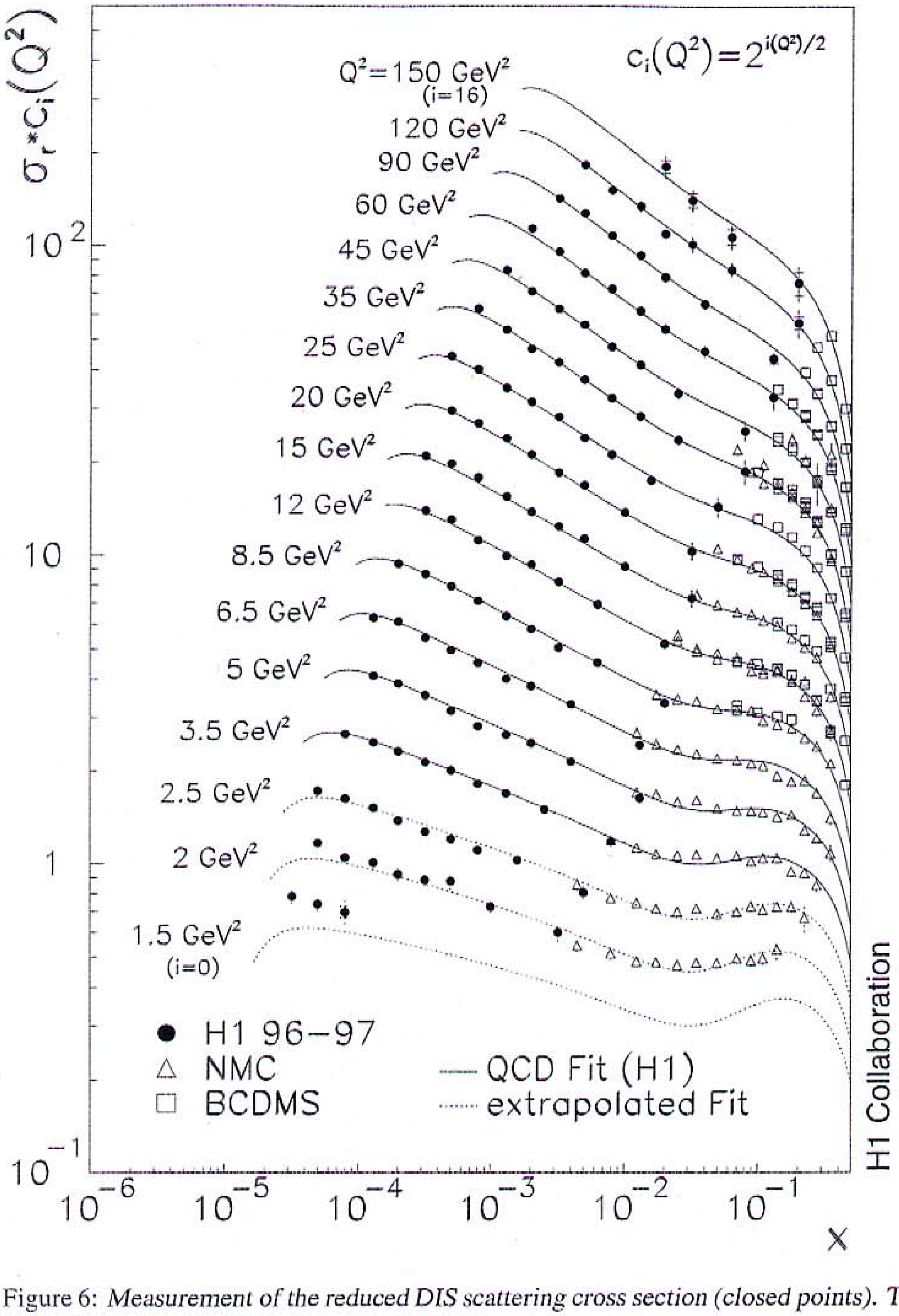


Figure 6: *Measurement of the reduced DIS scattering cross section (closed points). Triangles (squares) represent data from the NMC (BCDMS) muon-proton scattering experiments. The solid curves illustrate the cross section obtained in a NLO DGLAP QCD fit to the present data at low x, with $Q^2_{min} = 3.5$ GeV^2, and to the H1 data at high Q^2. The dashed curves show the extrapolation of this fit towards lower Q^2. The curves are labelled with the Q^2 value the data points belong to and scale factors are coveniently chosen to separate the measurements.*

I. Near the light cone QCD*

R. Naus (Hannover), T. J. Fields, J. P. Vary (Ames, Iowa) & H. J. P.
Phys. Rev. D 56 # 12 (1997) p. 8062, E. M. Ilgenfritz, Y. Ivanov, H. J. P. PRD 62 (2000) 0340XX

Near light cone coordinates:

$$x^+ = \frac{1}{\sqrt{2}}\left[\left(1+\frac{\eta^2}{2}\right)x^0 + \left(1-\frac{\eta^2}{2}\right)x^3\right] \qquad \text{near l.c. "time"}$$

$$x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^3\right) \qquad \text{"space"}$$

Quantization on a space like finite interval L in x^-:

$$\Delta s^2 = \Delta x^- \Delta x^+ + \Delta x^+ \Delta x^- - \frac{2\eta^2}{2}\Delta x^{-2} - \Delta x_\perp^2 = -\eta^2 L^2;$$

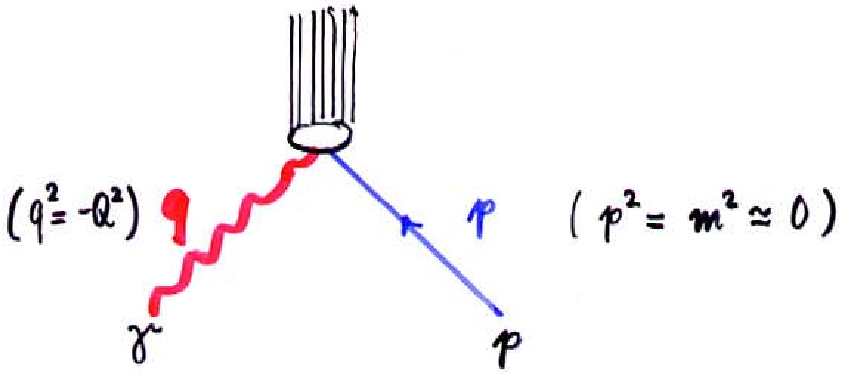
η-formalism can always be related to equal time results with a "boost":

$$\beta = \frac{1-\eta^2/2}{1+\eta^2/2} \qquad \text{or} \qquad \gamma = \frac{1}{\sqrt{2}}\frac{1}{\eta}$$

---

* First proposed by:
E.V. Prokhvatilov & V.A. Franke Sov. J. Nucl. Phys. 49 (688) 1989
See also: S. Hellermann & J. Polchinski hep-th/9711037

γ^*-p Scattering



Bjorken $x = \frac{Q^2}{s}$;

Low $x \iff$ very high cm energy^2 $s = (p+q)^2$

Define two light like vectors: $e_i^2 = 0$

$$e_1 = q - \frac{q^2}{2pq}\,p = q + xp\,;$$

$$e_2 = p\,;$$

Then the photon direction e_η is nearly light like

$$e_\eta = e_1 - \frac{\eta^2}{2}\,e_2$$

$$e_\eta = q + xp - \frac{\eta^2}{2}\,e_2$$

$$\boxed{x = \frac{\eta^2}{2}}$$

Low x scattering $\iff$ near light cone QCD

(η small)

Modified Light Cone Gauge $\partial_- A_- = 0$ and the resolution of Gauss Law

see also: F. Lenz, M. Thies, R. Naus Ann. of Phys. 233, 17, 317 (1994)

$$\mathcal{L} = \frac{1}{2} F_{+-}^{a} F_{+-}^{a} + \sum_{i=1,2} \left(F_{+i}^{a} F_{-i}^{a} + \frac{\eta^2}{2} F_{+i}^{a} F_{+i}^{a} \right)$$

$$- \frac{1}{2} F_{12}^{a} F_{12}^{a} - \rho \cdot A$$

→ Hamiltonian: only possibility to do scattering

① Weyl gauge $A_+^a = 0$ ↷ Fulfill Gauss Law

Canonical momentum: (Electric fields)

$$\Pi_-^{a} = \frac{\delta \mathcal{L}}{\delta F_{+-}^{a}} = F_{+-}^{a}$$

$$\Pi_i^{a} = \frac{\delta \mathcal{L}}{\delta F_{+i}^{a}} = F_{-i}^{a} + \eta^2 F_{+i}^{a}$$

② Hamiltonian $\mathcal{H} = \Pi \dot{Q} - \mathcal{L}$

$$\mathcal{H} = \frac{1}{2} \Pi_-^{a} \Pi_-^{a} + \frac{1}{2} F_{12}^{a} F_{12}^{a} + \rho A + \frac{1}{2} \eta^2 F_{+i}^{a} F_{+i}^{a}$$

$$\mathcal{H} = \underbrace{\tfrac{1}{2}\Pi_-^a \Pi_-^a} + \tfrac{1}{2} F_{12}^a F_{12}^a + \frac{1}{2\eta^2}\left(\Pi_i^a - F_{-i}^a\right)^2 ;$$

③ Modified Axial Gauge : $\partial_- A_- = 0$

Cannot make $A_- = 0$, because Polyakov $P(x_\perp)$.

$$P = \frac{1}{N_c}\,\mathrm{tr}\, e^{\,ig\int_0^L dx_- \, A_-^a(x_\perp, x_-)\frac{\tau^a}{2}}$$ is gauge invariant

Because of gauge freedom choose zero mode in 3-dir.

$$a_-^3(x_\perp) = \frac{1}{L}\int_0^L dx_- \, A_-^3(x_\perp, x_-) . \quad (SU2_{\text{color}})$$

$(SU^3_{\text{color}} \oplus a_-^8)$

④ Eliminate Π_-^a with the help of Gauss Law:

Vacuum $\quad Q^3 = \int \rho^3(x^-, \vec{t})\, d\vec{t}\, dx^- = 0 ;$

$$D_- \Pi_-^a |\phi\rangle = \left(D_\perp \Pi_\perp^a + g\rho\right)|\phi\rangle = G_\perp(x_\perp, x^-)\,|\phi\rangle$$

Solution = sol. homogeneous eq. + sol. inhomog. eq.

$$\Pi_-(x_\perp, x^-) = \underline{p_-^3(x_\perp)} + \underbrace{\left(\frac{1}{\partial_- - iga_-^3}\right) G_\perp(x_\perp, z)}_{\text{Coulomb Pot.}}$$

zero mode conjugate to $a_-^3(x_\perp)$

Nachtmann, Dosch, Krämer, ... H.J.P. → soft pomeron

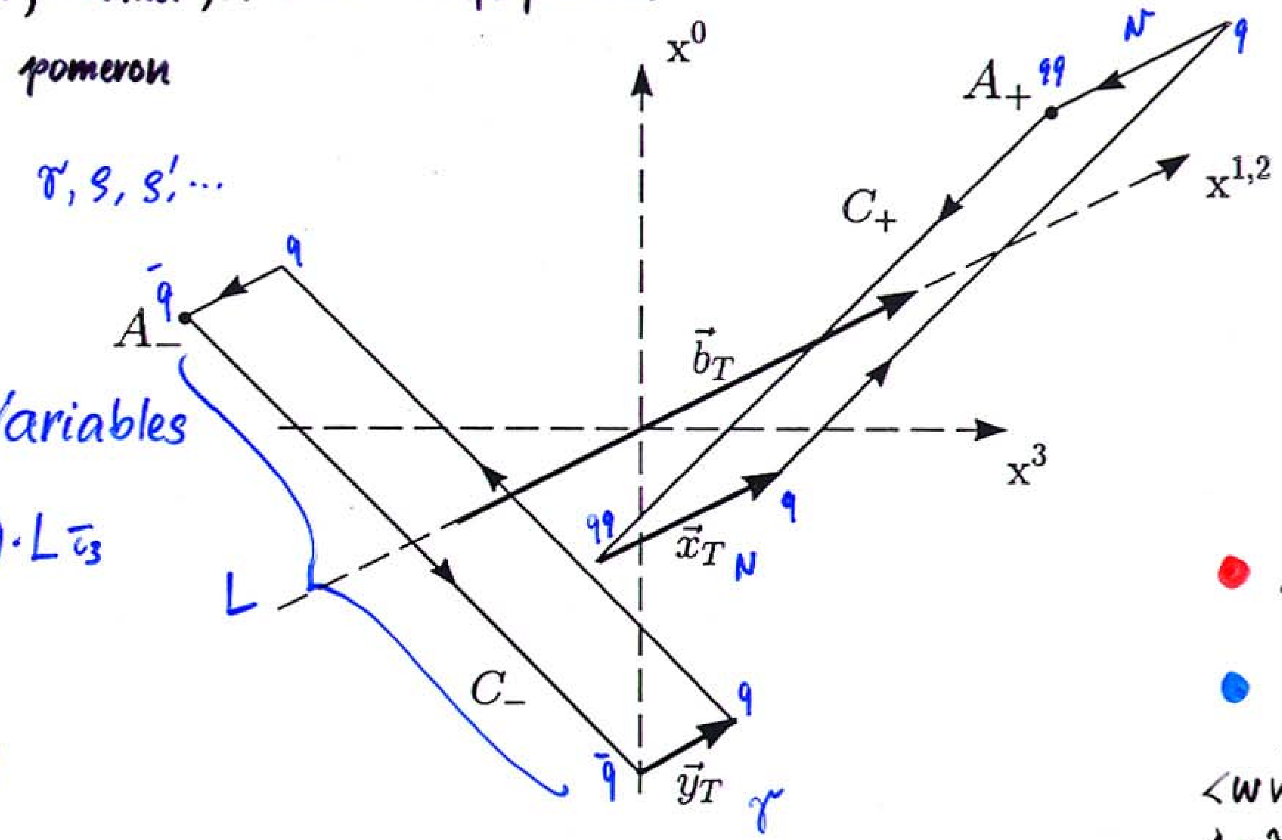
Balitsky → hard pomeron

$\gamma, \rho, \rho', \ldots$

"Collective" Variables

$e^{ig\cdot\frac{1}{2}a_-^3(x_\perp)\cdot L\tau_3}$

$e^{i\varphi(x_\perp,\tau)\tau_3}$



New

● Minimal Surfaces

● Cumulant Expansion
in SU(3)×SU3 space
$\langle WW\rangle = \frac{2}{3}e^{-i\frac{1}{3}\chi} + \frac{1}{3}e^{-i\frac{2}{3}\chi}$
$\frac{\lambda}{2}\otimes\frac{\lambda}{2} = \frac{1}{3}P_{\square\square} - \frac{2}{3}P_{\boxminus}$

Abbildung 1.4: Die lichtartigen, aufgeschnittenen Wegner-Wilsonloops im Minkowskiraum, $C_\pm$, bestehend aus zwei lichtartigen Linien in den Hyperebenen $x_\mp = 0$ und Verbindungsstücken an den Enden. Die Loops sind jeweils an einer Ecke A_+ bzw. A_- aufgeschnitten. Die Mittelpunkte der Loops im transversalen Raum liegen bei $\pm\boldsymbol{b}_T/2$, die Vektoren von den Antiquarks (diquark) zu den Quarks (quark) sind gegeben durch $\boldsymbol{x}_T$ bzw. $\boldsymbol{y}_T$.

● Add perturbative part: χ_P

Im Rahmen der Hochenergienäherung ergibt sich daher folgendes Bild: Die Quarks laufen auf lichtartigen Trajektorien durch ein gemeinsames Eichpotential $\tilde{G}$ und sam-

Put $\mathcal{H}^\eta$ on a lattice with lattice constant a:

$$h_{lat} = \sum_{b_\perp, c_0} \Big\{ - g_{eff}^2 \frac{1}{J} \frac{\delta}{\delta\varphi^{c_0}(b_\perp)} J \frac{\delta}{\delta\varphi^{c_0}(b_\perp)}$$
$$+ \frac{1}{g_{eff}^2} \sum_{\vec{\varepsilon}} \left(\varphi^{c_0}(\vec{b}) - \varphi^{c_0}(\vec{b}+\vec{\varepsilon})\right)^2$$
$$- \frac{4a}{\eta L} \langle \psi_-^\dagger \psi_- \rangle^{c_0} \varphi^{c_0}(b_\perp) \Big\}$$

g_{eff}^2 = effective coupling of dimensional reduced theory

$$= \frac{g^2 \cdot L \cdot \eta}{4a}$$

$\varphi^{c_0}(b_\perp) = \frac{1}{2} g L a_-^{c_0}(b_\perp)$ "angle variables"

defined in the fundamental domain

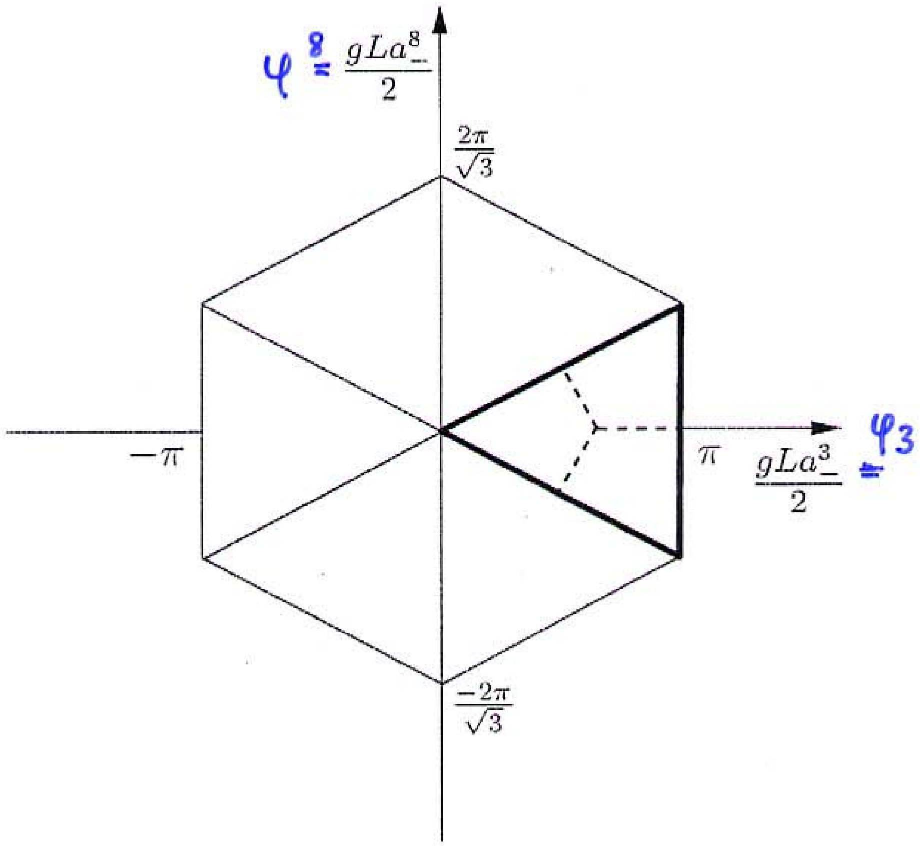
J = Jacobian for SU3

$$= \sin^2(\varphi^3) \sin^2\left(\tfrac{1}{2}(\varphi^3 - \sqrt{3}\varphi^8)\right) \sin^2\left(\tfrac{1}{2}(\varphi^3 + \sqrt{3}\varphi^8)\right)$$

$(= \sin^2\varphi^3$ for SU2$)$

Fundamental domain of gauge fields φ^3, φ^8

FIGURES



The fundamental domain of gauge field variables $\varphi_3 = \frac{gLa^3_-}{2}$ and $\varphi_8 = \frac{gLa^8_-}{2}$.

Kinetic Energy: $g^2_{eff} \cdot \vec{E}^2$ ~ rotational velocity^2

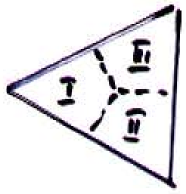
$$\frac{1}{J} \frac{\delta}{\delta\varphi} J \frac{\delta}{\delta\varphi}$$

Potential Energy: $\frac{1}{g^2_{eff}} \vec{B}^2$ ~ (nearest neighbour distance)^2

External field: $\langle \psi^+_- \psi_- \rangle^{c_0}$

Lattice calculation in SU2 gave a
Ilgenfritz, Ivanov & HJP PRD 62 (2000)

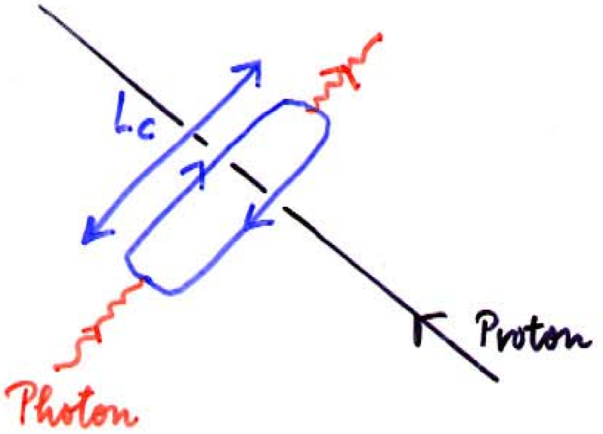
2nd order phase transition without external field

Symmetry of gauge fields	Spin-model
$[0, \pi/2] \leftrightarrow [\pi/2, \pi]$	up $\leftrightarrow$ down
$\mathcal{H}_\eta$ (SU2)	Ising Modell $Z(2)$
	three spins
$\mathrm{I} \leftrightarrow \mathrm{II} \leftrightarrow \mathrm{III}$	
$\mathcal{H}_\eta$ (SU3)	$Z(3)$

Extrapolate finding from SU2 to SU3, relevant Universality Class of Wilson lines is the same as $Z(3)$ spin theory; which has a second order endpoint for coupling and external field variation.
Study $\langle P(x_\perp)\, P(0) \rangle \sim e^{-x_\perp/\xi}$: Wilson line correlation function.

Match near light cone Hamiltonian

$\mathcal{H}^{\eta}$ with low x-scattering : $\eta = \sqrt{2x}$



Color coherence length of dipoles in photon:

$$L_c = \frac{1}{\Delta k^+} = \frac{1}{Q}\frac{1}{\sqrt{x}} \; ; \quad \text{and} \quad a \approx \frac{1}{Q} \; ;$$

Choose lattice $\frac{L}{a} = \frac{1}{g^2}\left(N_0 + \frac{c}{aQ\sqrt{x}}\right) \Rightarrow$

$$g^2_{eff} = \frac{g^2 L \eta}{4a} \longrightarrow g^{*2}_{eff} = \frac{c}{2\sqrt{2}}$$

$$\hat{t} = \frac{g^2 L \eta}{4a} - g^{*2}_{eff} \approx t_0 \sqrt{x}$$

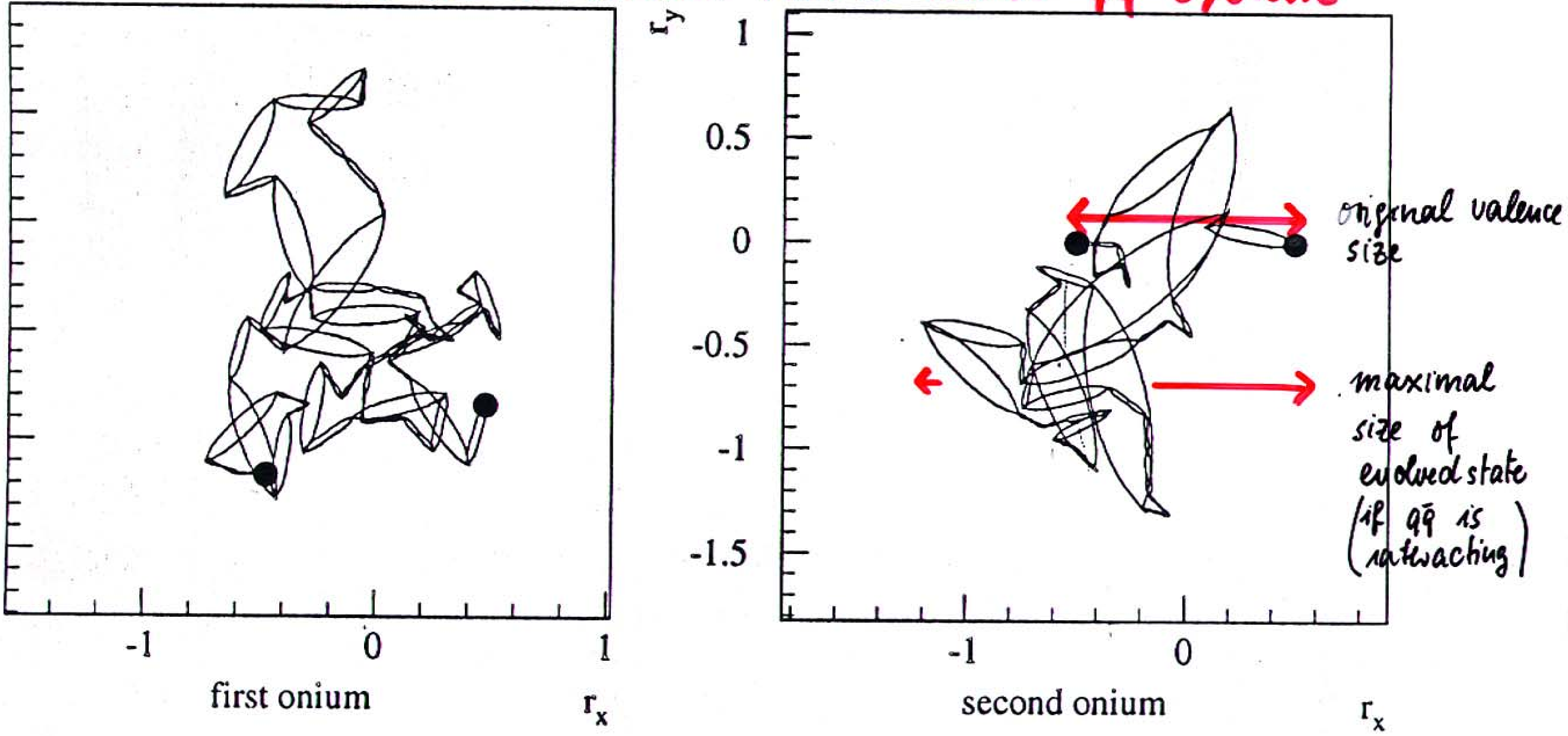
$$h = \frac{4a \langle \psi_-^{+} \psi_- \rangle}{\eta L} \longrightarrow h^* = \frac{4g^2 \langle \psi_-^{+} \psi_- \rangle}{\sqrt{2}\, c}$$

$$\hat{h} = |h - h^*| \approx h_0 \sqrt{x}$$

G.P. Salam/Nuclear Physics B 461 (1996) 512–538

A. H. Mueller / NP B 415 (1994) 373

Low x - evolution into a dense $q\bar{q}$ system



Valenz state $(Q\bar{Q})$ [• •] develops into $Q\bar{Q}$ $q\bar{q}$ $q\bar{q}$ $q\bar{q}$ $q\bar{q}$ ... (large N_c)

as $x \to 0$. Picture shows evolution into many dipole state $(y = 10)$ $(\sqrt{s} = m_t e^{y})$

Correlation length ξ

Substituting $\hat{h} = h_0 \sqrt{x}$ & $\hat{t} = t_0 \sqrt{x}$ gives

$$\boxed{\begin{aligned} \xi/a &= \left(\frac{x}{x_0}\right)^{-\frac{1}{2\lambda_2}} f_h\left(d \cdot x^{0.18}\right) \\ &\approx \left(\frac{x}{x_0}\right)^{-\frac{1}{2\lambda_2}} f_h(0) \end{aligned}}$$

$Z(3) \leftrightarrow$ Ising (3dim) : $\lambda_2 = 2.52$

$f_h(r)$ has been measured (J. Engels) and

$$f_h(r) \xrightarrow[r \to 0]{} \text{const}$$

Wilson line correlations

$$\langle P(x_\perp)\, P(0) \rangle \propto \frac{1}{x_\perp^{1+n}} \qquad (n = 0.04)$$

for $1/Q < x_\perp < \xi$

Power behavior responsible for critical opalescence in liquid-gas transition

$$\langle P(x_\perp)\, P(0) \rangle \propto e^{-x_\perp/\xi}$$

for $x_\perp > \xi$

Effective Photon wave function :

$$\rho_\gamma^T = |\Psi_\gamma^T(z, x_\perp)|^2 = \frac{6\alpha}{4\pi^2} \sum_f \hat{e}_f^2 \, \varepsilon^2 [z^2 + (1-z)^2] \, F_T(\varepsilon x_\perp)$$

$$\rho_\gamma^L = \frac{6\alpha}{4\pi^2} \sum_f \hat{e}_f^2 \, 4Q^2 z^2 (1-z)^2 \, F_L(\varepsilon x_\perp)$$

$$\varepsilon = \sqrt{Q^2 z (1-z)}$$; perturbative inverse size of photon dipole

$$\boxed{\xi = \left(\frac{x}{x_0}\right)^{-\frac{1}{2\lambda_2}} \cdot \frac{1}{\varepsilon}}$$

determines effective size for $x < x_0$

- **Perturbative region:**

$$x_\perp < \frac{1}{\varepsilon} \qquad F_{T/L}(\varepsilon x_\perp) = |K_{1/0}(\varepsilon x_\perp)|^2$$

- **Scaling region:**

$$\frac{1}{\varepsilon} < x_\perp < \xi \qquad F_{T/L}(\varepsilon x_\perp) = \frac{|K_{1/0}(1)|^2}{(\varepsilon x_\perp)^{2+2n}}$$

- **Exponentiell decay**

$$x_\perp > \xi \qquad F_{T/L}(\varepsilon x_\perp) = \frac{|K_{1/0}(\frac{x_\perp}{\xi})|^2}{(\varepsilon \xi)^{2+2n}}$$

$\gamma^* p$ - Cross section :

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha} \left(\sigma_{\gamma p}^{T} + \sigma_{\gamma p}^{L} \right)$$

$$\sigma_{\gamma p}^{T/L} = \int d^2 x_\perp \int dz \; \rho_\gamma^{T,L}(x_\perp, z)\, \sigma_{dip}(x_\perp)$$

Use dipole - proton cross section at fixed

$x_0 = 10^{-2}$

- Golec-Biernat Wüsthoff

$$\sigma_{GBW}(x_\perp, R_0) = \sigma_0 \left(1 - e^{-x_\perp^2/4R_0^2} \right)$$

$$R_0 = \frac{1}{1\,GeV} \left(\frac{x_0}{3 \cdot 10^{-4}} \right)^{0.145} = 0.33 \text{ fm}$$

$$\sigma_0 = 23 \text{ mb}$$

- GBW - simplified

$$\sigma(x_\perp) = \sigma_0 \left(\underbrace{\frac{x_\perp^2}{4R_0^2}\, \theta(2R_0 - |x_\perp|)}_{\text{dipole growth}} + \underbrace{\theta(|x_\perp| - 2R_0)}_{\text{constant}} \right)$$

Proton Structure function $F_2(x, Q^2)$

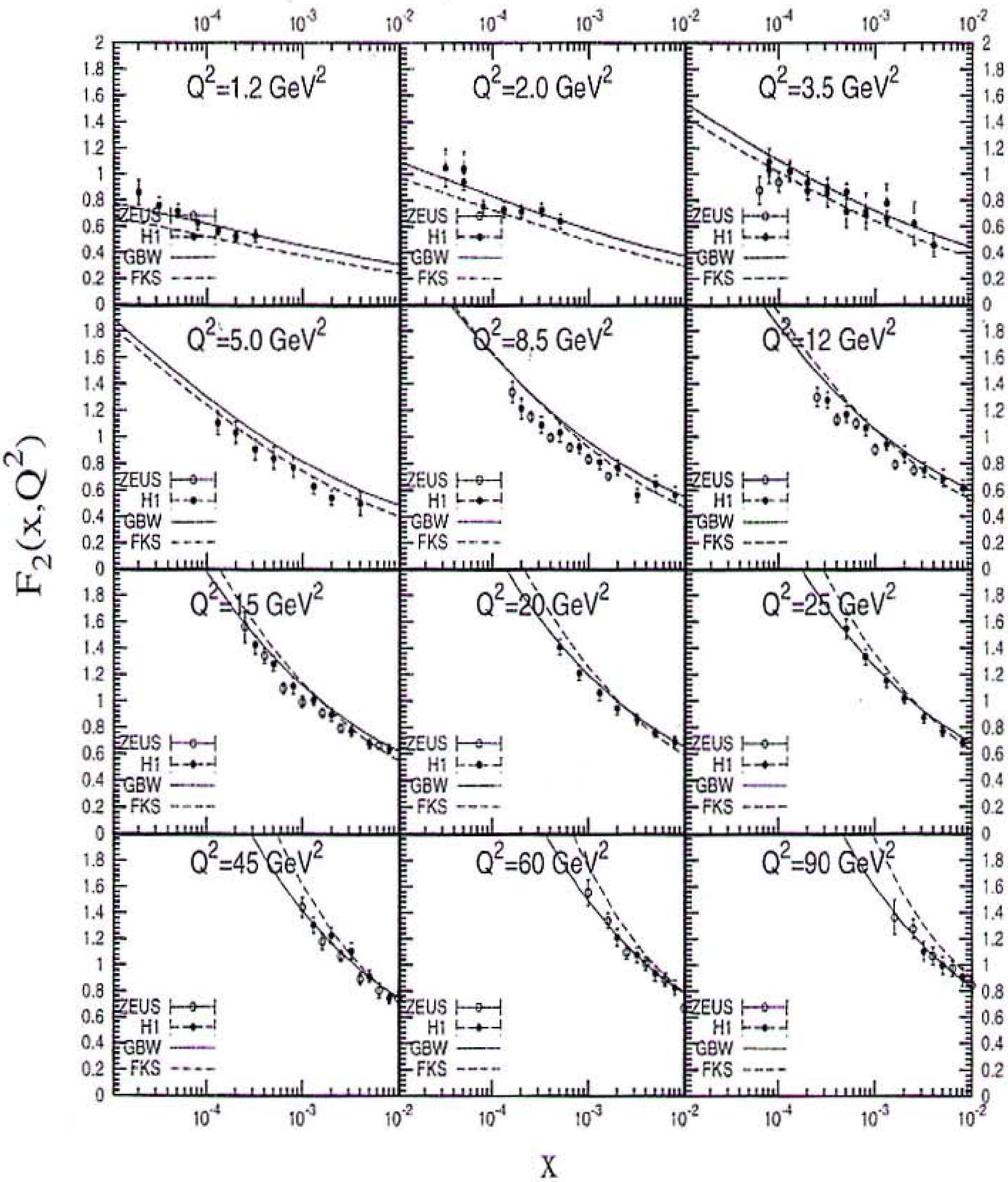


FIG. 2. Proton structure function F_2 as a function of x for various Q^2. The experimental data are from H1 and ZEUS at HERA [27,28]. The curves are the theoretical results of the critical theory with the functions $F_{T/L}(\epsilon x_\perp)$ of Eq. (30).

Data : H1 & Zeus

Theory : Critical dynamics with $\sigma^{GBW}(x_\perp, x_0)$ ——

with $\sigma^{\text{Foreshaw et al}}(x_\perp, s) = \sigma^{FKS}(x_\perp \frac{Q^2}{x_0})$ - - - -

Try other dipole - proton cross section:

σ Foreshaw, Kerley & Shaw $(x_1, s) =$

$\sigma_{soft}(s, r) \quad + \quad \sigma_{hard}(s, r)$

$\sigma_{soft}(s, r) = g_1(r)\, s^{0.06}$

$\sigma_{hard}(s, r) = g_2(r)\, s^{0.38}$

For $\sigma^{\text{GBW-simple}}$ get scaling with $\tau = Q^2 R_0^2 \left(\frac{x}{x_0}\right)^{\frac{1}{\lambda_2}}$

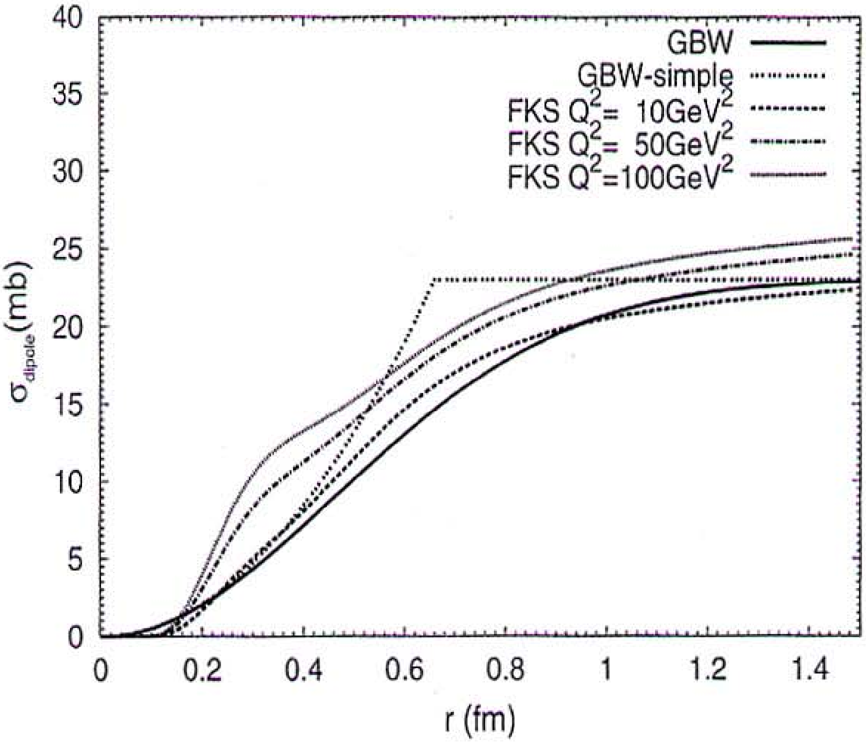


FIG. 3. Different dipole-proton cross sections as a function of the dipole size r. The full drawn curve is the Golec-Biernat-Wuesthoff cross section [26] at $x_0 = 10^{-2}$,the dashed curves are the Forshaw et al. [31] cross sections at the same $x_0 = 10^{-2}$ but different $Q^2 = 10\text{GeV}^2, 50\text{GeV}^2$ and 100 GeV^2. The curve marked (GBW-simple) gives the approximation in Eq. (35) to the GBW

Approximate calculation of $\gamma^* p$ cross section:

- Use $\sigma^{\text{GBW- simple}}$
- $n = 0.04 \to 0$
- neglect exponentially small piece of w.f.

$$\sigma_T^{\gamma^* p} = \frac{3\alpha}{\pi} \sum e_f^2 \, Q^2 \sigma_0 \int dz \, (z^2 + (1-z)^2) \int r\,dr \, \frac{z(1-z)}{r^2 \varepsilon^2}$$

$$\theta(1 - r^2/\xi^2) \; \frac{\sigma(r)}{\sigma_0}$$

Rescale: $\quad r' = r \left(\frac{x}{x_0}\right)^{\frac{1}{2\lambda_2}}$

$$\sigma_T^{\gamma^* p} = \frac{3\alpha}{\pi} \sum \hat{e}_f^2 \, Q^2 \sigma_0 \int dz \, (z^2 + (1-z)^2) \int r' dr' \, \frac{z(1-z)}{r'^2 \varepsilon^2}$$

$$\theta(1 - \varepsilon^2 r'^2) \; \frac{\sigma^{\text{GBW}}(r', R(x))}{\sigma_0}$$

We understand $\quad R(x) = R_0 \left(\frac{x}{x_0}\right)^{\frac{1}{2\lambda_2}} \quad$ in GBW cross section

$$\frac{1}{2\lambda_2} = 0.2$$

& $Q^2 R^2(x)$ scaling of the cross section $\sigma_{\gamma^* p}$

Conclusions

- Microscopic Understanding of Low x Scattering with Wilson Lines

- Z(3) symmetry determines growth of $\gamma^* - p$ cross section

- Quantitative description of F_2

- New critical point in QCD at infinite energies

- Running $R(x)$ in σ^{GBW} and scaling of cross section with $Q^2 R(x)^2$

- Clarify fermion zero modes
  Full calculation with the proton source