Oscillating Hadron and Jet multiplicity moments

Wolfgang Ochs
(Max Planck Institute, Munich)

1. **Theoretical approach**
2. **Multiplicity moments for hadrons and jets**
3. **QCD calculations**
   - **DLA**
   - **MLLA**
   - **Monte Carlo $\longleftrightarrow$ experiment**
4. **Conclusions**

Recent work with
Matt Buican & Clemens Förster
hep-ph/0307234 to appear in Eur.Phys.J.C

Motivation

- Simple ansatz for hadronization
  "Local Parton Hadron Duality (LPHD)"

Azimov, Dokshitzer
Khoze, Troyan

inclusive spectra in $\xi = \ln(1/x)$

$$D(\xi)|_{\text{hadron}} = K \times D(\xi)|_{\text{parton}}$$

- Analytical results for jet structure
  and soft particle production

Deep principle or accident?

$\downarrow$

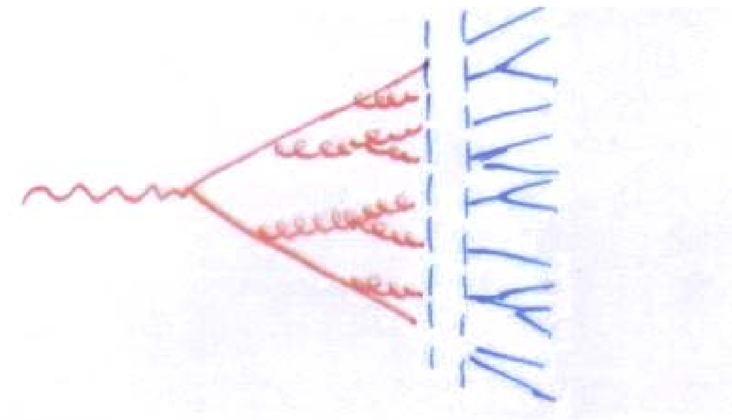
$\rightarrow$ check more complex observables

- correlations between hadrons
- jet resolution (y_{cut}) dependence

$\rightarrow$ increase accuracy of QCD calculations

Theoretical approach

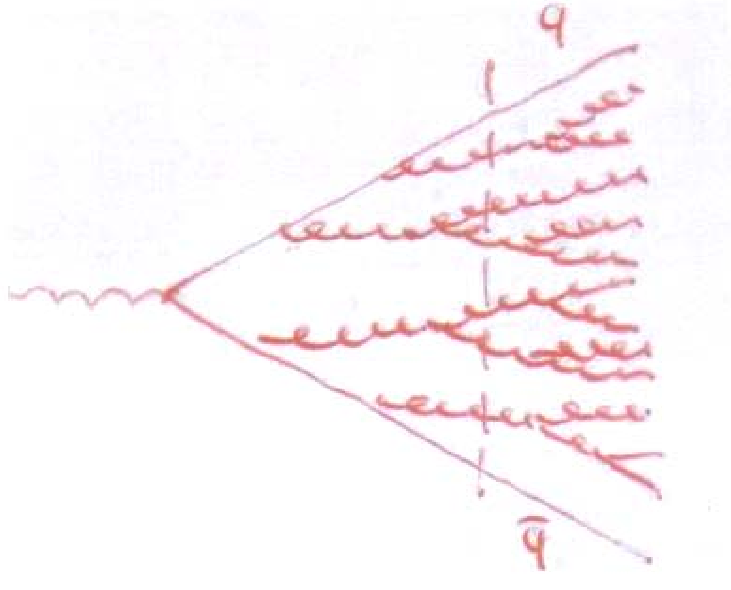
Hadronization model



hadrons
resonances

parameter Λ $\qquad$ $Q_0 \sim 1 - 2$ GeV

Parton Hadron Duality



hadrons
$\sim$ partons

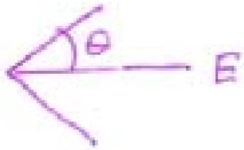
parameter Λ $\qquad$ $k_T \geq Q_0 > \Lambda$ (few 100 Mev)

QCD cascade evolves towards lower scale
results directly compared to hadron final state

Perturbative QCD predictions

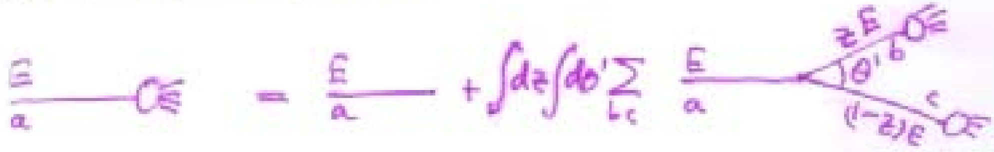
Multiplicity generating function *Dokshitzer et al.*

$$Z(Q,u) = \sum_{n=1}^{\infty} P_n(Q) u^n$$



$$Q = E\Theta$$

integral evolution equation



differential evolution equation

$$\begin{aligned}
\frac{d}{dY_c} Z(Y_c,u) &= \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_T)}{2\pi} P_{gg}(z) \times \\
&\quad \{Z(Y_c + \ln z, u) Z(Y_c + \ln(1-z), u) - Z(Y_c, u)\} \\
Z(0,u) &= u
\end{aligned}$$

with $\quad Y_c = \ln \frac{E}{Q_c}, \quad \tilde{k}_T = \min(z, 1-z)E$

- non perturbative cut-off $\quad k_T \geq Q_0$
- running coupling $\alpha_s(k_T)$ 1-loop

asymptotic solutions

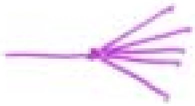
- DLA: $P_g^{gg}(z) \sim \frac{1}{z}$ for gluon emission
- MLLA: include next to leading $\log$ terms

full solution of evolution equation

- numerical solution of evolution eqn.
  (Complete up to next to leading $\log$)
- Monte Carlo generation of parton cascade
  ARIADNE-D
  $\Lambda = 400$ MeV $\lambda = \ln \frac{Q_0}{\Lambda} = 0.01$
  only light quarks
  full matrix element one-loop

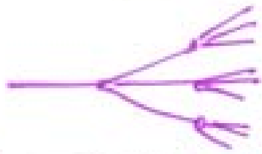
Multiplicity moments

hadrons



n_{had}

jets



n_{jet}

factorial moments

$f_q = < n(n-1)\ldots(n-q+1) >$

$F_q = f_q / < n >^q$

kumulant moments (genuine correlation) $k_q,\ K_q$

$$F_q = \sum_{m=0}^{q-1} \binom{q-1}{m} K_{q-m} F_m$$

$$K_2 = F_2 - 1, \quad K_3 = F_3 - 3F_2 + 2, \ldots$$

H_q-moments $\quad H_q = \frac{K_q}{F_q}$

Double log approximation

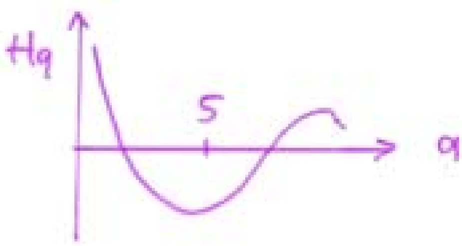
$$\bar{n} \sim \exp(2\beta\sqrt{\ln Q/Q_0})$$

$$f_q \sim < n >^q \qquad F_q \to \text{const}(q)$$

$$H_q \sim 1/q^2$$

MLLA + higher orders

Dremin et al.



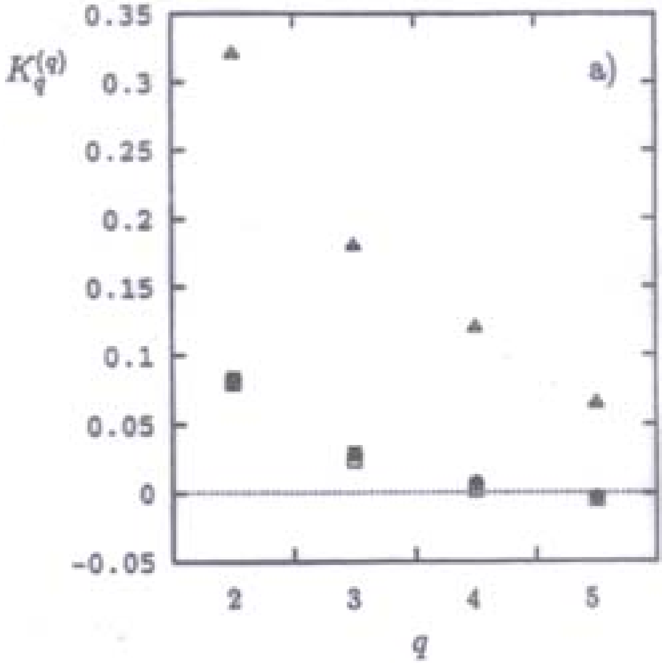
$$q_{min} = \frac{1}{h_1\gamma_0} + \frac{1}{2} + \mathcal{O}(\gamma_0)$$

$$\gamma_0 \sim \sqrt{\alpha} \quad h_1 = \frac{1}{24}$$

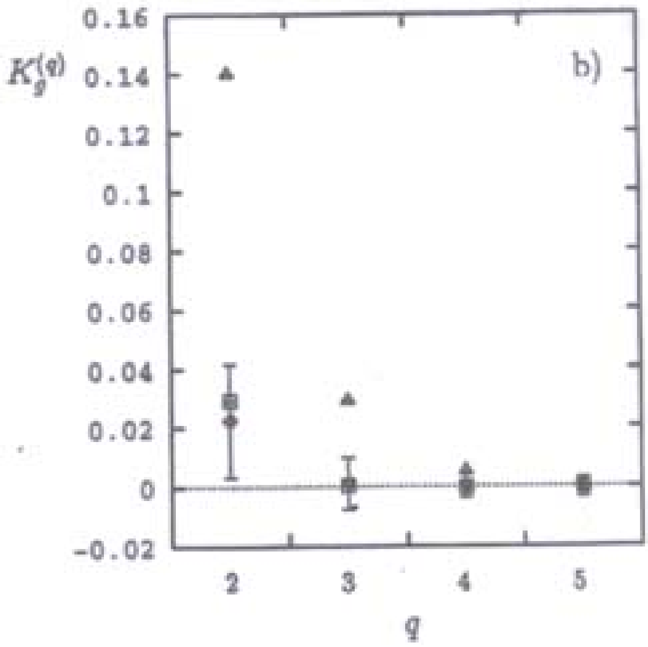
Kumulant multiplicity moments

- ◇ Asymptotic LL. expansion in $\sqrt{\alpha_s}$ *Dremin*
- ◇ numerical solution of ev.eqn. *Lupia Ochs*

OPAL data: quark jets



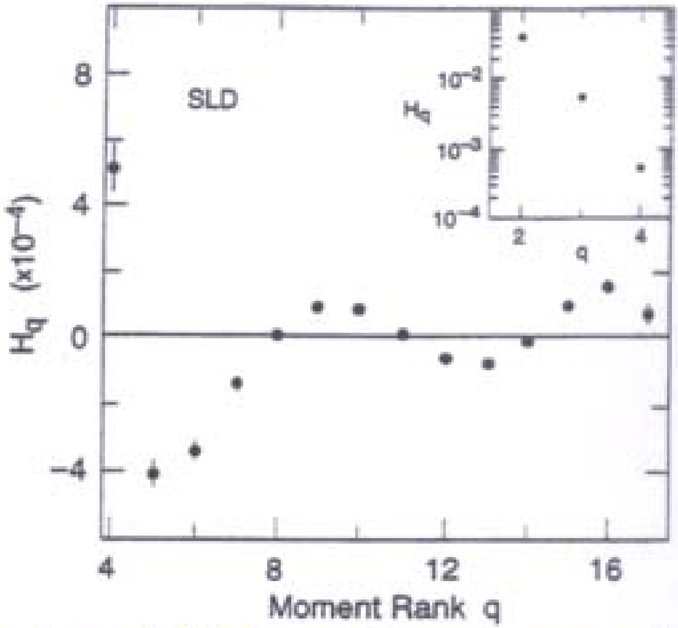
gluon jets



H_q Moments at $Q = 91$ GeV

Hadrons

SLD 1996, SLAC



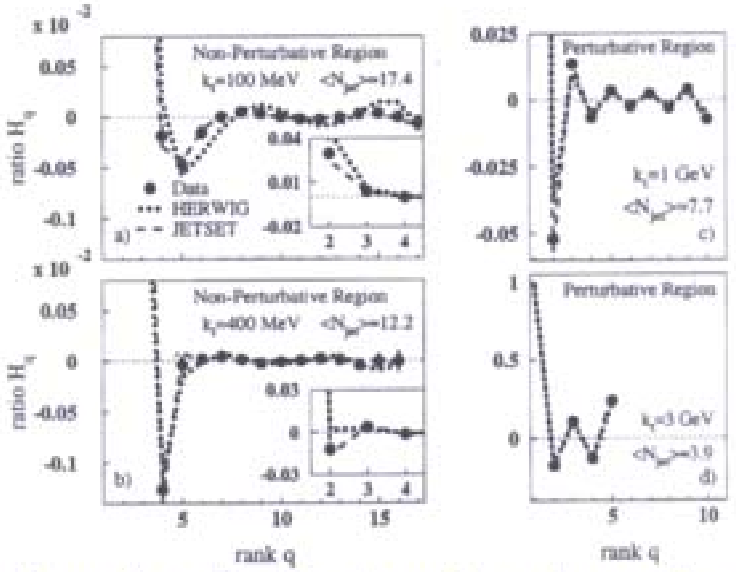
but: normalization of H_q far off from asymptotic predictions

Jets at resolution $y_{\rm cut}$
(Durham algorithm $y_{\rm cut} = k_T^2/Q^2$)

L3 2001, CERN

$k_T=$
100 MeV

400 MeV



1 GeV

3 GeV

strong variation of oscillation length and amplitude with y_{cut}

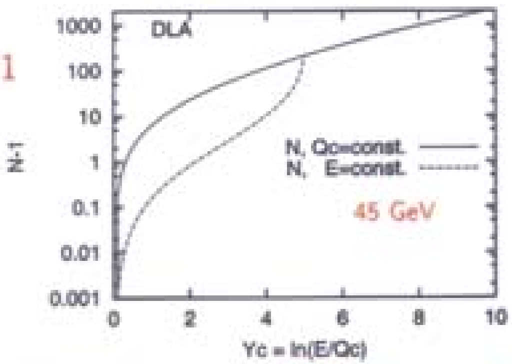
Double Logarithmic approximation (DLA)

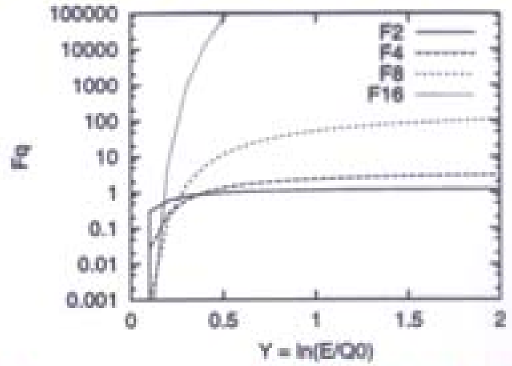
mean multiplicity $\bar{N}$ of partons in jet

$k_T \geq Q_0 \ (\sim \Lambda)$ partons $\sim$ hadrons

$k_T \geq Q_c > Q_0$ partons $\sim$ jets ($y_{\text{cut}} = Q_c^2/Q^2$)

$\bar{N}$-1





energies: $\uparrow_{\text{LEP}}$ $\uparrow_{10^7\text{GeV}}$ $Q_0 = 300$ MeV

exact solution of DLA ev. eqn.

$$N(Y_c) = 2\beta\sqrt{Q_c + \lambda}\left[I_1(2\beta\sqrt{Y_c+\lambda})K_0(2\beta\sqrt{\lambda}) + K_1 I_0\right]$$

$$Y_c = \ln\frac{E_{\text{jet}}}{Q_c}, \quad \lambda = \ln\frac{Q_c}{\lambda}, \quad \beta = \beta(n_f, N_c)$$

high energies E_{jet} $N \sim \exp(2\beta\sqrt{\ln E/Q_c})$
($Q_c = Q_0 =$ const.) $N(0) = 1$

limit $Q_c \to Q_0$ (jet $\to$ hadron) $N \sim \ln 1/\lambda_c$
E_{jet}=const. $\lambda_c \to 0$ Landau pole
fixed coupling α_s: $N \cong (E/Q_c)^{\gamma}$

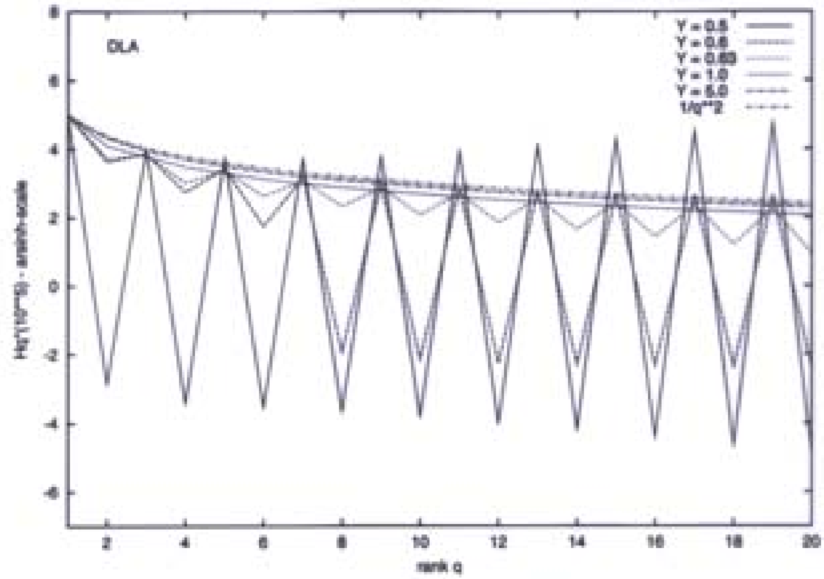
Multiplicity moments

- threshold: $F_q = 0$ for $q > 1$, $K_q = (-1)^{q-1}(q-1)!$
- high energies: $F_q \to \text{const}(q)$, $H_q \sim 1/q^2$

DLA results for ratios $H_q = K_q/F_q$

Hadrons vs. E_{jet}

Ash$_{10}$ $(H_q \cdot 10^5)$



$Y = \ln \frac{E_{\text{jet}}}{Q_0}$

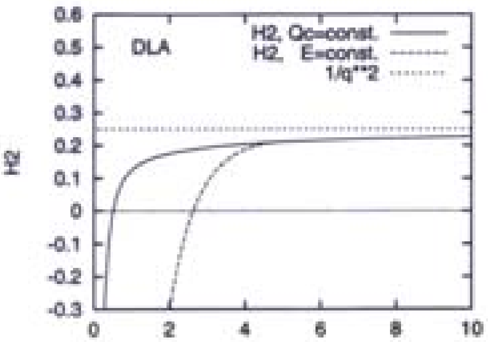
rank q

Ash$_{10}(y)$ = Arsinh$(y/2)/\ln 10$

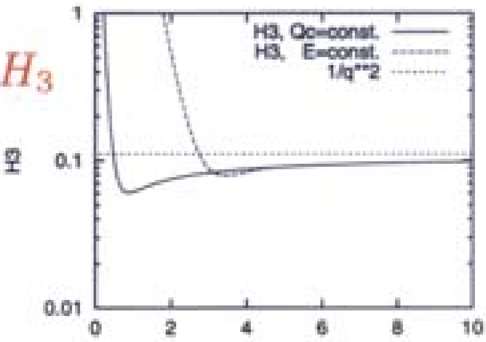
Ash$_{10}(\pm y) \to \pm \log_{10}(y)$ for large y

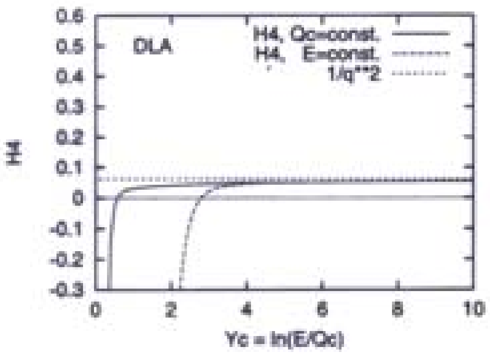
Jets vs. cut-off Q_c

H_2



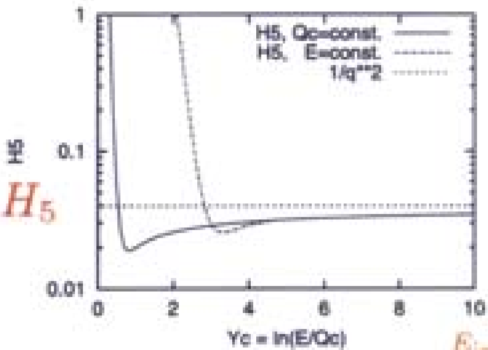
H_3





H_4

H_5

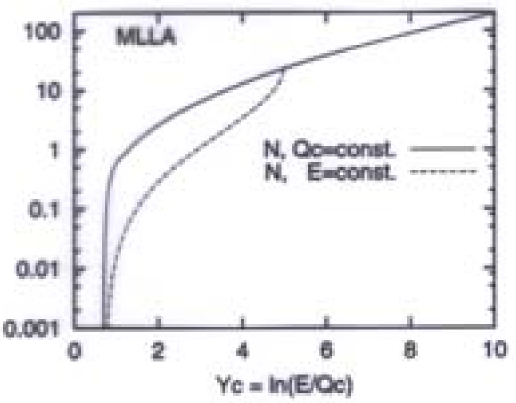


$Y_c = \ln \frac{E_{\text{jet}}}{Q_c}$

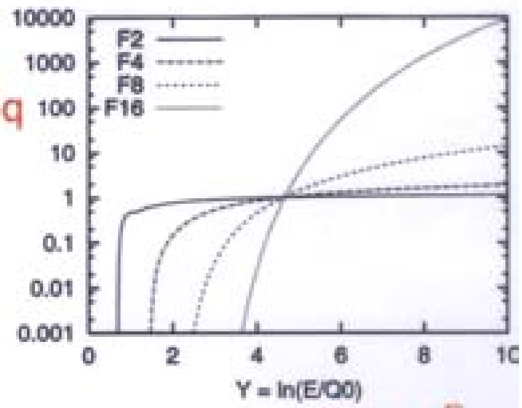
Modified leading log Approx. (MLLA)

mean multiplicity N factorial moments

N-1



Fq



$Y_c = \ln \frac{E}{Q_c}$

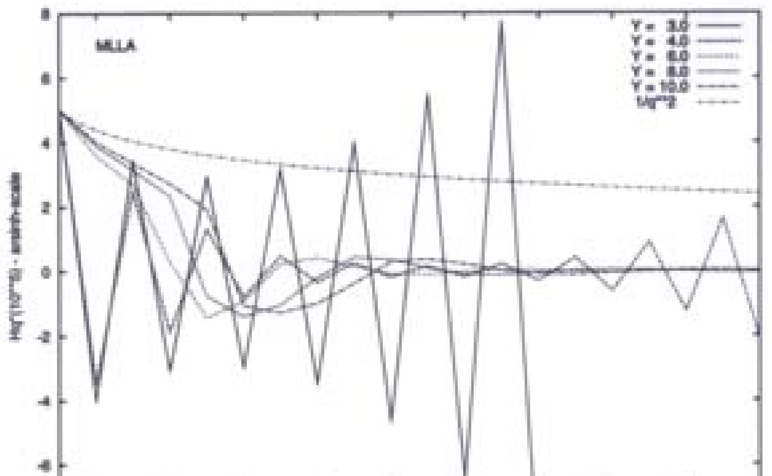
numerical solutions of MLLA Ev. Eq.

- Threshold: $F_1 = 0$ for $E = q \cdot Q_0$
- Poissonian transition point $(F_1 = 1)$ at $Y \approx 4.3$

hadrons: $Q_c = Q_0$

Ash$_{10}$

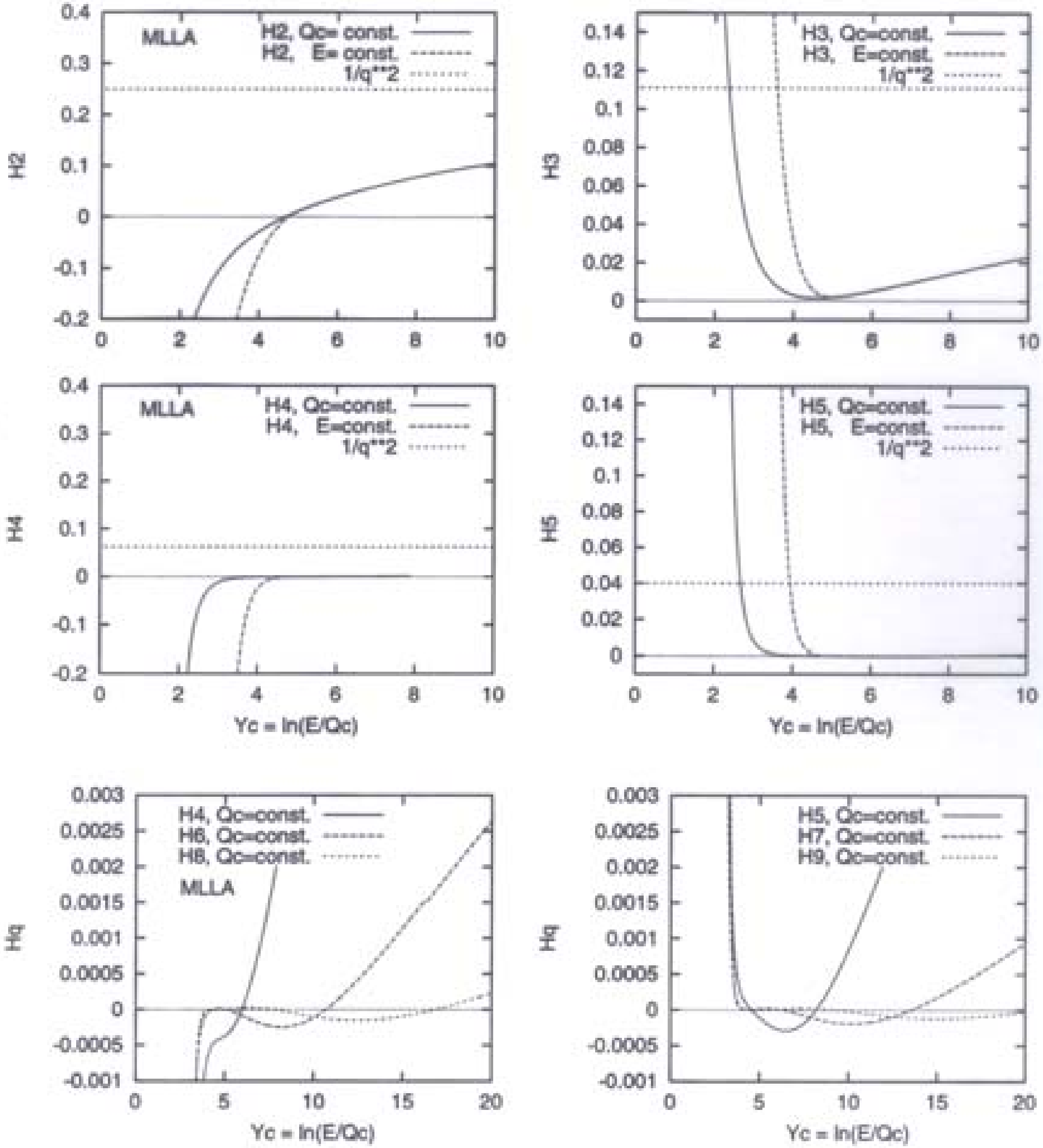
$(H_q \cdot 10^5)$



$\rightarrow$ rank q

- oscillation length increases with E_{jet}
- far away from asymptotic limit

(MLLA cont.'d)

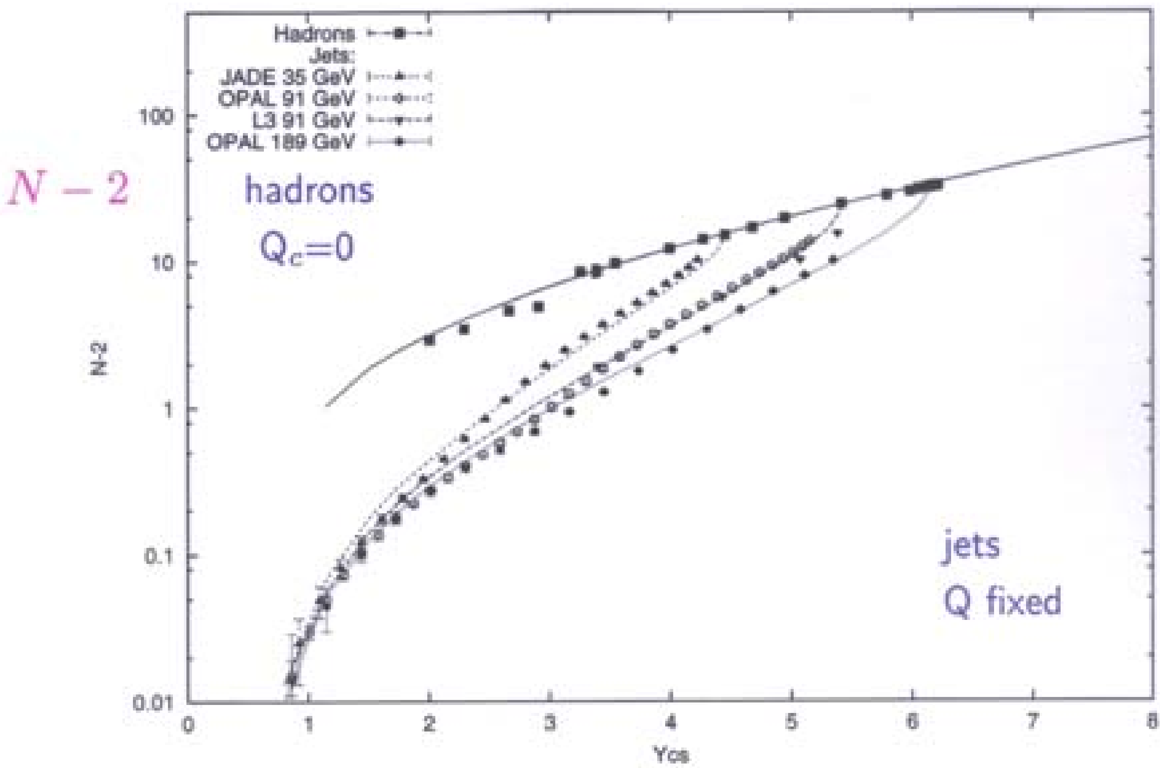


- more minima/maxima than in DLA
- slow approach towards asymptotics (if any)
  ($Y_c = 30 \to\sim 10^{13}$ GeV)

Multiplicities of hadrons and jets in e^+e^-

See also
Lupia/W.O., '98

hadrons: $Y_{cut} = 0$ jets: $Y_{cut} > 0$



$$Y_{cs} = \ln \frac{Q^2}{Q_c^2 + Q_0^2} = \ln \frac{1}{y_{cut} + \frac{Q_0^2}{Q_c^2}}$$

- data include b-decays, computations don't
- splitting of curves due to $\alpha_s(k_T)$

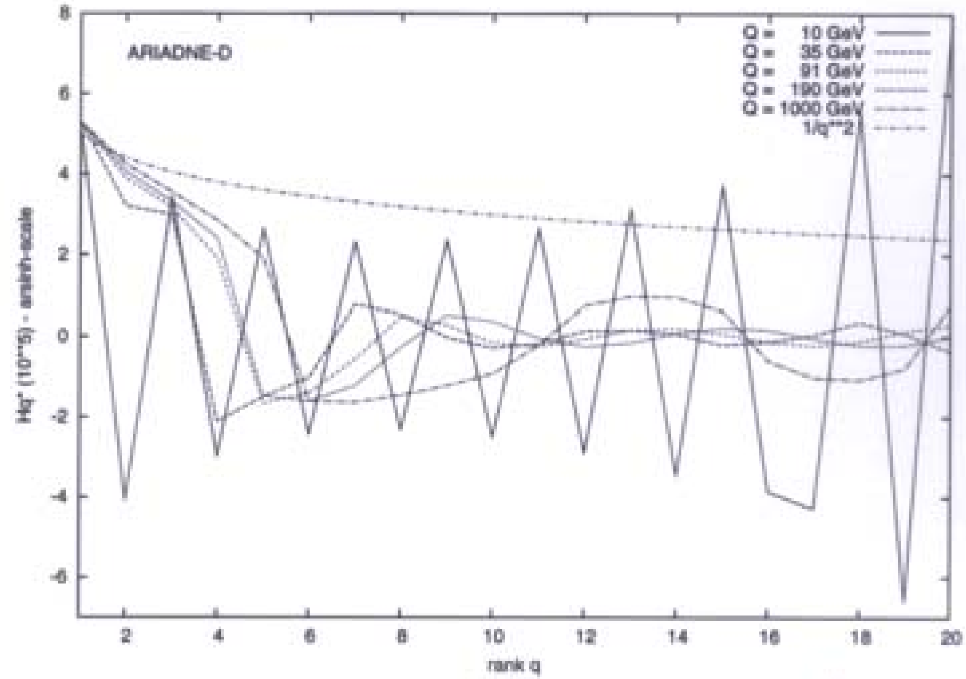
ARIADNE-D $\Lambda = 400$ MeV $Q_0 = 404$ MeV

$N_{had} = K_{ch} N_{ch}$ $K_{ch} = 1.25$

H_q moments

Ariadne - D

Ash$_{10}$
$(H_q \cdot 10^5)$



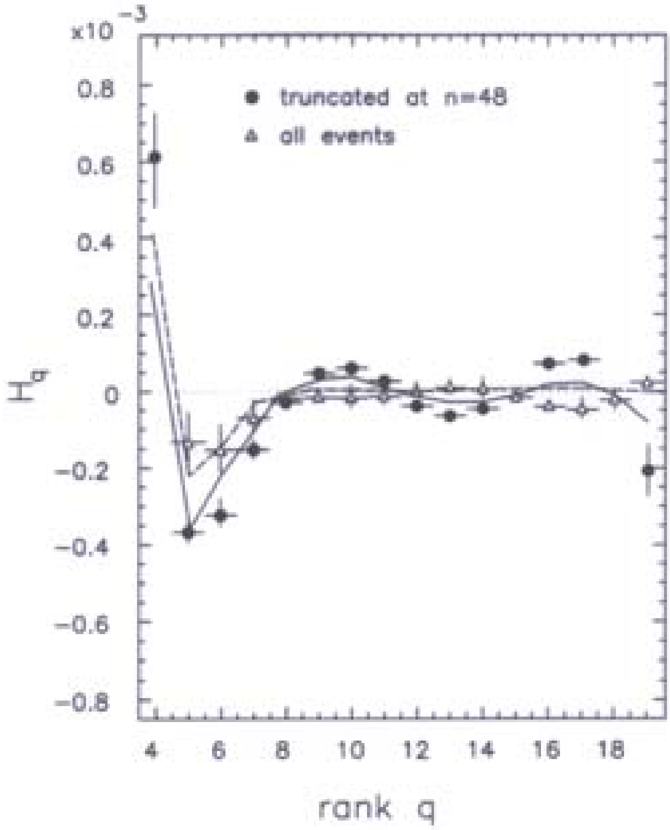
$\rightarrow$ rank q

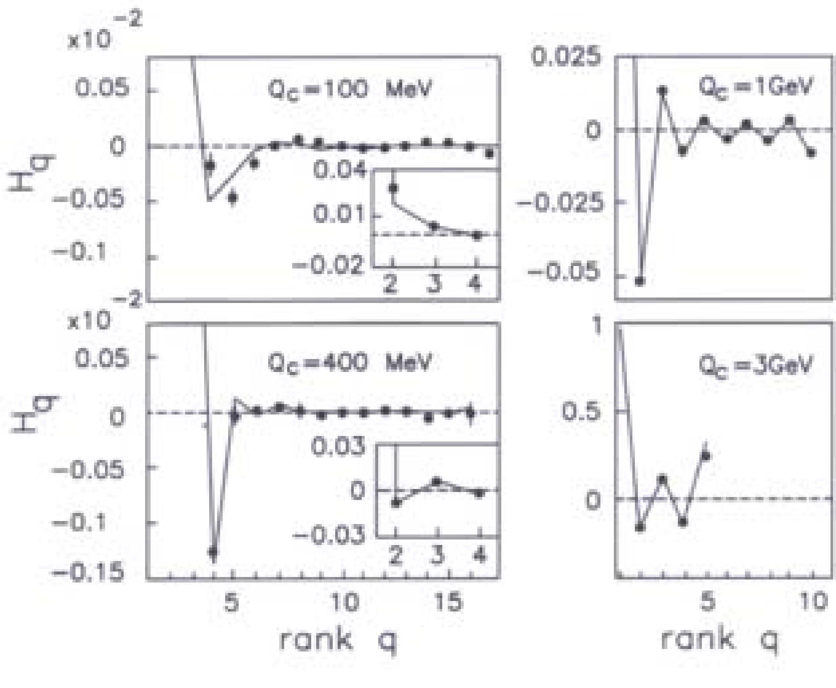
- qualitatively similar to MLLA ev. eq. results

H_q moments

Ariadne - D

L3-data

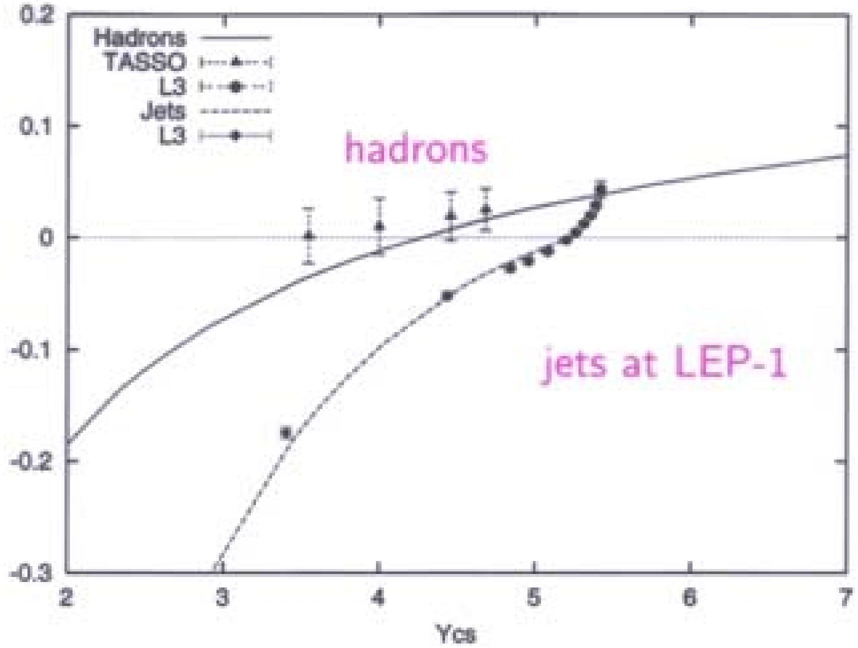




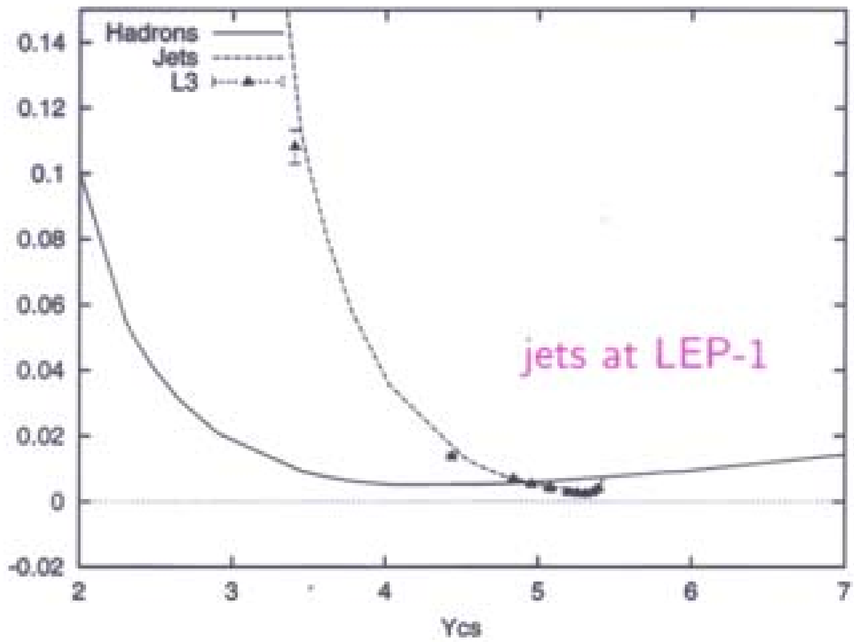
H_q moments

Ariadne - D

H_2



H_3



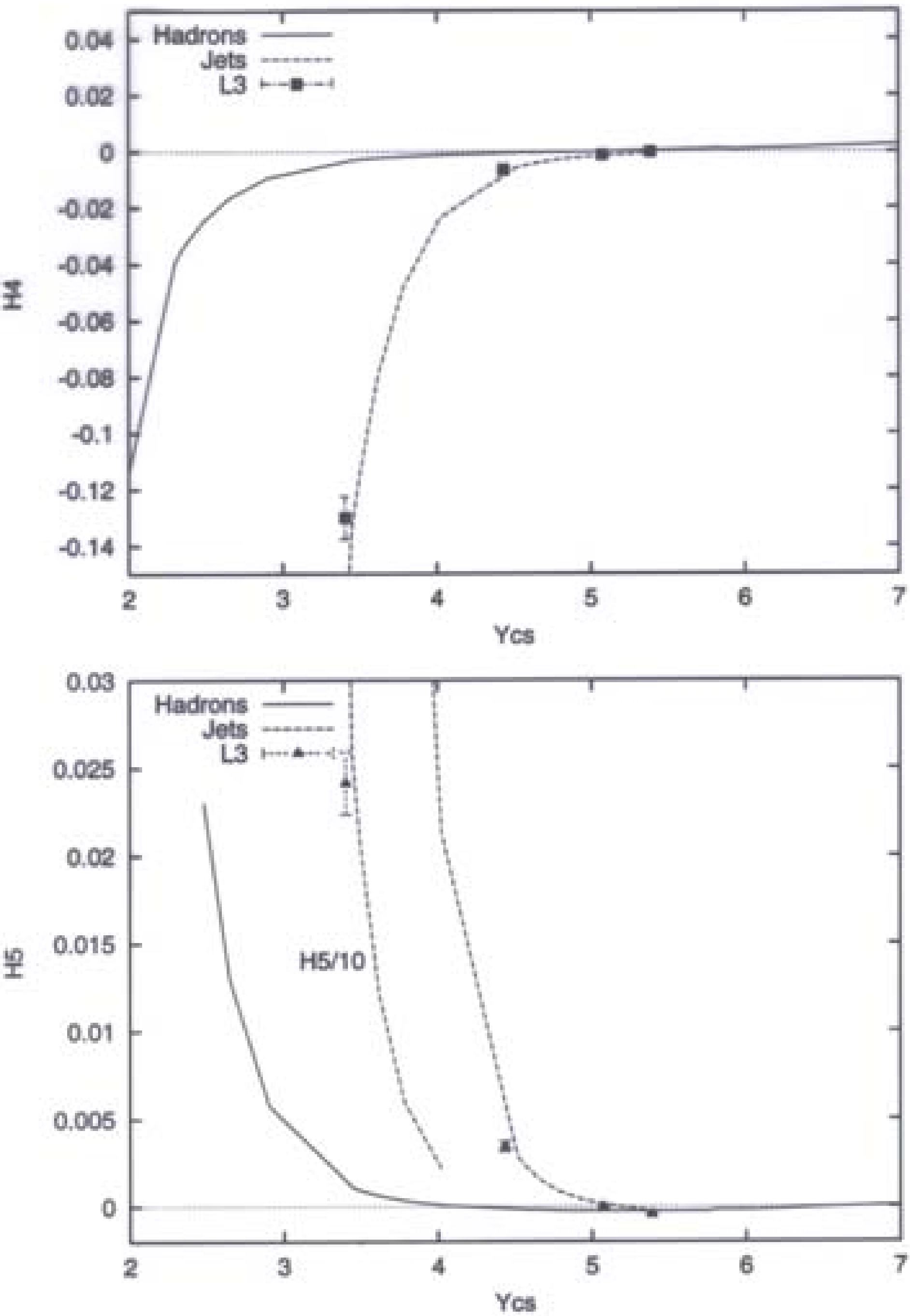
$$Y_{cs} = \ln \frac{1}{y_{cut} + Q_0^2/Q_c^2}$$

H_q moments

Ariadne - D

H_4

H_5

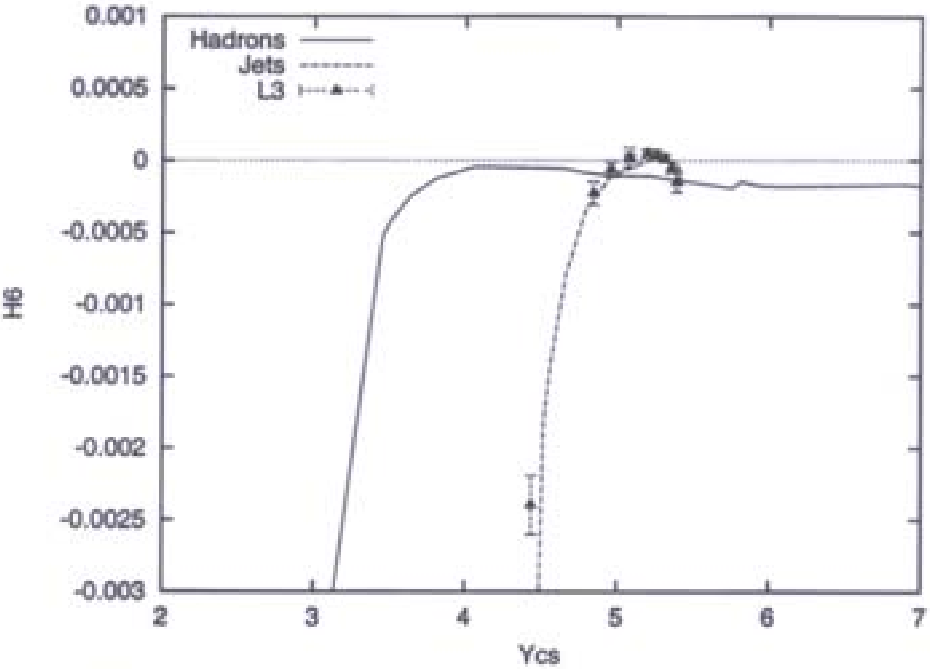


$$Y_{cs} = \ln \frac{1}{y_{cut} + Q_0^2/Q_c^2}$$

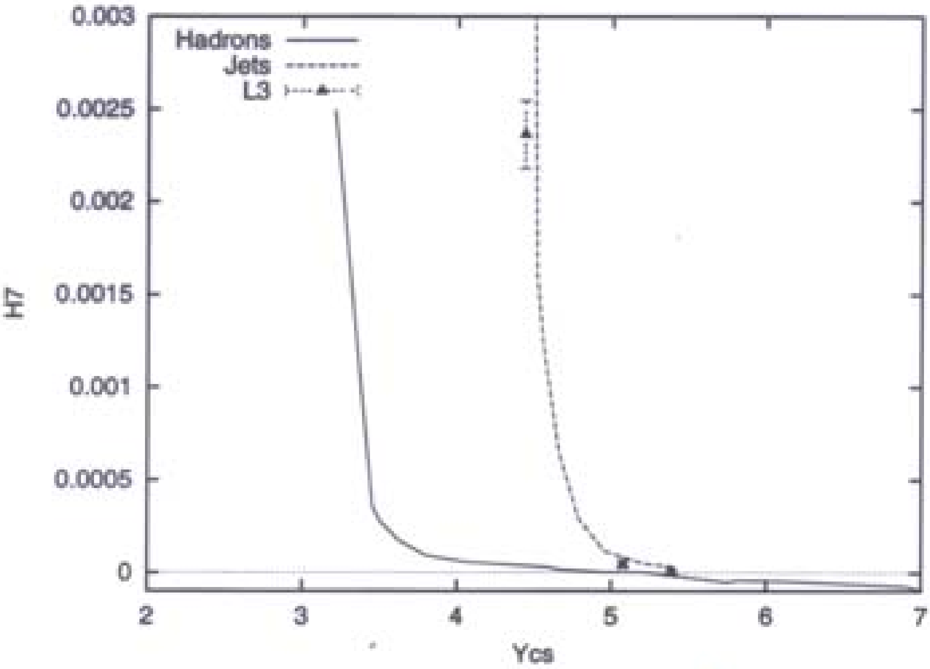
H_q moments

Ariadne - D

H_6



H_7



$$Y_{cs} = \ln \frac{1}{y_{cut} + Q_0^2/Q_c^2}$$

Conclusions

* Transition jet → hadron for $y_{\rm cut} \to 0$
  - qualitatively in DLA, MLLA
  - quantitatively in parton MC with low cut-off
    $k_T \geq Q_0 \geq \Lambda \sim 400$ MeV
    ⇒strong variation of multiplicity N and Hq
    near $y_{cut} \to 0$ from $\alpha_s(k_T) \gtrsim 1$

* Explanation / prediction
  - Poissonian transition point Y_P
    for particular energy or $y_{\rm cut}$
  - rapid oscillations below Y_P with length $\Delta q = 2$
  - oscillation length increases with energy
  - secondary extrema of Hq with energy
    in MLLA, MC

* Perturbative approach also applicable to correlations,
  but needs high accuracy
  $H_2 > 0$:
  resonance/cluster decay ↔ gluon Bremsstrahlung

* Possible improvements:
  include b-quark, 2-loop results

* Normalization $\qquad K \approx 1$

  ⇒ A hadron looks like a parton at scale $Q \sim \Lambda$