NON-PERTURBATIVE GLUON EVOLUTION, SQUEEZING, CORRELATIONS AND CHAOS IN JETS

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Introduction

Effects which are not connected with small $\alpha_s(Q^2)$
=NP effects in jets
Role of NP effects in large. Among them:

- confinement and hadronization
- exact YM field equations, solutions, ex. Instantons, vacuum properties
- long distances, soft collisions, diffraction
- power corrections
- NP evolution
- MC hadronization models, LPHD are not connected with QCD

Jets give example of separation between P and NP stages

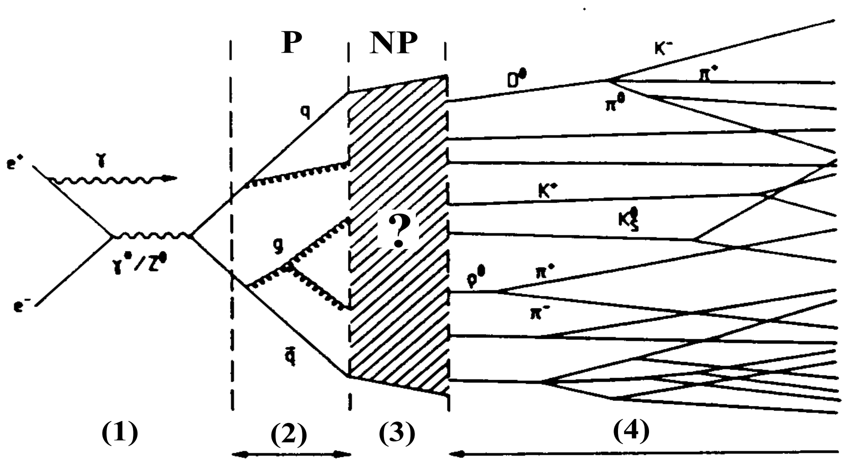

Figure 1: Jet evolution

Here we consider time gluon field evolution

Demonstrate under simple assumptions quantum squeezing

Consider chaos in jet in classical limit and chaos-squeezing connection.

Time gluon evolution in jet

Consider gluon self-interaction Hamiltonian

$$V = -g \iint f_{abc} \mathbf{E}_a \mathbf{A}_b A_c^0 \, d^3x + \frac{g}{2} \int f_{abc} \mathbf{B}_a [\mathbf{A}_b \mathbf{A}_c] d^3x + \\ + \frac{g^2}{2} \int \left(f_{abc} \mathbf{A}_b A_c^0 \right)^2 d^3x + \frac{g^2}{8} \int \left(f_{abc} [\mathbf{A}_b \mathbf{A}_c] \right)^2 d^3x \tag{1}$$

($\mathbf{E}_a = -\nabla A_a^0 - \partial_0 \mathbf{A}_a, \mathbf{B}_a = [\nabla A_a], A_a^\mu$ is a potential of the gluon field with colour a=$\overline{1,8}$; f_{abc} is the structure constant of the $SU_c(3)$ group; g is a coupling constant.)

Take jet ring with cone angle $\theta \in [\theta, \theta + d\theta]$

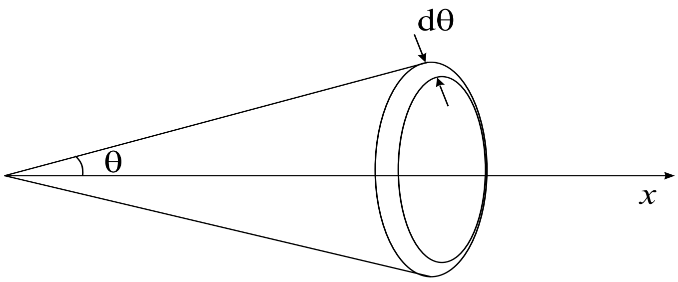

Figure 2: Jet ring

In terms of annihilation (creation) operators we have

$$V = \frac{k_0^4}{4(2\pi)^3} \left(1 - \frac{q_0^2}{k_0^2}\right)^{3/2} g^2 \pi f_{abc} f_{adf} \left\{ \left(2 - \frac{q_0^2}{k_0^2}\right) \left[a_{1212}^{bcdf} + a_{1313}^{bcdf} \right] + \right. \\ \left. + a_{2323}^{bcdf} + \frac{\sin^2\theta}{2} \left(1 - \frac{q_0^2}{k_0^2}\right) \left[2a_{2323}^{bcdf} - a_{1212}^{bcdf} - a_{1313}^{bcdf} \right] \right\} \sin\theta \, d\theta. \tag{2}$$

Here $a_{lmlm}^{bcdf} = a_l^{b+} a_m^{c+} a_l^d a_m^f + a_l^{b+} a_m^c a_l^{d+} a_m^f + a_l^b a_m^{c+} a_l^{d+} a_m^f + h.c.$,

$a_l^b (a_l^{b+})$ are annihilation (production) operators, k_o is a gluon energy, q_0 is a gluon virtuality.

For simplicity we put at the and of P cascade

- energies and virtualities of gluons are equal
- terms $\sim A^3$ are not written because they don't give contribution to the squeezing conditions (it'll be seen further)

(the same we obtain if assume collinearity of final gluon momenta)

The Hamiltonian determines the evolution of gluon state vectors.

- For small time evolution t we have final state

$$|\,\mathrm{f}\,\rangle \simeq |\,\mathrm{in}\,\rangle - \; i\; t\; \hat{V}_g\,|\,\mathrm{in}\,\rangle \tag{3}$$

- any state can be explained into raw on coherent state $|\alpha_l^b\rangle$, b - colour index, l is a polarization index.

It is natural state for quantum system consideration.

Also at the end of perturbative cascade we have multiplicity distribution close to $|\,\mathrm{NBD}\,\rangle = \sum\limits_i \omega_i\,|\alpha_i\rangle$

Therefore we study $|\alpha_l^b\rangle$ evolution under $\hat{V}_{\mathrm{int}}$

Some hints and guess:

- Hamiltonian $\hat{V}_{\rm int}$ has squares of operators of annihilation and creation. As it is known from QM and QO such structures in evolution Hamiltonian are nessesary condition of SS production because squeezing operator $S(z)$ has such operators:

$$S(z) = \exp\Big\{\frac{z^*}{2}a^2 - \frac{z}{2}(a^+)^2\Big\}. \quad (4)$$

- $f_2^{cc} = (\langle n^c(n^c-1)\rangle - \langle n^c\rangle^2) < 0$ (sub-poissonian distribution) when $\sqrt{s} \sim 3$ GeV corresponds SS

- confinement needs pairing of partons (SS?)

- Guess: gluon self-interaction can produce SS and play role of external nonlinear device in QO transforming coherent state to SS

- Task: to study evolution of $|\alpha\rangle$ under $\hat{V}_{\rm int}$ and search possibility of quantum squeezing.

Gluon SS production

||**Kuvshinov, Shaporov**
|| APP, 30, 59, 1999

To check whether final gluon state describes SS we should by analog to quantum optics to introduce operators

$$(\hat{X}_l^b)_1 = \left[\hat{a}_l^b + (\hat{a}_l^b)^+\right]/2 \quad \text{and} \quad (\hat{X}_l^b)_2 = \left[\hat{a}_l^b - (\hat{a}_l^b)^+\right]/2i$$

and to find out that dispersion of one then is smaller than that for coherent state.

Some properties of SS in QO:

- Usual uncertainty relations:

$$[\hat{X}_1, \hat{X}_2] = \frac{i}{2}, \quad \left\langle \left(\triangle \hat{X}_1\right)^2 \right\rangle \left\langle \left(\triangle \hat{X}_2\right)^2 \right\rangle \geq \frac{1}{16}, \tag{5}$$

$$\triangle X = X - \langle X \rangle$$

- for coherent state

$$\left\langle \left(\triangle \hat{X}_1\right)^2 \right\rangle = \left\langle \left(\triangle \hat{X}_2\right)^2 \right\rangle = \frac{1}{4} \tag{6}$$

— most close to classical state

- For SS

$$\left\langle \left(\triangle \hat{X}_1\right)^2 \right\rangle \left\langle \left(\triangle \hat{X}_2\right)^2 \right\rangle = \frac{1}{16}, \quad \text{(ideal squeezing)} \tag{7}$$

but ! one of component has

$$\left\langle \left(\triangle \hat{X}_i\right)^2 \right\rangle < \frac{1}{4}$$

||**F.Walls, Nature 306, 141, 1983**

- Pure quantum state (nonclassical analog)
- More organized then coherent state (entropy is small)
- Can have sub-Poisson multiplicity distribution (antibunching, or super-Poisson for bunching)
- Pairing of photons
- Can decrease quantum noise
- Can be obtained from CS by nonlinear interaction with out side devices
- Can be detected by interaction with controlling CS

Condition of squeezing

$$\left\langle \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle = \left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle + \frac{1}{4} \; < \; \frac{1}{4}$$

$$\text{or} \qquad \left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle < \; 0. \tag{8}$$

Averaging goes through the vector which appear as a result of evolution

$$\prod_{c=1}^{8}\prod_{l=1}^{3} | \, \alpha_l^c(t) \, \rangle \simeq \prod_{c=1}^{8}\prod_{l=1}^{3} | \, \alpha_l^c(0) \, \rangle - \; itV \; \prod_{c=1}^{8}\prod_{l=1}^{3} | \, \alpha_l^c(0) \, \rangle . \tag{9}$$

- time begins from the state $\prod_{c=1}^{8}\prod_{l=1}^{3} | \, \alpha_l^c(t) \, \rangle$ prepared by previous development
- example end of P cascade
- we have super position of coherent state, for simplicity we take one
- example colour index b=1, vector index b - any

$$\left\langle N \left(\triangle (X_l^1)_{\frac{1}{2}} \right)^2 \right\rangle = \pm \, 4\pi \, u_2 \, t \, \sin\theta \, d\theta \Big\{ (1+u_1) \Big[\delta_{l1}(Z_{33}+Z_{22}) + +(1-\delta_{l1})Z_{11} \Big] + \delta_{l2} Z_{33} + \delta_{l3} Z_{22} + +u_1 \sin^2\theta \left[-\tfrac{1}{2}\delta_{l1}(Z_{22}+Z_{33}) + \delta_{l2}(Z_{33} - \tfrac{1}{2}Z_{11}) + \delta_{l3}(Z_{22} - \tfrac{1}{2}Z_{11}) \right] \Big\} \neq 0. \tag{10}$$

Here $Z_{mn} = \sum_{k=2}^{7}{}' \langle (X_m^k)_1 \rangle \langle (X_n^k)_2 \rangle (m,n = 1,2,3)$,

$$\sum_{k=2}^{7}{}'(\,) = \sum_{k=2}^{3}(\,) + \tfrac{1}{4}\sum_{k=4}^{7}(\,), u_1 = \left(1 - \frac{q_0^2}{k_0^2}\right), \; u_2 = \frac{k_0^4}{4(2\pi)^3}\frac{g^2}{2}\sqrt{u_1^3}.$$

We have phase squeezed state if

$$\left\langle (X^k_m)_1 \right\rangle < 0, \left\langle (X^k_m)_2 \right\rangle < 0$$
or
$$\left\langle (X^k_m)_1 \right\rangle > 0, \left\langle (X^k_m)_2 \right\rangle > 0,$$

‖P.F.Walls, G.J.Milburn,
‖Quantum Optics,
‖Camb. Univ. Pr.1992

$k \neq 1, m \neq l$ We have amplitude squeezing state if

$$\left\langle (X^k_m)_1 \right\rangle > 0, \left\langle (X^k_m)_2 \right\rangle < 0$$
or
$$\left\langle (X^k_m)_1 \right\rangle < 0, \left\langle (X^k_m)_2 \right\rangle > 0$$

$$k \neq 1, m \neq l$$

- The conditions cover all possible cases ⇒ SS - should exist
- The same is true for other colours

‖ Thus vector $\prod_{c=1}^{8}\prod_{l=1}^{3} | \, \alpha^c_l(t) \, \rangle$ describes SS

We can estimate parameter of squeezing

$$r = \mp 2 \left\langle N \left(\triangle X \right)^2 \right\rangle \tag{11}$$

$$\begin{aligned} r = & - 8\pi \, u_2 \, t \, \sin\theta \, d\theta \Big\{ (1+u_1) \Big[\delta_{l1}(Z_{33}+Z_{22}) + (1-\delta_{l1}) Z_{11} \Big] + \\ & + \delta_{l2} Z_{33} + \delta_{l3} Z_{22} + \\ & + u_1 \sin^2\theta \left[-\tfrac{1}{2}\delta_{l1}(Z_{22}+Z_{33}) + \delta_{l2}(Z_{33} - \tfrac{1}{2}Z_{11}) + \delta_{l3}(Z_{22} - \tfrac{1}{2}Z_{11}) \right] \Big\} \neq 0. \end{aligned} \tag{12}$$

It can be shown that $\sim A^3$ terms don't lead to squeezing. In fact, the squeezing condition may be write as

$$\left\langle N\left(\triangle(X^h_{(\lambda)})_{\frac{1}{2}}\right)^2\right\rangle = \mp\frac{i\,t}{4}\{\langle\alpha\,|\,[a^h_{(\lambda)}\,(k)\,,[a^h_{(\lambda)}\,(k)\,,V]\,]|\alpha\rangle- \\ -\langle\alpha\,|[\,[V,a^{+h}_{(\lambda)}\,(k)],a^{+h}_{(\lambda)}\,(k)]\} \tag{13}$$

Hamiltonian of the three-gluon self-interaction in momentum representation has the next form

$$V = ig(2\pi)^3 f_{bcd}\sum_{\lambda_1,\lambda_2,\lambda_3}\int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3(\vec{k}_1\vec{\varepsilon}_{(\lambda_2)}(k_2))(\varepsilon^{(\lambda_1)}_{\nu}(k_1)\,\varepsilon^{\nu}_{(\lambda_3)}(k_3))\times \\ \times\left\{\left[a^b_{(\lambda_1)}(k_1)\,a^c_{(\lambda_2)}(k_2)\,a^d_{(\lambda_3)}(k_3) - a^{b+}_{(\lambda_1)}(k_1)\,a^{c+}_{(\lambda_2)}(k_2)\,a^{d+}_{(\lambda_3)}(k_3)\right]\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\times\right. \\ \times\left[a^b_{(\lambda_1)}(k_1)\,a^{c+}_{(\lambda_2)}(k_2)\,a^{d+}_{(\lambda_3)}(k_3) - a^{b+}_{(\lambda_1)}(k_1)\,a^c_{(\lambda_2)}(k_2)\,a^d_{(\lambda_3)}(k_3)\right]\delta(\vec{k}_1-\vec{k}_2-\vec{k}_3)\times \\ \times\left[a^b_{(\lambda_1)}(k_1)\,a^{c+}_{(\lambda_2)}(k_2)\,a^d_{(\lambda_3)}(k_3) - a^{b+}_{(\lambda_1)}(k_1)\,a^c_{(\lambda_2)}(k_2)\,a^{d+}_{(\lambda_3)}(k_3)\right]\delta(\vec{k}_1-\vec{k}_2+\vec{k}_3)\times \\ \left.\times\left[a^b_{(\lambda_1)}(k_1)\,a^c_{(\lambda_2)}(k_2)\,a^{d+}_{(\lambda_3)}(k_3) - a^{b+}_{(\lambda_1)}(k_1)\,a^{c+}_{(\lambda_2)}(k_2)\,a^d_{(\lambda_3)}(k_3)\right]\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3)\right\} \tag{14}$$

Obviously, that

$$[a^h_{(\lambda)}\,(k)\,,[a^h_{(\lambda)}\,(k)\,,V]\,]=0,\quad [\,[V,a^{+h}_{(\lambda)}\,(k)],a^{+h}_{(\lambda)}\,(k)]=0\quad\text{because}\quad f_{hhb}=0$$

The second normalized correlation function is the following

$$K_{(2)}(\theta_1,\theta_2) \;=\; \frac{C_{(2)}(\theta_1,\theta_2)}{\rho_1(\theta_1)\rho_1(\theta_2)}, \tag{15}$$

where $C_{(2)}(\theta_1,\theta_2) = \rho_2(\theta_1,\theta_2) - \rho_1(\theta_1)\rho_1(\theta_2)$,
$\rho_2(\theta_1,\theta_2)$ ($\rho_1(\theta)$) — two (one) particle inclusive distribution

$$K^b_{l(2)}(\theta_1,\theta_2) \;=\; \frac{\rho^b_{l(2)}(\theta_1,\theta_2)}{\rho^b_{l(1)}(\theta_1)\rho^b_{l(1)}(\theta_2)} - 1. \tag{16}$$

$$|f(\theta_1,t), f(\theta_2,t)\rangle = \prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha^c_l(\theta_1,t),\alpha^c_l(\theta_2,t)\rangle$$

$$\left.\begin{array}{l}\rho_1(\theta) = \langle f(\theta,t)|a^+a|f(\theta,t)\rangle,\\ \rho_2(\theta_1,\theta_2) = \langle f(\theta_2,t), f(\theta_1,t)|a^+a^+a\,a|f(\theta_1,t), f(\theta_2,t)\rangle\end{array}\right\} \tag{17}$$

Then second gluon normalized function has the form

$$K^b_{l(2)}(\theta_1,\theta_2) = -\,M_1(\theta_1,\theta_2)/\{|\ \alpha^b_l\ |^4 - 2\ |\ \alpha^b_l\ |^2\ M_1(\theta_1,\theta_2) + M_2(\theta_1,\theta_2)\} \tag{18}$$

<u>Example</u>: for b = 1, and any l:

$$M_1(\theta_1,\theta_2) \;=\; 24\,t\,u_2\,\pi\ |\ \alpha\ |^2|\ \beta\ |^2 \sin\left(\delta+\frac{\pi}{2}\right)\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1})\times \times(\sin\theta_1\ +\sin\theta_2) - \frac{1}{2}u_1\,(3\delta_{l1}-1)(\sin^3\theta_1\ +\sin^3\theta_2)\Big\} \tag{19}$$

$$M_2(\theta_1,\theta_2) \;=\; 80\,t\,u_2\,\pi\ |\ \alpha\ |^3|\ \beta\ |^3 \sin\left(\frac{\delta}{2}+\frac{\pi}{4}\right)\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1})\times \times(\sin\theta_1\ +\sin\theta_2) - \frac{1}{2}u_1\,(3\delta_{l1}-1)(\sin^3\theta_1\ +\sin^3\theta_2)\Big\} \tag{20}$$

Here for simplicity we supposed that $\alpha_l^1 = |\alpha|\ e^{i\gamma_1}$, $l =$ any and $\alpha_l^b = |\beta|\ e^{i\gamma_2}$, when $b \neq 1$, for $\forall\, l$, $\gamma_1 - \gamma_2 = \delta/2 + \pi/4$ (phase δ defines the direction of squeezing maximum)

Comparison $K_2^{\gamma\gamma}$ and K_2^{gg} in QO

$$K_{l(2)} = g_l^{(2)} - 1 \ = \frac{\langle \hat{a}_l^+ \, \hat{a}_l^+ \, \hat{a}_l \, \hat{a}_l \rangle}{\left\langle \hat{a}_l^{1+} \, \hat{a}_l \right\rangle^2} - 1. \tag{21}$$

(Averaging over final state at moment t)

! For SS:

- $K_{l(2)} > 0 \Rightarrow$ bunching of photon
- $K_{l(2)} > 0 \Rightarrow$ antibunching (sub-Poisson multiplicity distribution)

For coherent field: $K_{l(2)} = 0$

For photon SS (when parameter of squeezing r is small)

$$K_{l(2)} = -\frac{r_l[\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}{\mid \alpha_l \mid^4 - 2r_l \mid \alpha_l \mid^2 [\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}. \tag{22}$$

In QCD

We had $K_{l(2)}^b(\theta_1, \theta_2)$ which include function $M_2(\theta_1, \theta_2)$ due to nonlinear combinations of creation and annihilation operators of gluons with different colours in Hamiltonian

Example:

Angle dependence of correlation function $K^1_{1(2)}(\theta_1, \theta_2 = 0)$ parameters: $b = 1$, $l = 1$, $t = 0.001$, $\theta_2 = 0$, $q_0^2 = 1$ GeV, $k_0 \sim \sqrt{s}/2 < n >_{\mathbf{gluon}}$, $\sqrt{s} = 91$ GeV

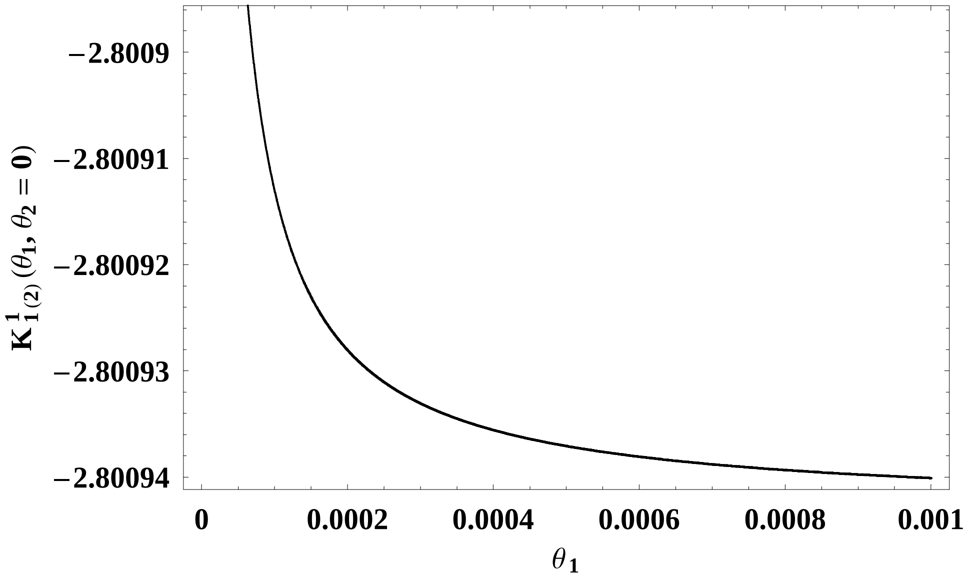

Figure 3: *The angular dependence of the squeezed gluon correlation function at* $|\alpha|^2 = 1, |\beta|^2 = 1, \delta = 0$

For synphase case:

- If $|\alpha| = |\beta|$ for any color and vector index $\Rightarrow K_2$ is in negative region = antibunching of gluons with sub-Poisson multiplicity distribution
- when $\theta_1 \to \theta_{\max}$ $K^1_{1(2)}(\theta_1, \theta_2 = 0) \to const = -2.80094$
  The behavior is similar to photon case $\|$**Hirota, Squeezed Light.** $\|$ − **Tokio, 1992**
- $|\alpha^{b=1}| > |\alpha^{b\neq 1}|$ K_2 has singularity at: a) $\theta_1 \approx 1.518928762 \times 10^{-9}$ at $|\alpha|^2 = 3, |\beta|^2 = 1$; b) $\theta_1 \approx 7.8873381715 \times 10^{-9}$ at $|\alpha|^2 = 10, |\beta|^2 = 1$

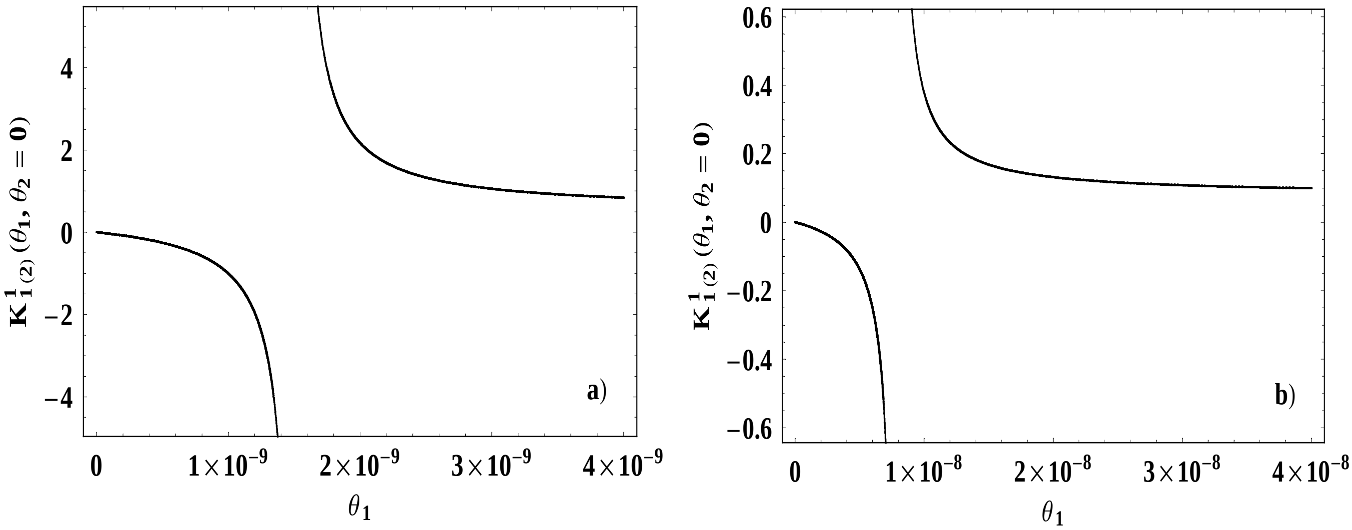

Figure 4: a) $|\alpha|^2 = 3$, $|\beta|^2 = 1$, b) $|\alpha|^2 = 10$, $|\beta|^2 = 1$.

For antiphase SS ($\delta = \pi$)

- Correlation function lies in positive region $\Rightarrow$ bunching of gluons with

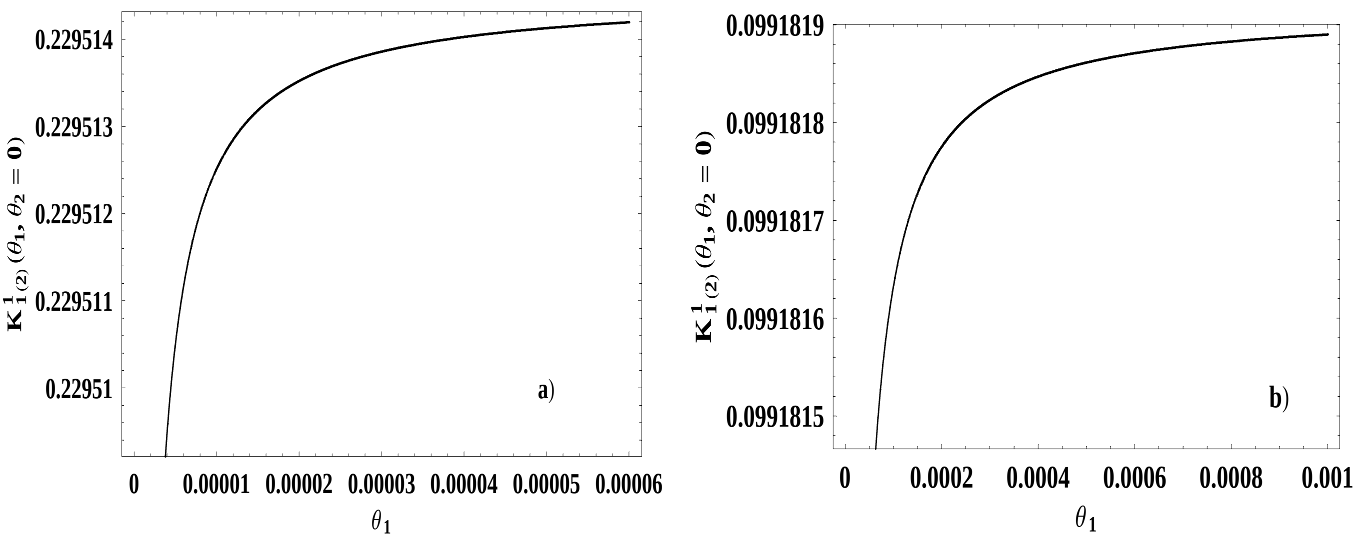

Figure 5: a) $|\alpha|^2 = 1$, $|\beta|^2 = 1$, b) $|\alpha|^2 = 3$, $|\beta|^2 = 1$.

Using well known transformation

$$\sin\theta = \sqrt{1 - \frac{\tanh^2 y}{u_1}}, \quad d\theta = -\frac{dy}{\cosh^2 y\,\sqrt{u_1 - \tanh^2 y}}, \tag{23}$$

we can obtain correlation function of SS in terms of rapidity

For synphase

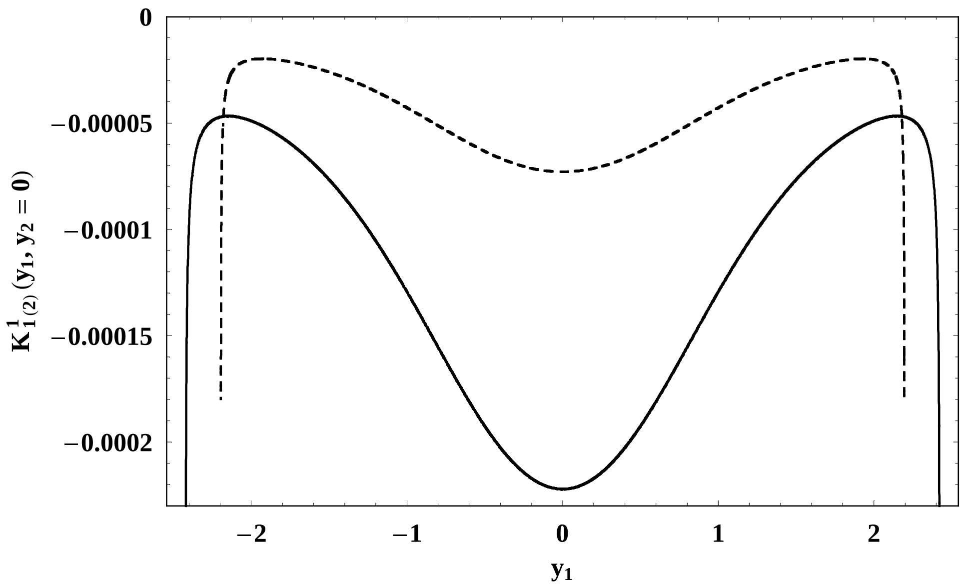

Figure 6: The rapidity dependence of the squeezed gluon correlation function at $y_2 = 0$: $|\alpha|^2 = 1, |\beta|^2 = 1-$ solid line, $|\alpha|^2 = 3, |\beta|^2 = 1-$ dotted line

- rapidity correlation lies in negative region
- minimum in center $y_1 = y_2 = 0$ and two maxima

Behavior of second order correlation function could be one of criteria for gluon SS existence

Chaos in jets
Chaos and squeezing – coexistence

1) Role of C in QFT and HEP is a kind of challenge
There are a lot of footprint of C here:

- chaotic solutions of classical Yang-Mills field equations of all fundamental interactions. Chaos and order in classical YMH models ||**Savvidy PL (1983),**
||**Kawabe PRD (1983)**

- C assisted quantum tunneling: probability of tunneling between wells increases by several orders in presence of classical driving chaotic force

- quantum footprints of classical chaos in nuclear physics (energy level spacing distribution) and stochastic billiards
||**Zaslavskii,Sagdeev,**
|| **Introduction in nonlinear physics (1988)**

In semiclassics Gutzwiller formula gives connection between level spacing and classical phase trajectories

- Chaos simulates confinement ||**Savvidy, PL (1977)**

- Higgs field lead YMH system to order (in classics)

- Quantum fluctuations of YM field lead the system chaos-order transition ||**Kuvshinov, Kuzmin,**
||**JPA (2001)**

- chaos and squeezing are connected: roughly :

  the more chaos – the more squeezing ||**Alekseev, Perina**
  ||**chao-dyn/9804041**

2) chaos theory is developed in

- classical mechanics with finite number degrees of freedom and statistical physics

- in semi-classical regime of QM

- there is no generally recognized definition of chaos for QM and QFT ||**Kuvshinov, Kuzmin,**
  ||**PLA (2001)**

New quantum chaos criterion was suggested for QFT in terms of Green function, true also for QM and which in classical limit comes to known Toda criterion.

Does jet Hamiltonian giving squeezing lead to chaos?

Local instabilities. Toda criterion (in classical mechanics)

- If the distance between two phase space trajectories initially very close behaves with time as follows:

$$d(t) = e^{\sigma t},$$

$\sigma > 0-$ Lyapunov exponent $\Rightarrow$ the system is locally unstable, leads to mixing and to chaos. Regular stable motion is characterized by $\sigma = 0$

- Toda criterion based on Hamiltonian equation analysis

$$\frac{d\vec{q}}{dt} = \frac{\partial H}{\partial \vec{p}}, \quad \frac{d\vec{p}}{dt} = -\frac{\partial H}{\partial \vec{q}} \Rightarrow$$

$$\frac{d(\delta\vec{p})}{dt} = I\delta\vec{p} \quad \frac{d(\delta\vec{q})}{dt} = -S(t)\delta\vec{q}, \quad S = \frac{\partial^2 H_{int}}{\partial q_i \partial p_j}$$

$$G = \begin{pmatrix} 0 & I \\ -S(t) & 0 \end{pmatrix}$$ - instability matrix

- If at least one of eigenvalues of G is real than separation of neighboring trajectories grows exponentially and motion is unstable

- If all eigenvalues are imaginary than the motion is stable

Coexistence of chaos and squeezing conditions (Examples)

- Squeezing is pure quantum effect

- Condition of chaos is basically understood in classical systems ||**Shuster,**
||**Deterministic chaos (1984)**

- it was shown that effects of squeezing and chaos exist in semi-classical level ||**Alekseev, Perina,**
||**JETP (1998)**

- it is possible study chaos in classical system, when corresponding quantum system has squeezing we consider Hamiltonians:

1) SU(2) jet Hamiltonian

2) $H_2 = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{q_1^2}{2} + \frac{q_2^2}{2} + \frac{g}{2}(p_1q_1 + q_1p_1 + p_2q_2 + q_2p_2)$ - degenerate parameter amplifier

3) $H_3 = g(p_1q_2 + q_1p_2)$ - non-degenerate parameter amplifier

For H_1 (see below)

for H_2: $\lambda_{1,2} = \pm\sqrt{g^2 - 1}$, for H_3: $\lambda = g \Rightarrow H_1$ - chaotical if $g > 1$ and H_3 - always chaotical.

It is interesting that squeezing condition here has the form:

$$\langle\alpha|\frac{\partial^2 H}{\partial p_i \partial q_i}|\alpha\rangle \neq 0$$

$|\alpha\rangle$- is coherent state and is closely connected with instability matrix.

- analysis of components $\frac{\partial^2 H}{\partial p_i \partial q_i}$ if they are not equal zero — we have effect of the squeezing

For H_1, H_2, H_3- squeezing exists.

Thus S and C can coexist under some conditions.

Chaos and order in SU(2) jet

SU(2) $V_{\rm int}$ for jet:

$$V_{\rm int} = \frac{k_0^4}{4(2\pi)^3}\left(1 - \frac{q_0^2}{k_0^2}\right)^{3/2} g^2\,\pi \left\{\left(2 - \frac{q_0^2}{k_0^2} - \frac{\sin^2\theta}{2}\left(1 - \frac{q_0^2}{k_0^2}\right)\right)\times \right.$$
$$\times\left[a_{1212}^{bcbc} + a_{1313}^{bcbc} - a_{1212}^{bccb} - a_{1313}^{bccb}\right] + \left(1 + \sin^2\theta\left(1 - \frac{q_0^2}{k_0^2}\right)\right)\times \qquad (24)$$
$$\left.\times\left[a_{2323}^{bcbc} - a_{2323}^{bccb}\right]\right\}\sin\theta\, d\theta.$$

Analysis is made numerically.

(Power of computer was not enough for SU(3) case and for analytical SU(2) calculations).

- We come to classical Hamiltonian by keeping the order of operators a^+, a and consider them as c-numbers

- We have 18 variables and calculate matrix instability 18×18 for this case

- next step is calculation of its eigenvalues to find out whether they are real or imaginary

the result is:

1) If all variable a and a^*

- are real or

- are imaginary

than the system of gluons described by the above mentioned Hamiltonian is strictly ordered and effect of the squeezing is absent

2) If at least one of a or a^* is imaginary and other is real or at least one of a and a^* are real and other are imaginary - we have chaotical system.

- Experimental consequences of chaos in HEP are not known because there is no yet chaos theory for QFT (quantumness and infinite number degrees of freedom) is not yet developed

- Our quantum chaos criterion states, that we have chaos if Green function

$$G(x,y) \sim e^{-\lambda \Delta(x,y)}$$

(Suitable for any number degrees of freedom, corresponds symmetry breaking in classical field theory, can be used in QM and corresponds Toda criterion in classical mechanics)

- corresponds to confinement condition in Lattice Models

Conclusion

- Evolution of gluon field can lead to quantum squeezed gluon states in QCD jet

- GSS have many unusual properties, in particular can have second correlation function with angle singularity (experimental signature)

- Yang-Mills systems are chaotical at different energy and lead to chaos in QCD jet

- Chaos and squeezing coexist

- Gluon self-interaction leads to two-mode squeezed gluon states which are also entangled

- Role of chaos, quantum squeezing and entanglement in HEP processes

- Connections of chaos, entanglement and SS with confinement

- Experimental signatures and search C, E and SS

Photon entangled states

|| **Walls D. F., Milburn G. J.,**
|| ***Quantum Optics*, (Springer-Verlag), 1995**.

Two-mode photon SS are isomorphic to the Bell states

|| **Bell J.S., Physics, 1964, 1, 195.**

basis of which are

$$|\Phi^+\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 + |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2}$$
$$|\Phi^-\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 - |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2}$$
$$|\Psi^+\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2}$$
$$|\Psi^-\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2}$$

If one photon is registered with defined polarization, the other photon immediately becomes opposite polarized.
Measurement over one particle have an instantaneous effect on the other, possibly located at a large distance.

Two-mode squeezed state is one of the example of the entangled states || **De Wolf E. A., Progress in Optics, 2001, 42, P. 1**.

$$|\mathrm{f}\rangle = S(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle_1|n\rangle_2,$$

where $S(r) = \exp\{r(a_1^+a_2^+ - a_1a_2)\}$ is the operator of two-mode squeezing.

At small squeezing parameter r we have

$$|\mathrm{f}\rangle = |0\rangle_1|0\rangle_2 + r\,|1\rangle_1|1\rangle_2$$

The entanglement condition of considering photon state can be verified by investigation of the conditional probability $P\left(Y_j/X_i\right)\left(i,j=\overline{1,2}\right)$

$$P\left(Y_j/X_i\right)=\frac{\left\|\left\langle Y_j\right|\left\langle X_i\left|\mathrm{f}\right\rangle\right\|^2}{\left\langle\mathrm{f}\right|X_i\rangle\left\langle X_i\left|\mathrm{f}\right\rangle\right.}=\begin{cases}\delta_{ij}\\ \text{or}\\ 1-\delta_{ij},\end{cases}$$

where $\left|X_1\right\rangle=\left|0\right\rangle_1,\left|X_2\right\rangle=\left|1\right\rangle_1,\ \left|Y_1\right\rangle=\left|0\right\rangle_2,\left|Y_2\right\rangle=\left|1\right\rangle_2.$

Two-mode squeezed gluon states

|| **Walls D. F., Milburn G. J.,**
|| ***Quantum Optics*, (Springer-Verlag), 1995**.

The phase-sensitive Hermitian operators

$$(X_\lambda^{h,g})_1 = \left[b_\lambda^h + b_\lambda^g + b_\lambda^{h+} + b_\lambda^{g+}\right]/(2\sqrt{2}),$$
$$(X_\lambda^{h,g})_2 = \left[b_\lambda^h + b_\lambda^g - b_\lambda^{h+} - b_\lambda^{g+}\right]/(2i\sqrt{2})$$

are introduced.
Two-mode squeezing condition is

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle < 0.$$

Here N is the normal-ordering operator such as

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{1}{8}\Bigg\{\left\langle\left(b_\lambda^h\right)^2\right\rangle - \left\langle b_\lambda^h\right\rangle^2 + \left\langle\left(b_\lambda^g\right)^2\right\rangle - \left\langle b_\lambda^g\right\rangle^2 + \left\langle\left(b_\lambda^{h+}\right)^2\right\rangle - \left\langle b_\lambda^{h+}\right\rangle^2$$
$$+\left\langle\left(b_\lambda^{g+}\right)^2\right\rangle - \left\langle b_\lambda^{g+}\right\rangle^2 \pm 2\Big[\left\langle b_\lambda^{h+}b_\lambda^h\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^h\right\rangle + \left\langle b_\lambda^{g+}b_\lambda^g\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^g\right\rangle$$
$$+\left\langle b_\lambda^{h+}b_\lambda^g\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^g\right\rangle + \left\langle b_\lambda^{g+}b_\lambda^h\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^h\right\rangle\Big]$$
$$+2\Big[\left\langle b_\lambda^h b_\lambda^g\right\rangle - \left\langle b_\lambda^h\right\rangle\left\langle b_\lambda^g\right\rangle + \left\langle b_\lambda^{h+}b_\lambda^{g+}\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^{g+}\right\rangle\Big]\Bigg\}.$$

The expectation values of the creation and annihilation operators are taken over the vector $|\mathrm{f}\rangle$ which describes the evolution of the virtual gluon field during small time interval Δt within interaction representation

$$|\mathrm{f}\rangle \simeq |\mathrm{in}\rangle - i\,\Delta t\, H_{\mathrm{I}}(t_0)\,|\mathrm{in}\rangle,$$

where $H_{\mathrm{I}}(t_0) = H_{\mathrm{I}}^{(3)}(t_0) + H_{\mathrm{I}}^{(4)}(t_0)$.
We choose product of the gluon coherent states as initial state

vector

$|\mathrm{in}\rangle \equiv |\,\alpha\rangle = \prod_{\lambda=1}^{3}\prod_{b=1}^{8}|\alpha_\lambda^b\rangle$. $|\alpha_\lambda^b\rangle$ is the eigenvector of the annihilation operator b_λ^b with eigenvalue $\alpha_{\lambda_1}^b = |\alpha_{\lambda_1}^b|e^{i\,\gamma_{\lambda_1}^b}$.

$$H_{\rm I}^{(3)}(t) = ig(2\pi)^3 f_{abc} \sum_{\lambda_1,\lambda_2,\lambda_3} \int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3 \Big\{ \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

$$\times \left[b^a_{\lambda_1}(k_1) b^b_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\; e^{-2i(k_{01}+k_{02}+k_{03})t} - \; b^{a+}_{\lambda_1}(k_1) b^{b+}_{\lambda_2}(k_2) b^{c+}_{\lambda_3}(k_3)\; e^{2i(k_{01}+k_{02}+k_{03})t} \right]$$

$$+ \, b^{a+}_{\lambda_1}(k_1) b^b_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\; e^{2i(k_{01}-k_{02}-k_{03})t}\, \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3)$$

$$\times \Big[\vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_3}_{\nu}(k_3) + \vec{k}_2 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\nu}(k_2) - \; \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \Big]$$

$$+ \, b^{a+}_{\lambda_1}(k_1) b^{b+}_{\lambda_2}(k_2) b^c_{\lambda_3}(k_3)\; e^{2i(k_{01}+k_{02}-k_{03})t} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3)$$

$$\times \Big[\vec{k}_1 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\nu}(k_2) + \vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_3}_{\nu}(k_3) - \; \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon^{\nu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\nu}(k_3) \Big] \Big\},$$

$$H_{\rm I}^{(4)}(t) = \frac{g^2}{4}(2\pi)^3 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3 d\tilde{k}_4 \Big\{ \varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\mu}(k_3) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_4}_{\nu}(k_4) f_{abc} f_{ade}$$

$$\times \Big[\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \Big(b^b_{\lambda_1}(k_1) b^c_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\; e^{-2i(k_{01}+k_{02}+k_{03}+k_{04})t}$$

$$+ \, b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^{d+}_{\lambda_3}(k_3) b^{e+}_{\lambda_4}(k_4)\; e^{2i(k_{01}+k_{02}+k_{03}+k_{04})t} \Big)$$

$$+ \; 4 b^{b+}_{\lambda_1}(k_1) b^c_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\; e^{2i(k_{01}-k_{02}-k_{03}-k_{04})t} \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4)$$

$$+ \; 4 b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^{d+}_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\; e^{2i(k_{01}+k_{02}+k_{03}-k_{04})t} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}_4) \Big]$$

$$+ \, 2 b^{b+}_{\lambda_1}(k_1) b^{c+}_{\lambda_2}(k_2) b^d_{\lambda_3}(k_3) b^e_{\lambda_4}(k_4)\; e^{2i(k_{01}+k_{02}-k_{03}-k_{04})t}\, \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)$$

$$\times \Big[\varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_3}_{\mu}(k_3) \varepsilon^{\nu}_{\lambda_2}(k_2) \varepsilon^{\lambda_4}_{\nu}(k_4) (f_{abc} f_{ade} + f_{abe} f_{adc})$$

$$+ \, \varepsilon^{\mu}_{\lambda_1}(k_1) \varepsilon^{\lambda_2}_{\mu}(k_2) \varepsilon^{\nu}_{\lambda_3}(k_3) \varepsilon^{\lambda_4}_{\nu}(k_4) f_{abd} f_{ace} \Big] \Big\}.$$

Then two-mode squeezing condition is

$$\left\langle N\left(\Delta(X^{h,g}_{\lambda})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}\Big\{\langle\alpha|\big[\big[H_{\rm I}(0),b^{h+}_{\lambda}\big],b^{h+}_{\lambda}\big]|\alpha\rangle - \langle\alpha|\big[b^{h}_{\lambda},\big[b^{h}_{\lambda},H_{\rm I}(0)\big]\big]|\alpha\rangle$$
$$+\langle\alpha|\big[\big[H_{\rm I}(0),b^{g+}_{\lambda}\big],b^{g+}_{\lambda}\big]|\alpha\rangle - \langle\alpha|\big[b^{h}_{\lambda},\big[b^{h}_{\lambda},H_{\rm I}(0)\big]\big]|\alpha\rangle$$
$$+2\langle\alpha|\big[\big[H_{\rm I}(0),b^{h+}_{\lambda}\big],b^{g+}_{\lambda}\big]|\alpha\rangle - 2\langle\alpha|\big[b^{g}_{\lambda},\big[b^{h}_{\lambda},H_{\rm I}(0)\big]\big]|\alpha\rangle\Big\} < 0.$$

The three-gluon self-interaction doesn't lead to squeezing effect since

$$\big[\big[H^{(3)}_{\rm I}(0),b^{h+}_{\lambda}\big],b^{h+}_{\lambda}\big]=0,\quad \big[\big[H^{(3)}_{\rm I}(0),b^{g+}_{\lambda}\big],b^{g+}_{\lambda}\big]=0,$$
$$\big[\big[H^{(3)}_{\rm I}(0),b^{h+}_{\lambda}\big],b^{g+}_{\lambda}\big]=0,\quad \big[b^{g}_{\lambda},\big[b^{h}_{\lambda},H^{(3)}_{\rm I}(0)\big]\big]=0,$$
$$\big[b^{h}_{\lambda},\big[b^{h}_{\lambda},H^{(3)}_{\rm I}(0)\big]\big]=0,\quad \big[b^{g}_{\lambda},\big[b^{g}_{\lambda},H^{(3)}_{\rm I}(0)\big]\big]=0.$$

Only the four-gluon self-interaction can yield can a two-mode squeezing effect. Indeed, the two-mode squeezing condition can be written in explicit form as

$$\left\langle N\left(\Delta(X^{h,g}_{\lambda})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}g^2(2\pi)^3\int d\tilde{k}_1 d\tilde{k}_2\sum_{\lambda_1,\lambda_2}$$
$$\times\Big\{\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2)-b^{b}_{\lambda_1}(k_1)b^{c}_{\lambda_2}(k_2)|\alpha\rangle\Big[\delta(2\vec{k}-\vec{k_1}-\vec{k_2})-\delta(2\vec{k}+\vec{k_1}+\vec{k_2})\Big]$$
$$\times\Big[(f_{ahb}f_{ahc}+f_{agb}f_{agc}+2f_{ahb}f_{agc})\Big(\varepsilon^{\lambda_1}_{\mu}(k_1)\,\varepsilon^{\mu}_{\lambda_2}(k_2)\varepsilon^{\lambda}_{\nu}(k)\,\varepsilon^{\nu}_{\lambda}(k)$$
$$-\varepsilon^{\lambda_1}_{\mu}(k_1)\varepsilon^{\mu}_{\lambda}(k)\varepsilon^{\lambda_2}_{\nu}(k_2)\,\varepsilon^{\nu}_{\lambda}(k)\Big) - 2f_{ahg}f_{abc}\varepsilon^{\lambda_1}_{\mu}(k_1)\varepsilon^{\mu}_{\lambda}(k)\varepsilon^{\lambda_2}_{\nu}(k_2)\varepsilon^{\nu}_{\lambda}(k)\Big]$$
$$+2\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^{c}_{\lambda_2}(k_2)|\alpha\rangle\Big[\delta(2\vec{k}-\vec{k_1}+\vec{k_2})-\delta(2\vec{k}+\vec{k_1}-\vec{k_2})\Big]$$
$$\times(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahc}f_{agb}+f_{ahb}f_{agc})$$
$$\Big(\varepsilon^{\lambda_1}_{\mu}(k_1)\,\varepsilon^{\mu}_{\lambda_2}(k_2)\varepsilon^{\lambda}_{\nu}(k)\,\varepsilon^{\nu}_{\lambda}(k)-\varepsilon^{\lambda_1}_{\mu}(k_1)\,\varepsilon^{\mu}_{\lambda}(k)\varepsilon^{\lambda_2}_{\nu}(k_2)\,\varepsilon^{\nu}_{\lambda}(k)\Big)\Big\} < 0.$$

For collinear gluon corresponding squeezing condition is

$$\left\langle N\left(\Delta(X_{\lambda}^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\, t\,\frac{\alpha_s\pi}{4k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc} + f_{agb}f$$

$$\times\sum_{\lambda_1\neq\lambda}|\alpha^b_{\lambda_1}||\alpha^c_{\lambda_1}|\sin(\gamma^b_{\lambda_1}+\gamma^c_{\lambda_1}) < 0.$$

Gluon entangled states

At small value of the squeeze factor we have

$$r = 2\left|\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle\right|.$$

As initial states we can take vector including $\mid 0_\lambda^h\rangle \mid 0_\lambda^g\rangle$ evolution of which at small time can be written in terms of the squeeze factor as

$$|\mathrm{f}\rangle = |0_\lambda^h\rangle|0_\lambda^g\rangle + r\,|1_\lambda^h\rangle|1_\lambda^g\rangle,$$

where the squeeze factor for the collinear gluons is defined as

$$r = t\,\frac{\alpha_s\pi}{2k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc} + f_{agb}f_{ahc}) \\ \times \sum_{\lambda_1\neq\lambda}|\alpha_{\lambda_1}^b||\alpha_{\lambda_1}^c|\sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c).$$

The entanglement condition of considering gluon states with colours h, g and polarization λ can be verified by investigation of the conditional probability $P\left(Y_j/X_i\right)(i,j = \overline{1,2})$ by analogy with corresponding condition for photons assuming $|X_1\rangle = |0_\lambda^h\rangle$, $|X_2\rangle = |1_\lambda^h\rangle$, $|Y_1\rangle = |0_\lambda^g\rangle$, $|Y_2\rangle = |1_\lambda^g\rangle$. It is not complicated to make sure that the condition is fulfilled for the cases as for the two-mode squeezing for the state vector $|\mathrm{f}\rangle$ which describes the non-perturbative evolution of the gluon fields during small time.

Thus by analogy with quantum optics as a result of four-gluon self-interaction we obtain two-mode squeezed gluon states which are also entangled.

Conclusion

- evolution of gluon field can lead to quantum squeezed gluon states in QCD jet

- the emergence of such remarkable states becomes possible owing to the self-interaction of gluon

- the form of the normalized correlation function $K^b_{l(2)}$ for cophased squeezed states specifies the gluon antibunching effect if the amplitudes of all gluon fields (with various colour and vector components) are equal to one another

- in contrast to the normalized correlation function known in quantum optics, the correlation function of the gluon SS has a singularity if the amplitude for the fixed-colour gluon field being studied is greater than the amplitudes for gluon fields with other colour indices

- non-perturbative gluon evolution makes a contribution to the parton distribution prepared by the perturbative stage of jet evolution in the form of a sub-Poissonian (cophased squeezed states) or a super-Poissonian (antiphased squeezed states) distributions

- four-gluon self-interaction leads to two-mode squeezed gluon states which are also entangled

- the greater are both the amplitudes of the initial gluon coherent fields with different colour and polarization indexes and coupling constant, the greater are squeezing and entanglement effects of the colour gluons.