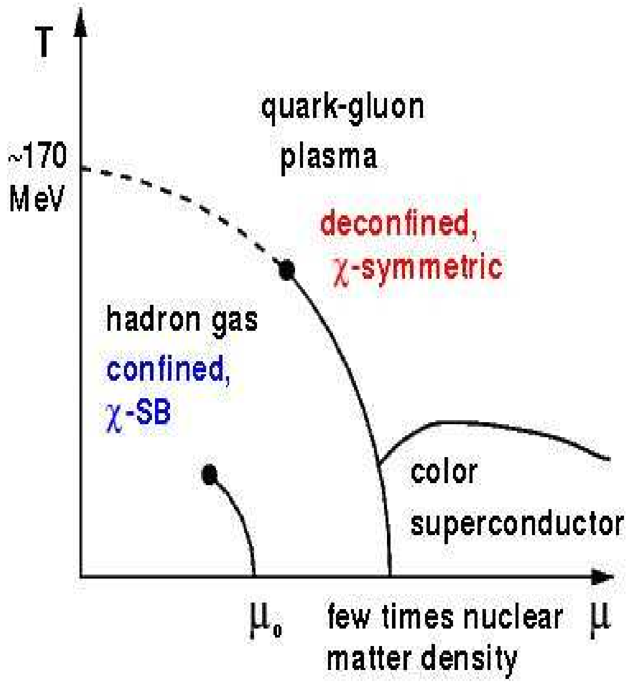
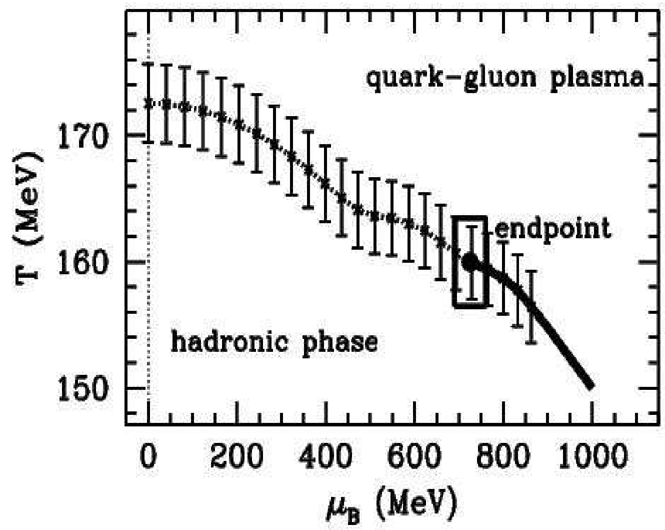
Event-by-Event Fluctuations

- What are E-by-E fluctuations?
- Fluctuations in a thermal system
- What can be and is being measured?
- Some results and their interpretation

Phase diagram



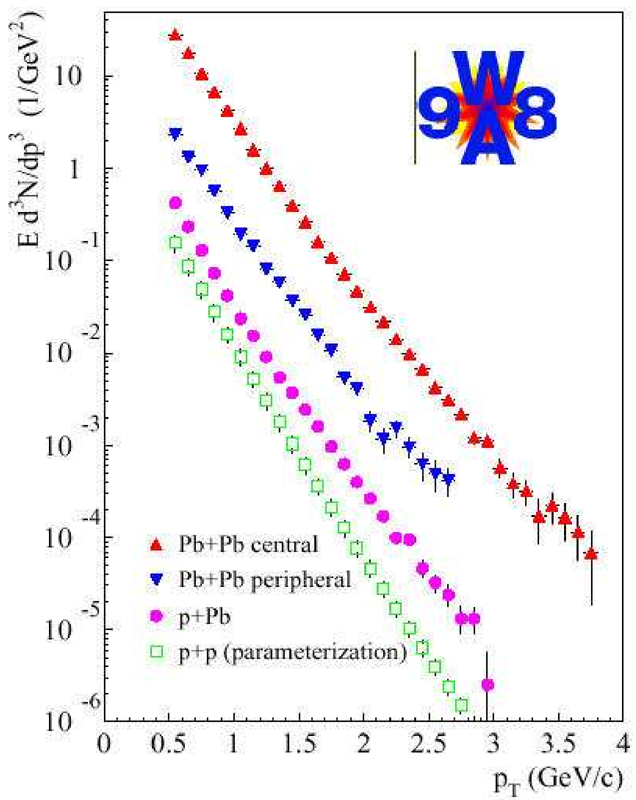


Z. Fodor et al., PLB

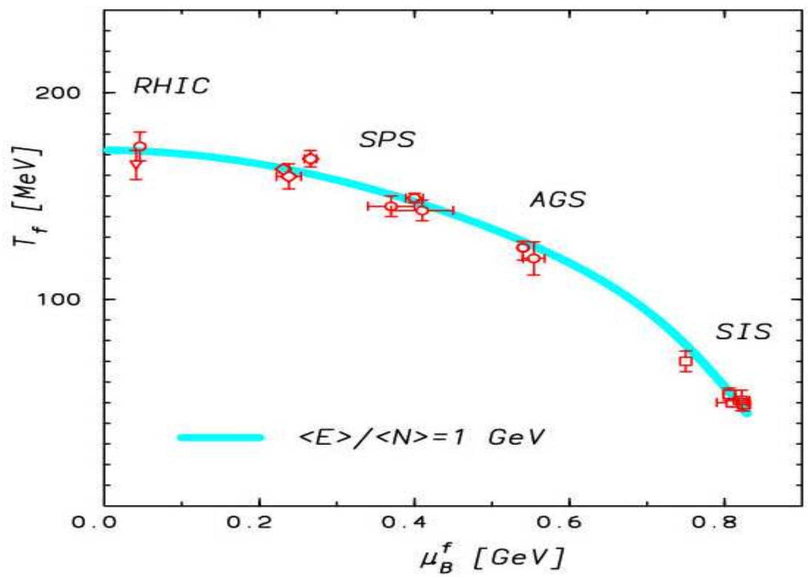
How to study

- Characterize the system
  - Spectra, collective expansion (QGP)
  - Response functions (fluctuations) (QGP, Matter)
- Probe the system
  - Photons and di-leptons (QGP, chiral)
  - Charmonium (QGP)
  - Jets (QGP)

Particle Spectra

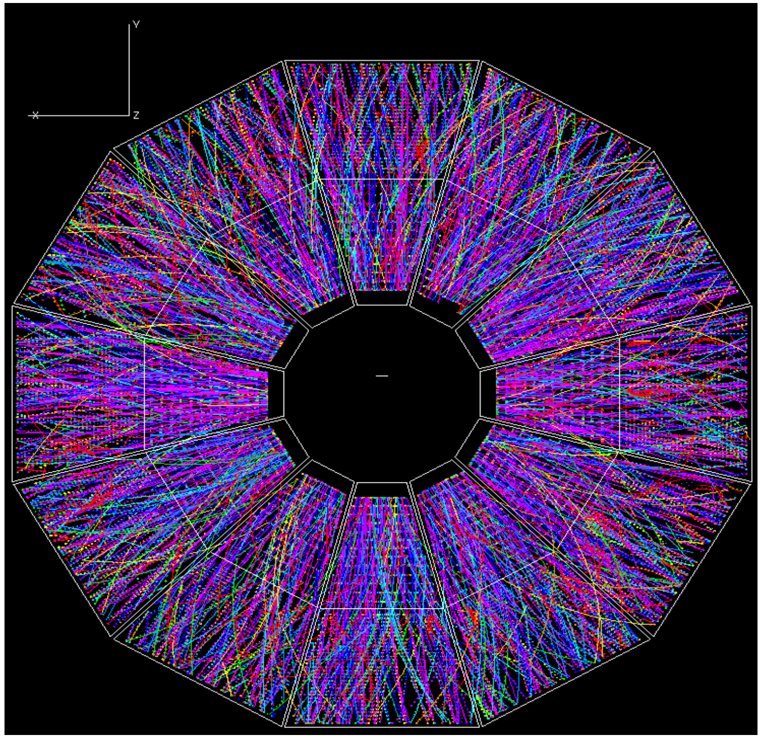


Particle Ratios
(Chemical equilibrium)



K. Redlich et al.

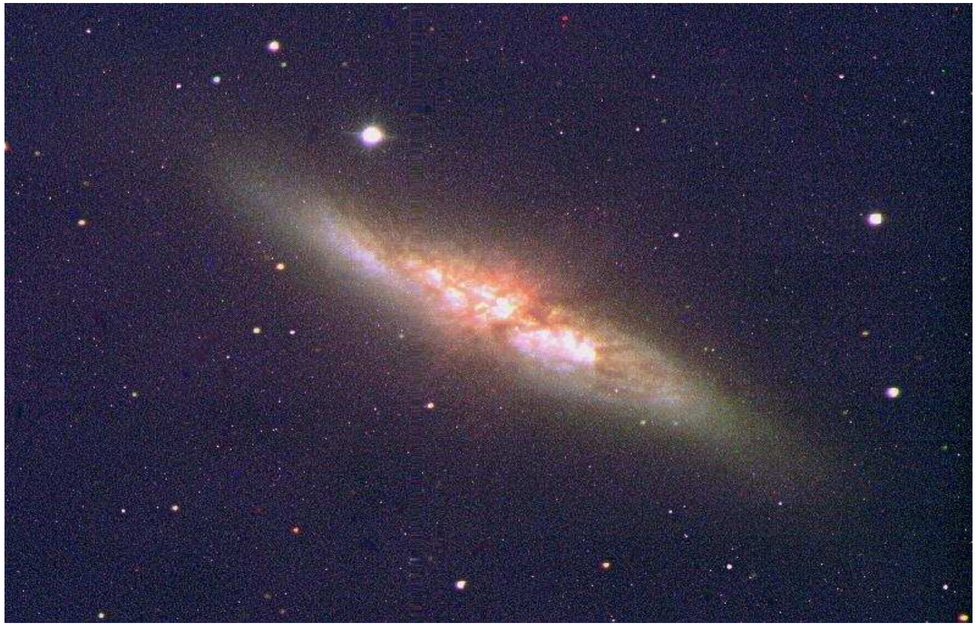
Statistical approach

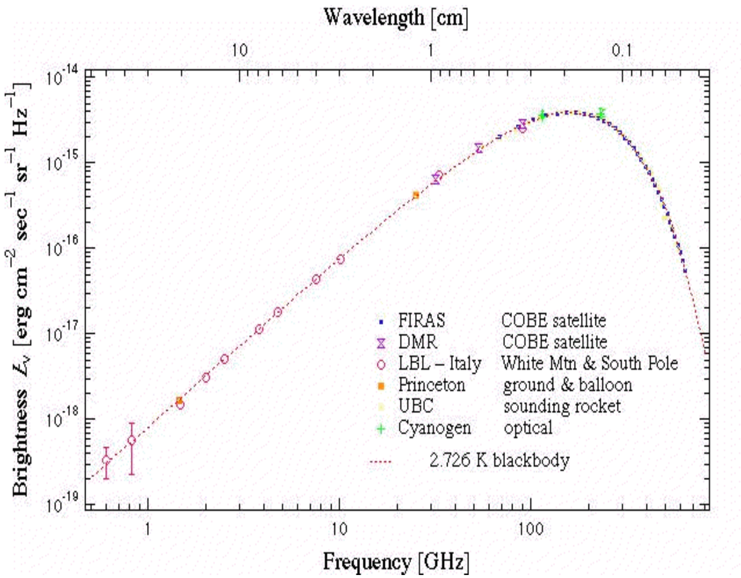


Au+Au

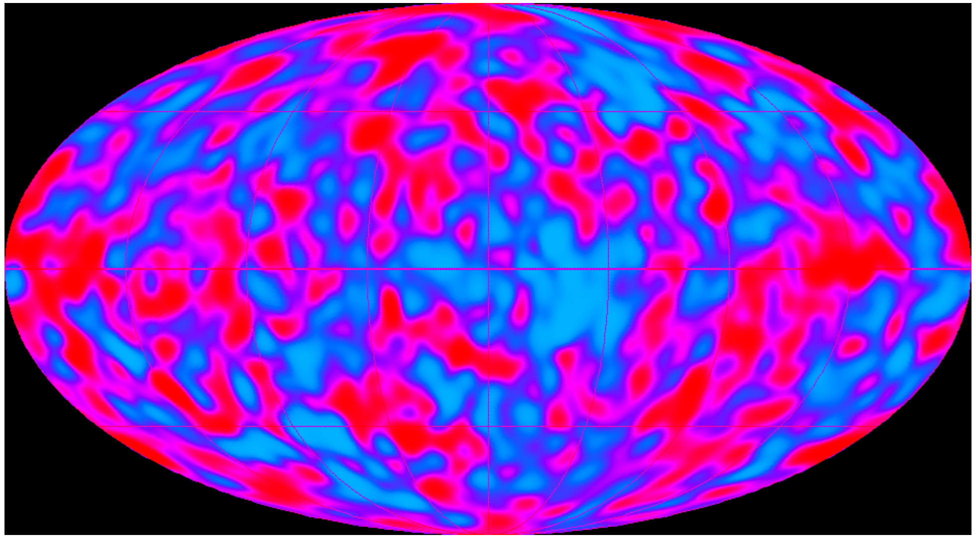
(STAR)

The mother of all thermal spectra



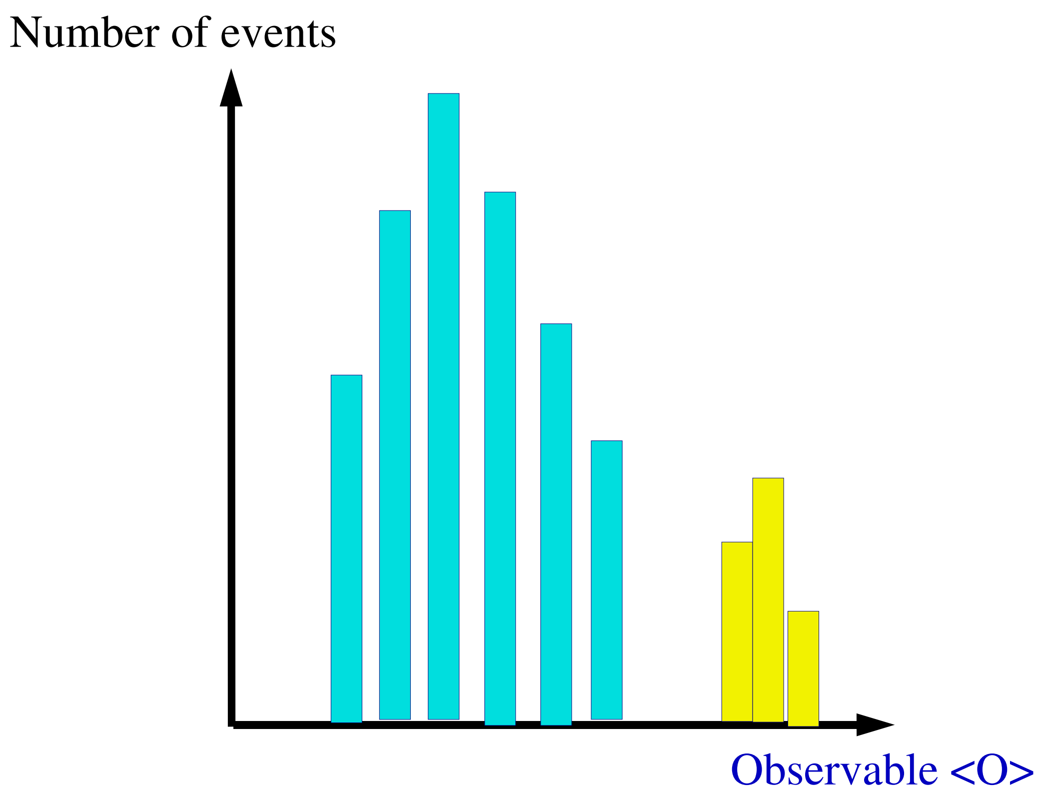


COBE



Fluctuations at the level of 10^{-5} !!!

Event-by-Event fluctuations



Event-by-Event fluctuations

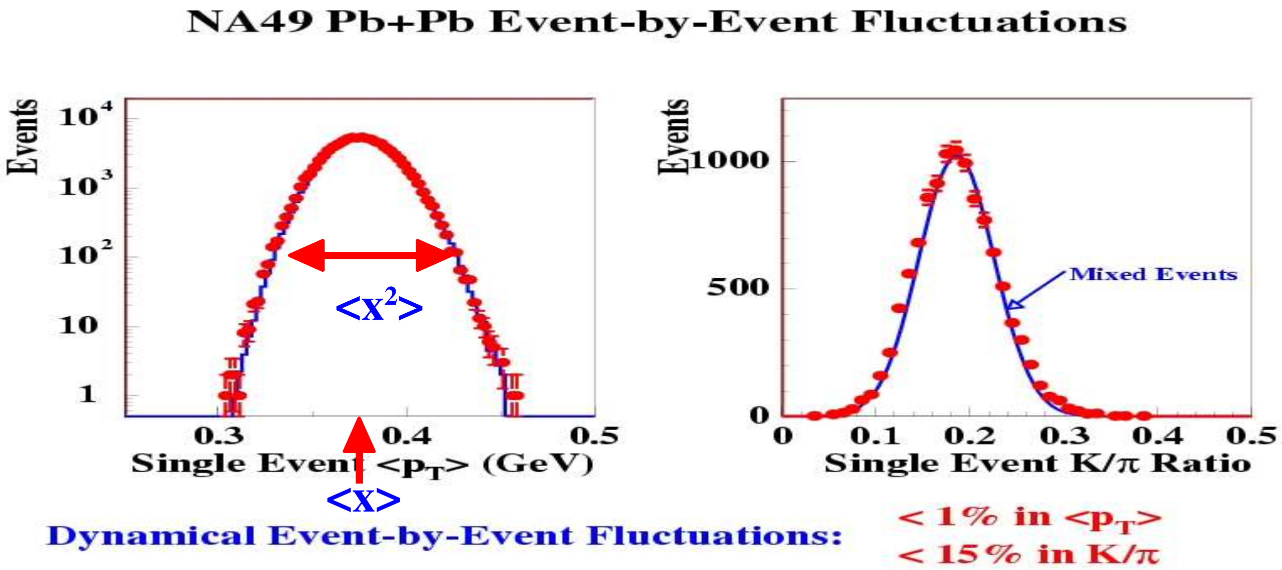
Old idea: distinct event classes

After several experiments (NA49, STAR...)

GAUSSIANS

Physics is in the WIDTH of the Gaussian!

Event-by-Event



The physics is in the width

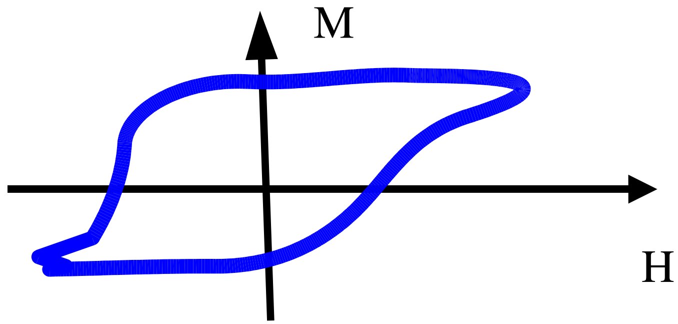
E-by-E measures
2-particle correlations

Susceptibilities

$$E = E_0 + mH + \mu Q$$

$$\langle m \rangle = \frac{dF}{dH}$$

$$\langle Q \rangle = \frac{dF}{d\mu}$$



Susceptibilities

$$x_m = \frac{d^2 F}{dH^2}$$

$$x_Q = \frac{d^2 F}{d\mu^2}$$

$$\langle \delta m \rangle = x_m \delta H$$

$$\langle \delta Q \rangle = x_Q \delta \mu$$

Linear response

$$\langle (\delta m)^2 \rangle = x_m$$

$$\langle (\delta Q)^2 \rangle = x_Q$$

Fluctuations

Fluctuations and correlations

(A. Bialas and v.k.)

Event-by-Event averages:

$$S(x) = \sum_{i=1}^{N} x(p_i) \equiv \sum_{i=1}^{N} x(i)$$

$$\langle S^k \rangle_{ebe} = \frac{1}{M} \sum_{m=1}^{N_{events}} \sum_{i_1=1}^{N_m} \ldots \sum_{i_k=1}^{N_m} x_m(i_1) \ldots x_m(i_k)$$

Inclusive averages:

$$\int dp_1 \ldots dp_n \; \rho(p_1, \ldots p_n) [x(p_1)]^{k_1} \ldots [x(p_n)]^{k_n} =$$

$$\frac{1}{M} \sum_{m=1}^{N_{events}} \sum_{i_1=1}^{N_m} \ldots \sum_{i_n=1}^{N_m} [x_m(i_1)]^{k_1} \ldots [x_m(i_n)]^{k_n}$$

Event-by-Event fluctuations measure many particle correlations

$$\langle S \rangle_{ebe} = \int dp \; \rho(p) x(p)$$

$$\langle S^2 \rangle_{ebe} = \int dp_1 dp_2 \; \rho_2(p_1, p_2) x(p_1) x(p_2) + \int dp \; \rho(p) [x(p)]^2$$

Fluctuations in thermal system

example: charge fluctuations

$$Z = Tr\left[\exp\left(-\beta\left(H - \mu Q\right)\right)\right]$$

Mean charge: $\langle Q \rangle = T \frac{\partial}{\partial \mu} \log(Z) = -\frac{\partial}{\partial \mu} F$

Variance: $\langle (\delta Q)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu^2} \log(Z) = -T \frac{\partial^2}{\partial \mu^2} F$

Susceptibility: $x_Q = -\frac{1}{V} \frac{\partial^2}{\partial \mu^2} F$

Relation to correlators: $x_Q = -\Pi_{00}(\omega = 0, q \rightarrow 0)$ $\quad \Pi_{\mu\nu} = i \int d^4x \exp(-ikx) \langle\langle j_\mu(x) j_\nu(0) \rangle\rangle$

Similar for energy fluctuations: $\langle (\delta E)^2 \rangle = T^2 C_V$

Fluctuations of rare particles

Charge conservation needs to be taken into account explicitly: Canonical treatment

Consider: $$F_2 \equiv \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}$$ $F_2 = 1$ for Poisson statistics

Equilibrium: $$P_{n,eq.} = \frac{\epsilon^n}{I_0(2\sqrt{\epsilon})\,(n!)^2}$$ $\epsilon = \langle N\rangle_{MB}^2$

$$\langle N\rangle_{eq.} = \sqrt{\epsilon}\,\frac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{2\epsilon})} = \epsilon - \frac{\epsilon^2}{2}$$

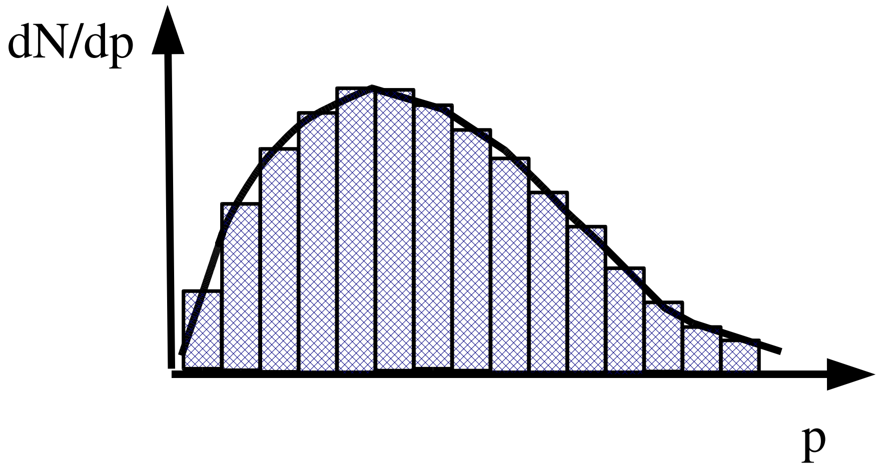
$$\langle N^2\rangle_{eq.} = \epsilon$$

$$F_2^{eq.} = \frac{1}{2} + \frac{\epsilon}{6} + \ldots \approx \frac{1}{2} + \frac{\langle N_{eq.}\rangle}{6} + \ldots$$

Towards real life

Remember ideal gas:

$$N = \sum_p \langle n(p) \rangle$$

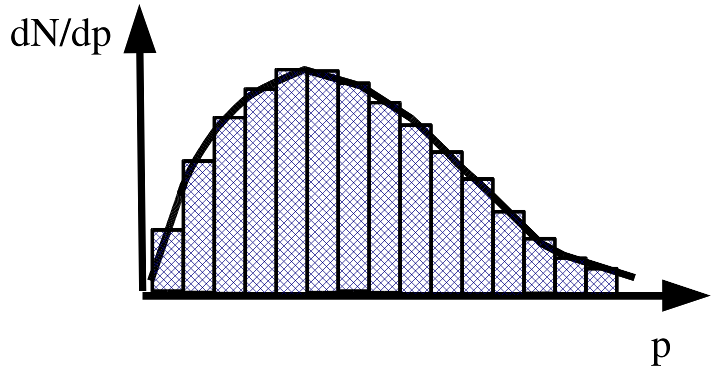


Fluctuations: $\delta n(p) = n(p) - \langle n(p) \rangle$

Global fluctuations are due to "local" fluctuations of the particle numbers in each momentum bin!

Towards real life II

Consider observable X: $X = \sum_p x(p)\, n(p)$



Mean: $\langle X \rangle = \sum_p x(p) \langle n(p) \rangle$

Fluctuations: $\delta X = \sum_p x(p)\, (\delta n(p))$ $\langle (\delta X)^2 \rangle = \sum_{p,q} \Delta(p,q)\, x(p)\, x(q)$

Basic correlator: $\Delta_{\alpha,\beta}(p,q) \equiv \langle \delta n_\alpha(p)\ \delta n_\beta(q) \rangle$

Basic Correlator

Basic correlator: $\Delta_{\alpha,\beta}(p,q) \equiv \langle \delta n_\alpha(p)\, \delta n_\beta(q) \rangle$

Ideal gas: $\Delta_{\alpha,\beta}(p,q) = \delta_{p,q}\, \delta_{\alpha,\beta}\, \omega_\alpha(p) \langle n_\alpha(p) \rangle$

$\omega_\alpha(p) = 1$ classical limit (NO correlations)

$\omega_\alpha(p) = (1 \pm \langle n_\alpha(p) \rangle)$ quantum gases

Relation to correlation function:

$$\Delta_{\alpha,\beta}(p,q) = \rho^2_{\alpha,\beta}(p,q) - \rho_\alpha(p)\rho_\beta(q) + \delta_{p,q}\,\delta_{\alpha,\beta}\,\rho_\alpha(p) = C_{\alpha,\beta}(p,q) + \delta_{p,q}\,\delta_{\alpha,\beta}\,\rho_\alpha(p)$$

Fluctuations of ratios

Most *intensive* observables are ratios: $\langle \frac{p_t}{N} \rangle$ $\langle \frac{K}{\pi} \rangle$ $\langle \frac{N_+}{N_-} \rangle$

To order 1/N: $$\langle R_{AB} \rangle = \frac{\langle A \rangle}{\langle B \rangle}$$

$$\langle \delta R_{AB}^2 \rangle = \frac{\langle A \rangle^2}{\langle B \rangle^2} \left(\frac{\langle \delta A^2 \rangle}{\langle A \rangle^2} + \frac{\langle \delta B^2 \rangle}{\langle B \rangle^2} - 2 \frac{\langle \delta A \, \delta B \rangle}{\langle A \rangle \langle B \rangle} \right)$$

With: $A = \sum_{p,\alpha} a_p^\alpha n_\alpha(p)$ $B = \sum_{p,\alpha} b_p^\alpha n_\alpha(p)$

This gives: $$\langle \delta R_{AB}^2 \rangle = \frac{\langle A \rangle^2}{\langle B \rangle^2} \sum_{p,q} \sum_{\alpha,\beta} \Delta_{\alpha,\beta}(p,q) \left(\frac{\langle a_p^\alpha a_q^\beta \rangle}{\langle A \rangle^2} + \frac{\langle b_p^\alpha b_q^\beta \rangle}{\langle B \rangle^2} - 2 \frac{\langle a_p^\alpha b_q^\beta \rangle}{\langle A \rangle \langle B \rangle} \right)$$

BASIC CORRELATOR

Statistical background

Finite number statistics:

subtract/divide/remove.... statistical background

Use un-correlated basic correlator $\Delta_{\alpha,\beta}(p,q)=\delta_{p,q}\delta_{\alpha,\beta}\langle n_\alpha(p)\rangle$

based on measured inclusive single particle spectrum $\langle n_\alpha(p)\rangle=\frac{dN}{dp}$

$\rightarrow$ $\Phi, \ \sigma_{dynamic}, \ F \ldots$

All we need is ....

$$\Delta_{\alpha,\beta}(p,q)$$

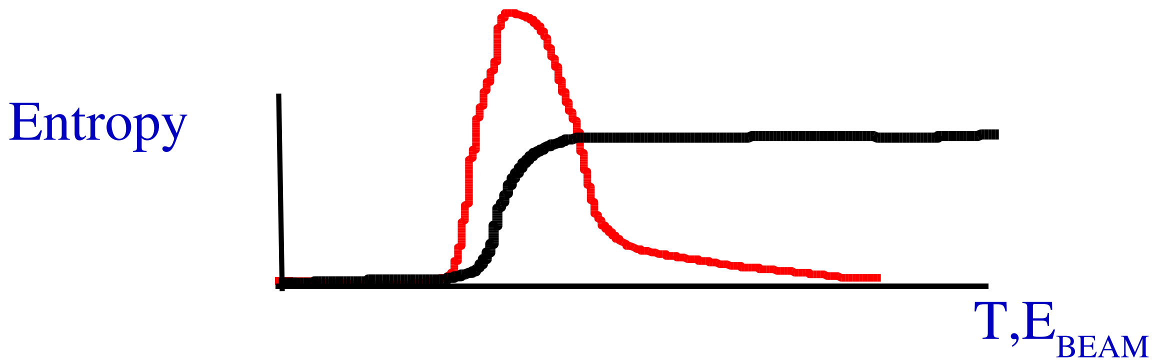
Or: one-particle and two-particle inclusive densities

P_t - Fluctuations

Related to energy fluctuations:

$$\langle (\Delta E)^2 \rangle = T^2 C_V$$

Landau, Vol. 5:

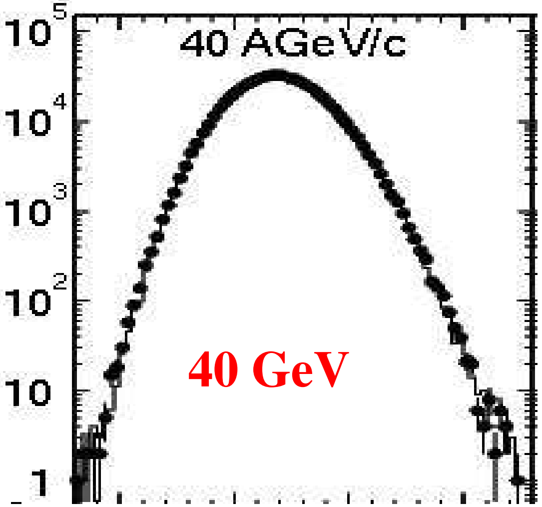


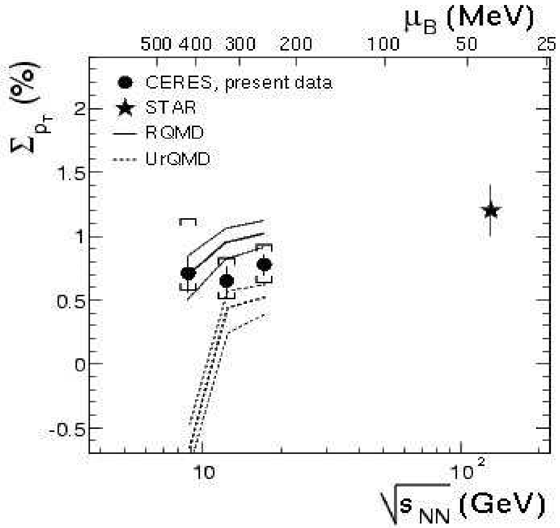
But many other sources as well

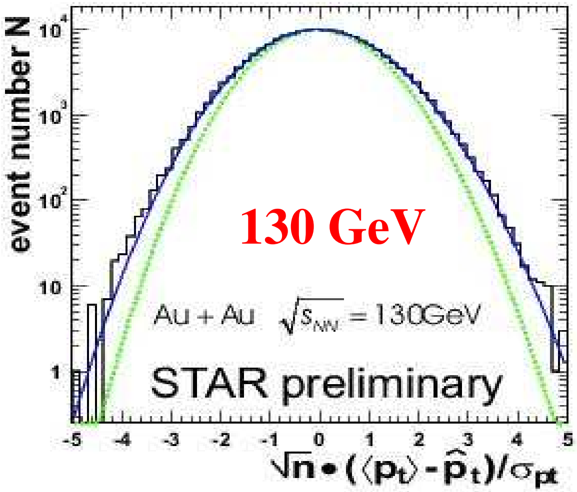
Need excitation function

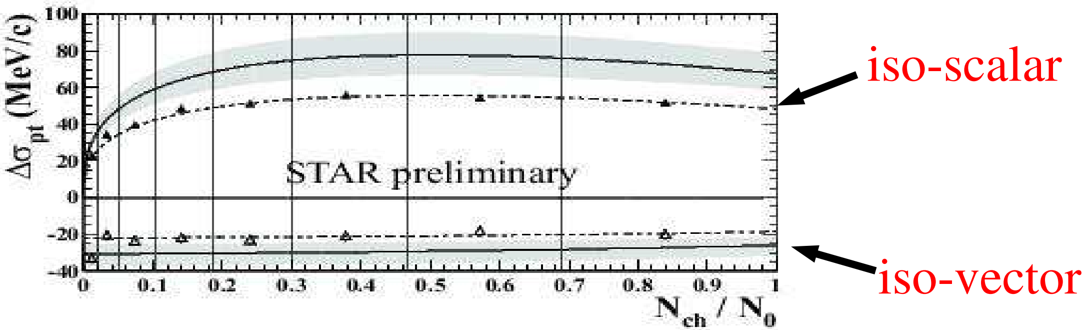
P_t-fluctuations

CERES, nucl-ex/0305002



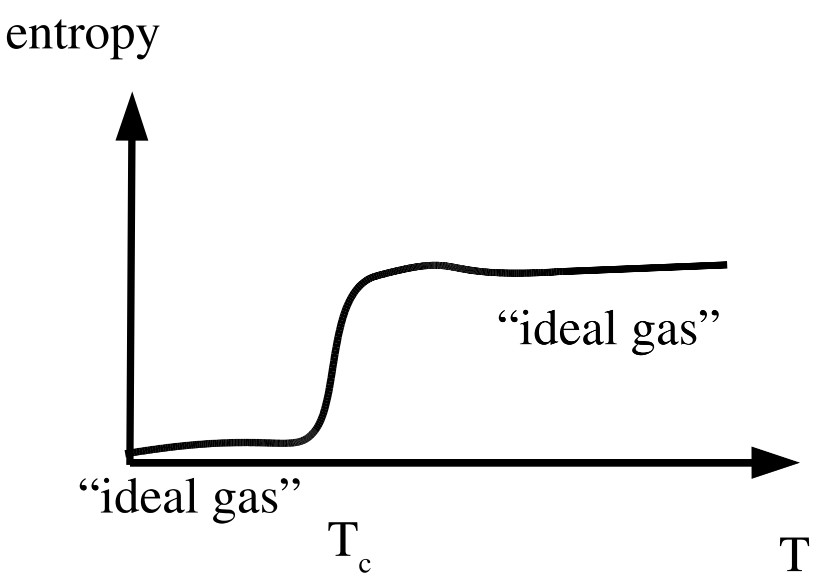


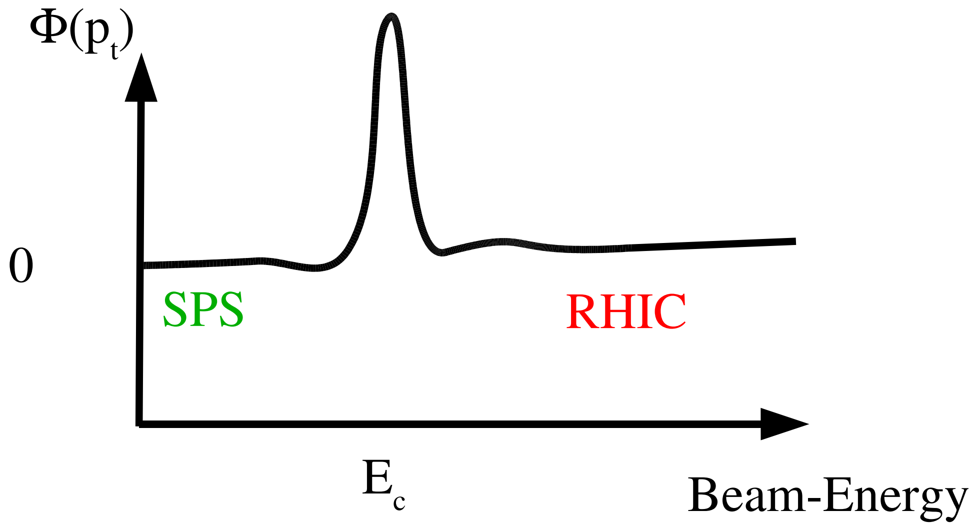




p_t-fluctuations

Are we missing the boat?





Charge Fluctuations

Poisson (classical ideal gas): $\langle (\delta N)^2 \rangle \equiv \langle N^2 \rangle - \langle N \rangle^2 = N$

Charge Fluctuations:

$$\langle (\delta Q)^2 \rangle \equiv \langle Q^2 \rangle - \langle Q \rangle^2 = q^2 (\langle N^2 \rangle - \langle N \rangle^2) = q^2 \langle (\delta N)^2 \rangle = q^2 N$$

Square of charge enters!!

Hadrons:

$q = \pm 1 ; \quad q^2 = 1$

Quarks:

$q = \pm \frac{1}{3}, \pm \frac{2}{3}; \quad q^2 = \frac{1}{9}, \frac{4}{9}$

Fluctuations are sensitive to

fractional charges

of quarks!!!!

Charge Fluctuations

(Asakawa et al, Jeon et al.)

$$\langle(\delta Q)^2\rangle \equiv \langle Q^2\rangle - \langle Q\rangle^2 = q^2 N$$

Need to get rid of particle number (N) dependence to be sensitive to fractional charges

Divide by entropy!

$$\frac{\langle(\delta Q)^2\rangle}{S} \propto q^2$$

Charge fluctuations, correlators and Balance functions

$$\langle (\delta Q)^2 \rangle = \sum_{p,q} \Delta^{++}(p,q) + \Delta^{--}(p,q) - 2\,\Delta^{+-}(p,q)$$

Balance function:
(Bass et al.)

$$B(y) = \frac{1}{2}\left[\frac{\rho_{+-}(y)}{N_+} + \frac{\rho_{-+}(y)}{N_-} - \frac{\rho_{++}(y)}{N_+} - \frac{\rho_{--}(y)}{N_-}\right]$$

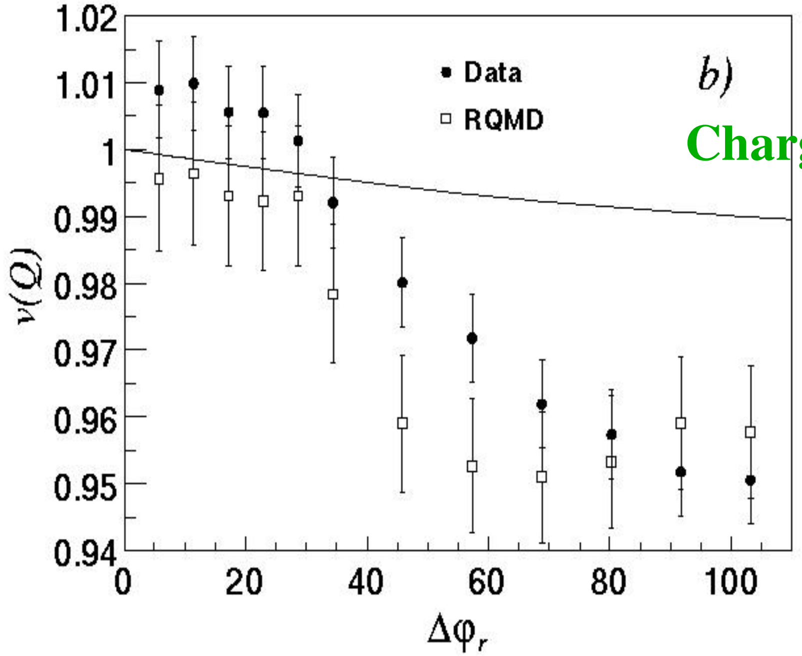
For charge symmetric system $N_+ = N_-$: $B(y) = \frac{1}{2N_+}\left[2\,\Delta_{+-}(y) - \Delta_{++}(y) - \Delta_{--}(y)\right] - 1$

$$\frac{\langle (\delta Q)^2 \rangle}{N_{charge}} = 1 - \int dy\, B(y)$$

Charge Fluctuations

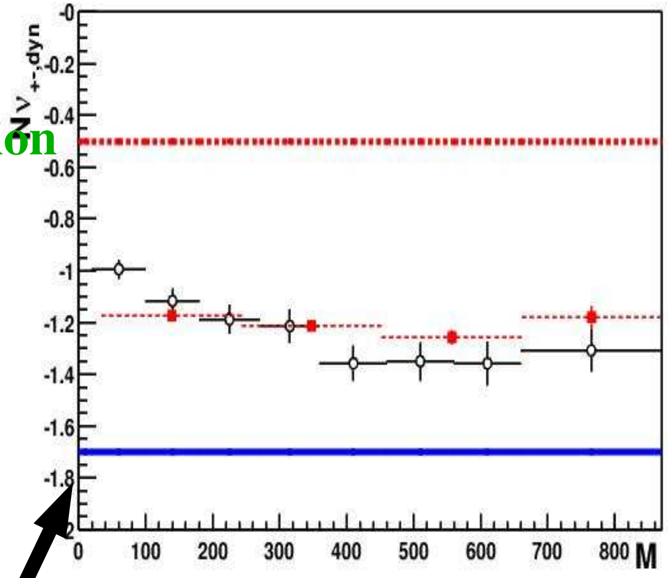
Exp. Data

Phenix, PRL



STAR

Charge conservation

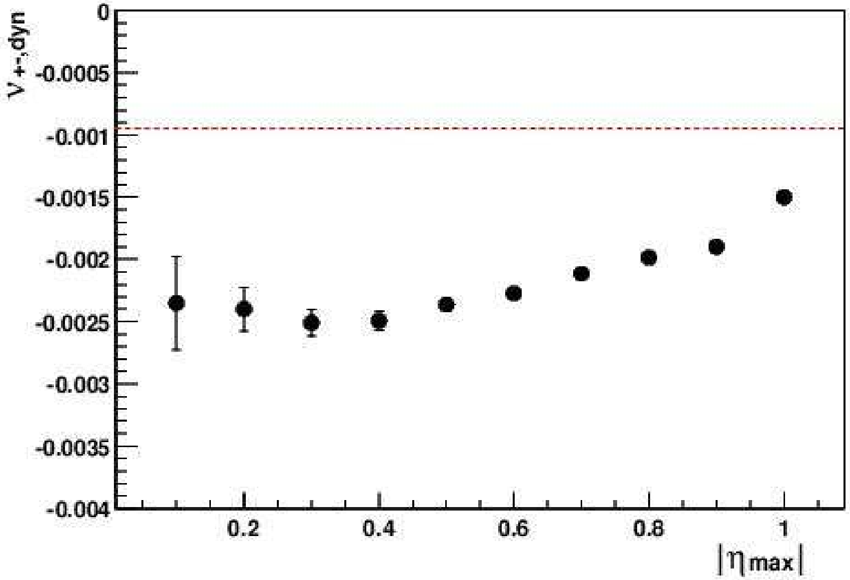


Centrality

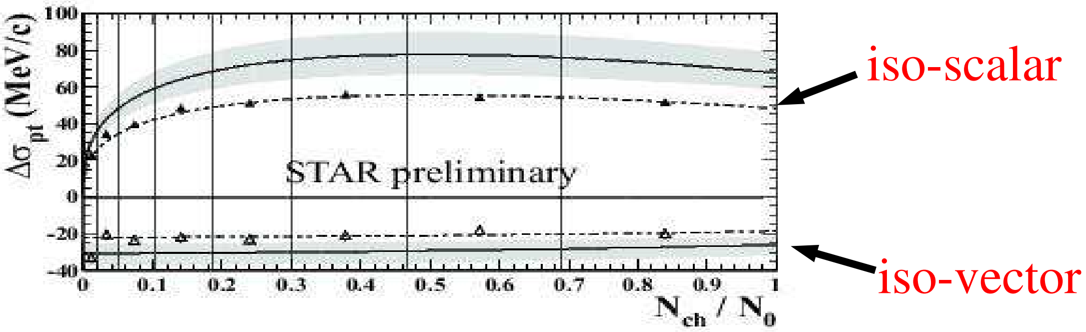
Resonance gas
(is probably somewhat higher, M. Doering)
Constituent quarks? (Bialas)

Charge Fluctuations

STAR



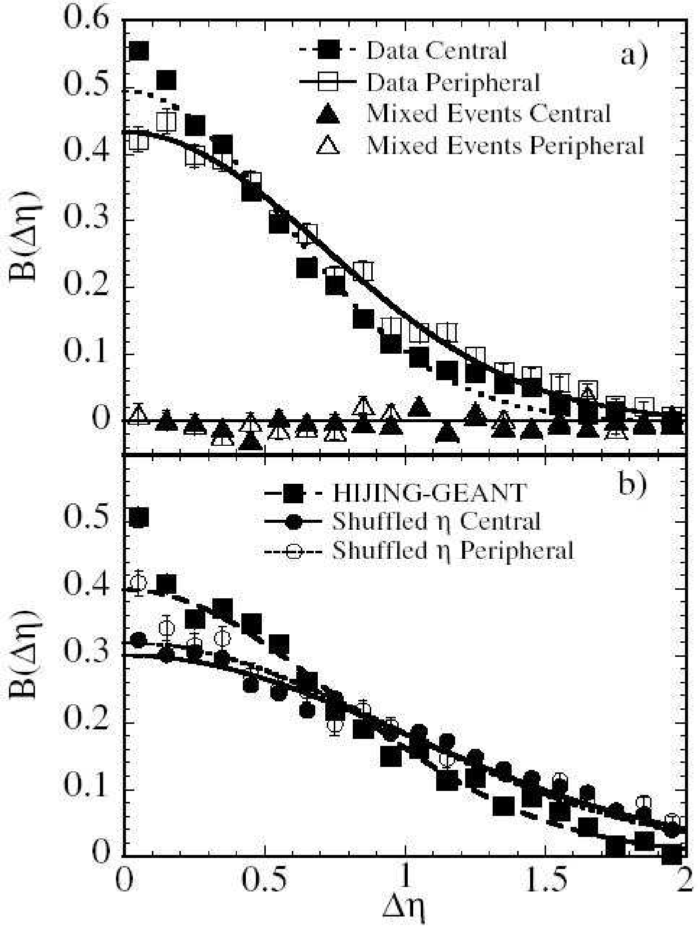
P_t-fluctuations



Iso-vector p_t fluctuations are related to charge fluctuations!

Balance Functions

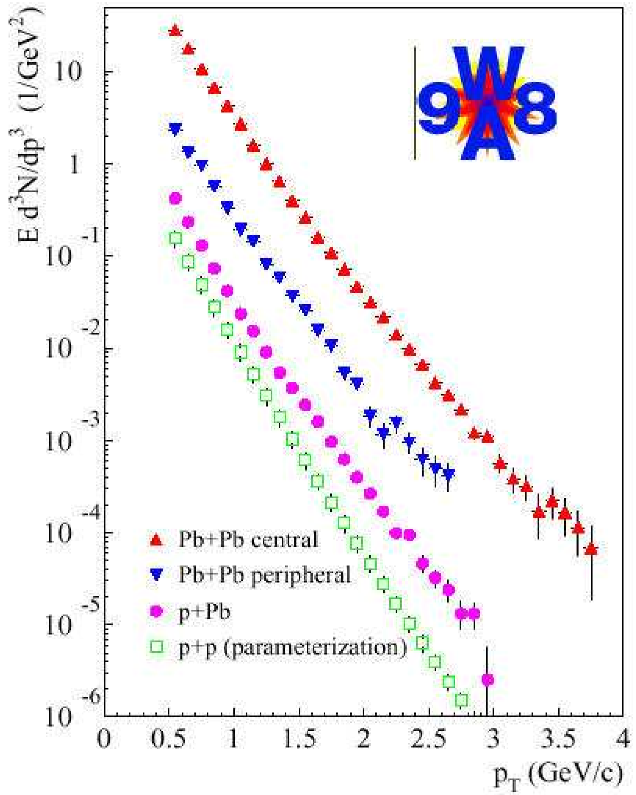
STAR

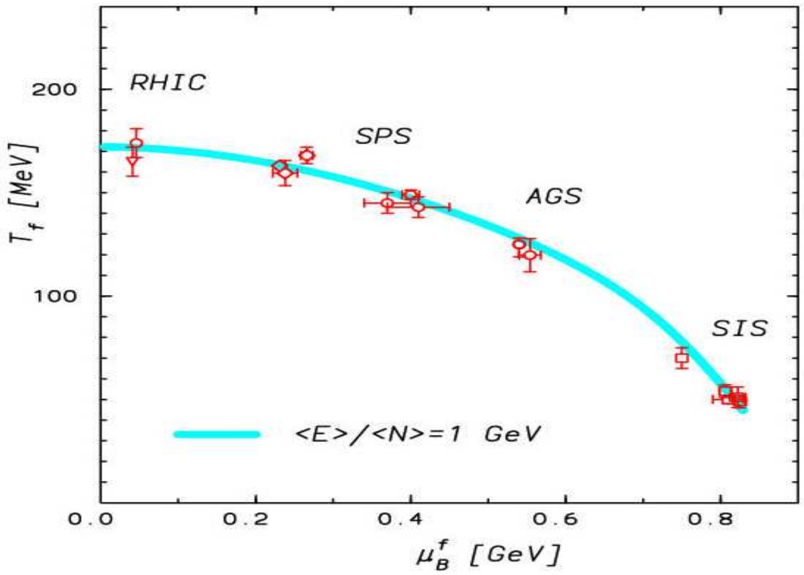


trend consistent with charge fluctuations

clusters of constituent quarks?
(Bialas)

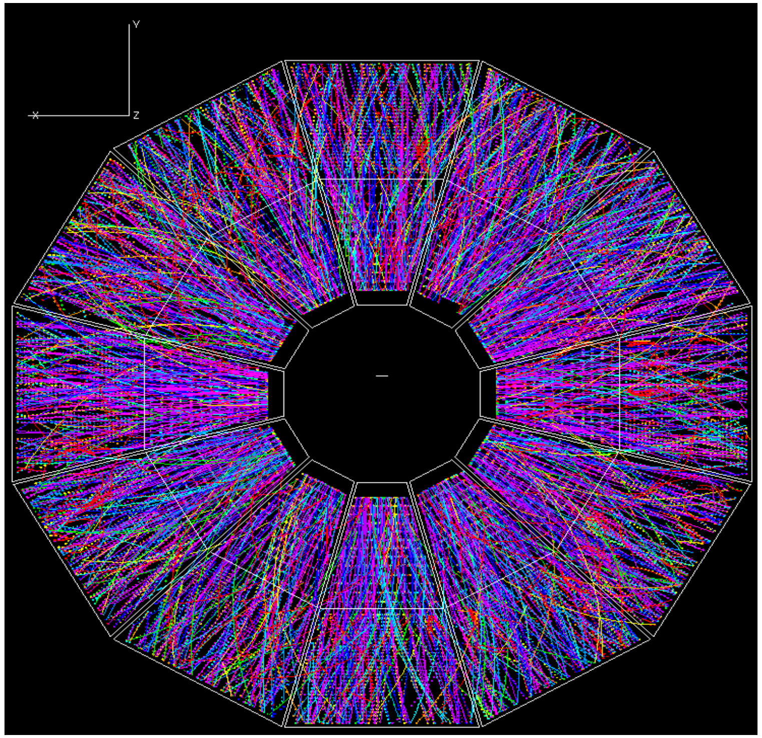
Equilibrium?





K. Redlich et al.

Statistical approach

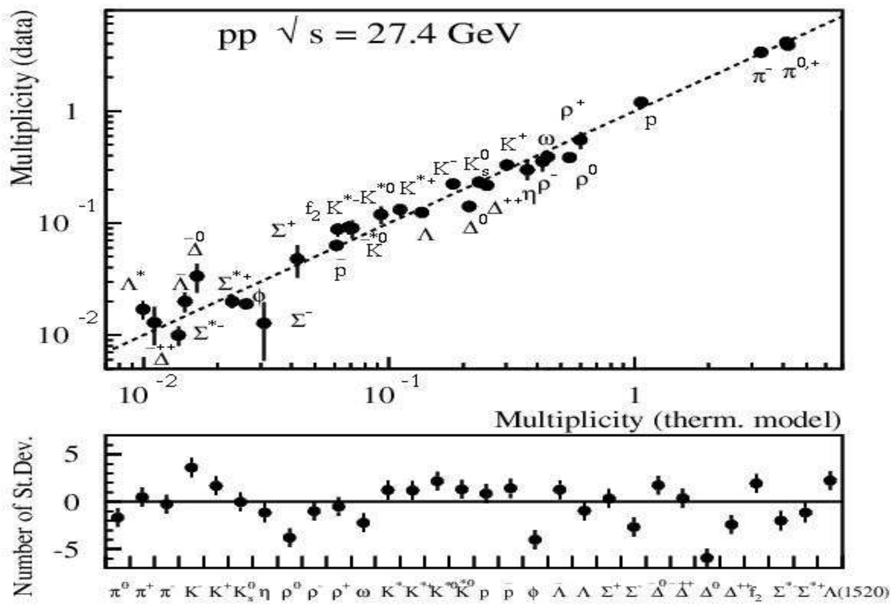


Au+Au

(STAR)

Who has ordered that??

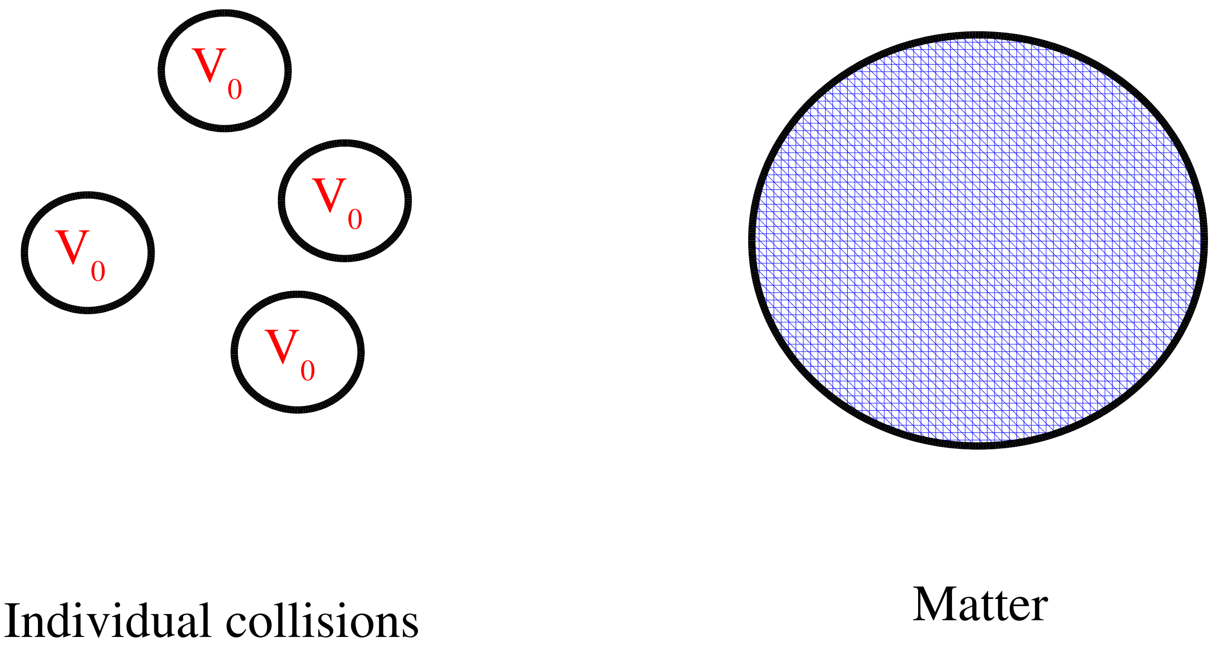
(Becatini et al)



Proton Proton

also: e^+e^-

Matter or not?

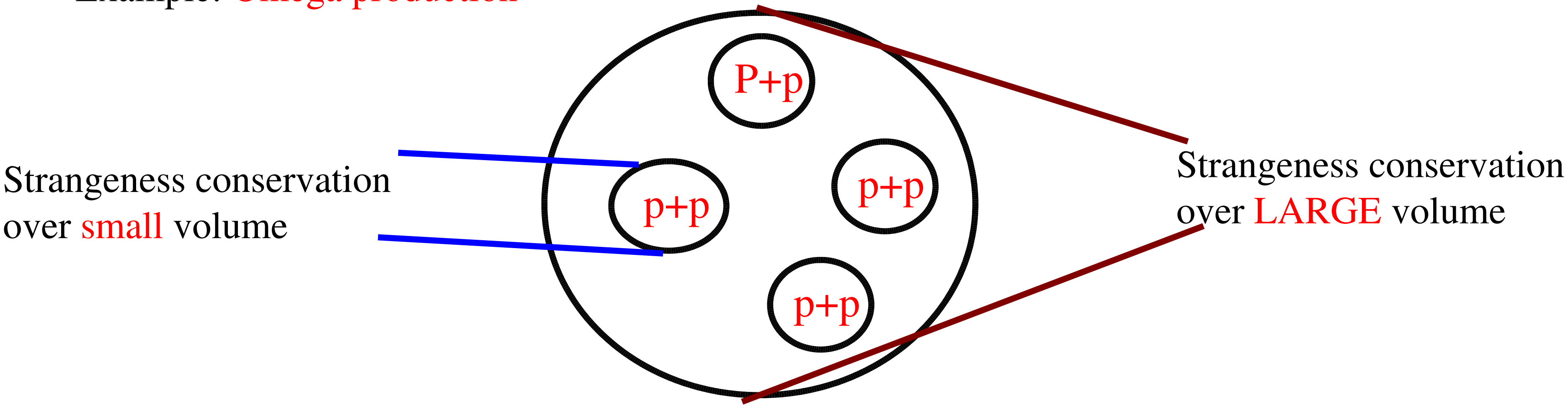


Do we have Matter?

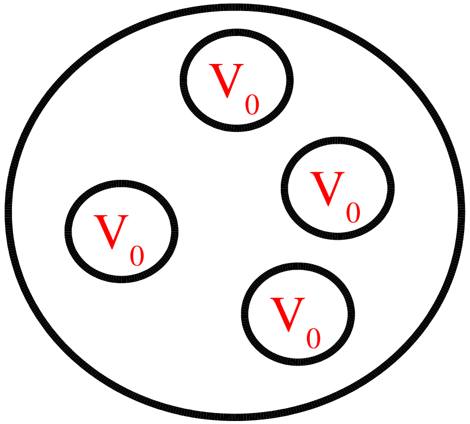
$$\Phi_{A+A} \gg \Phi_{p+p}$$

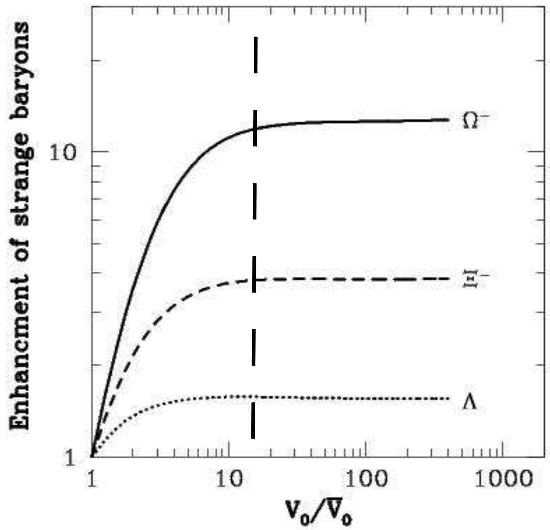
Look at observables which are phase space suppressed in p+p

Example: Omega production



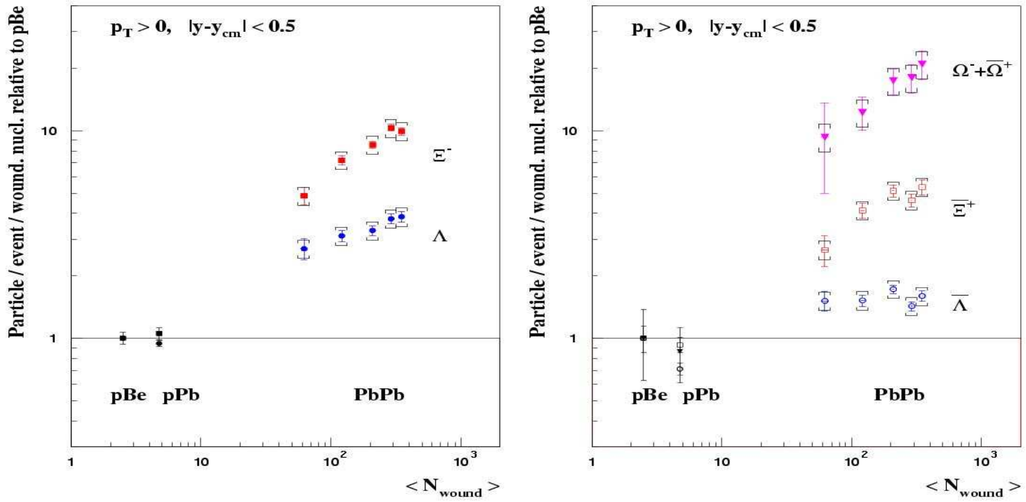
Strangeness equilibrium at SPS ?





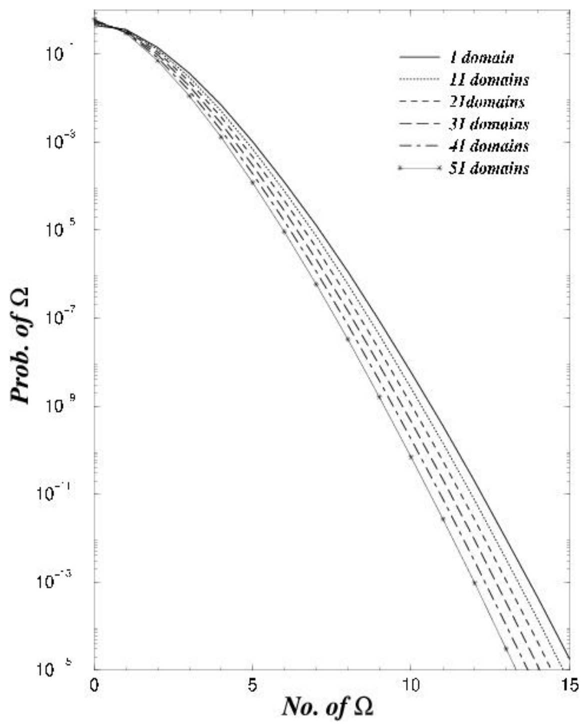
Redlich et al.

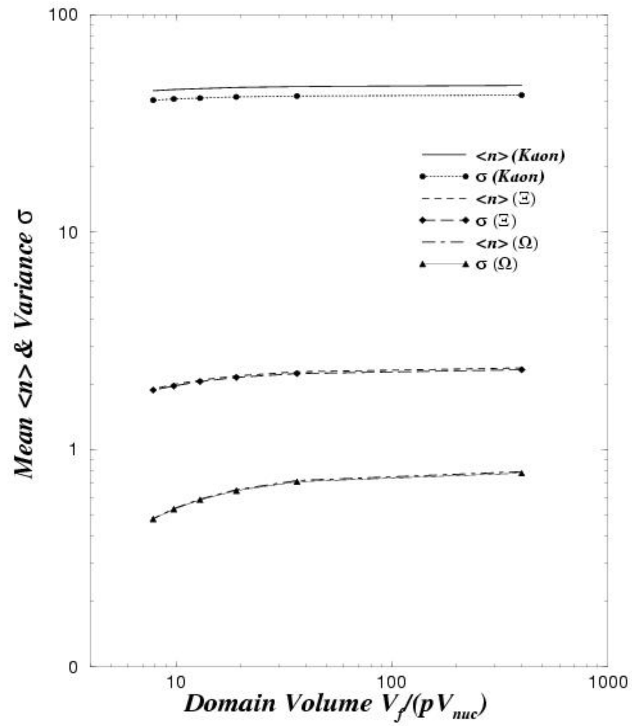
NA57



Strangeness equilibrium at RHIC ?

(A. Majumder)

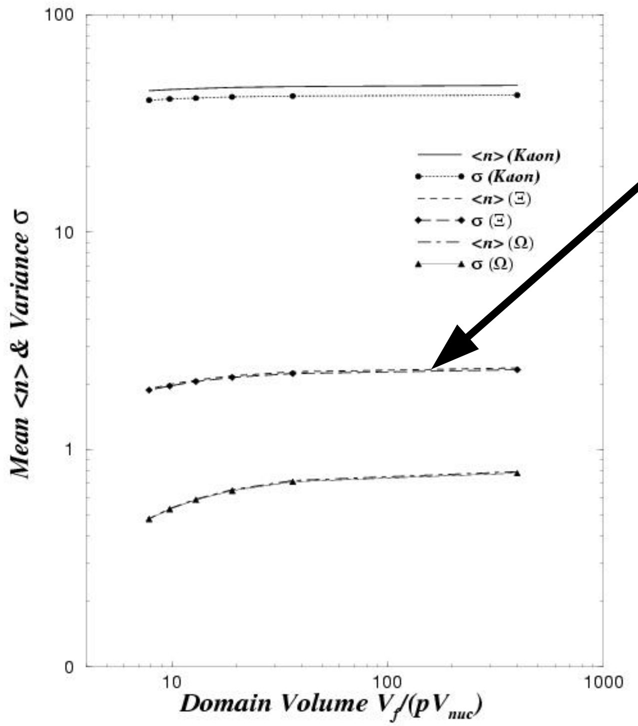




Need to measure 5 OMEGAS per event!!!!

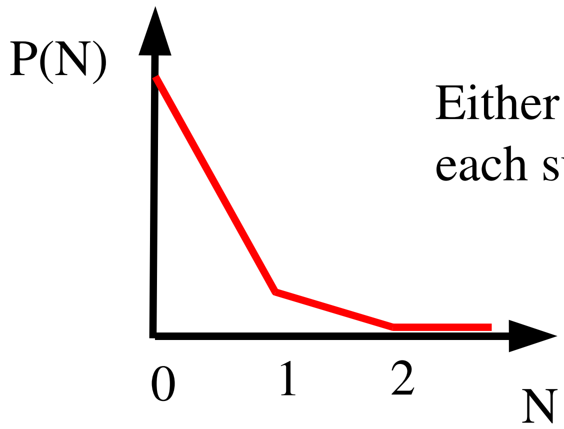
2-particle correlations are misleading

(A. Majumder)



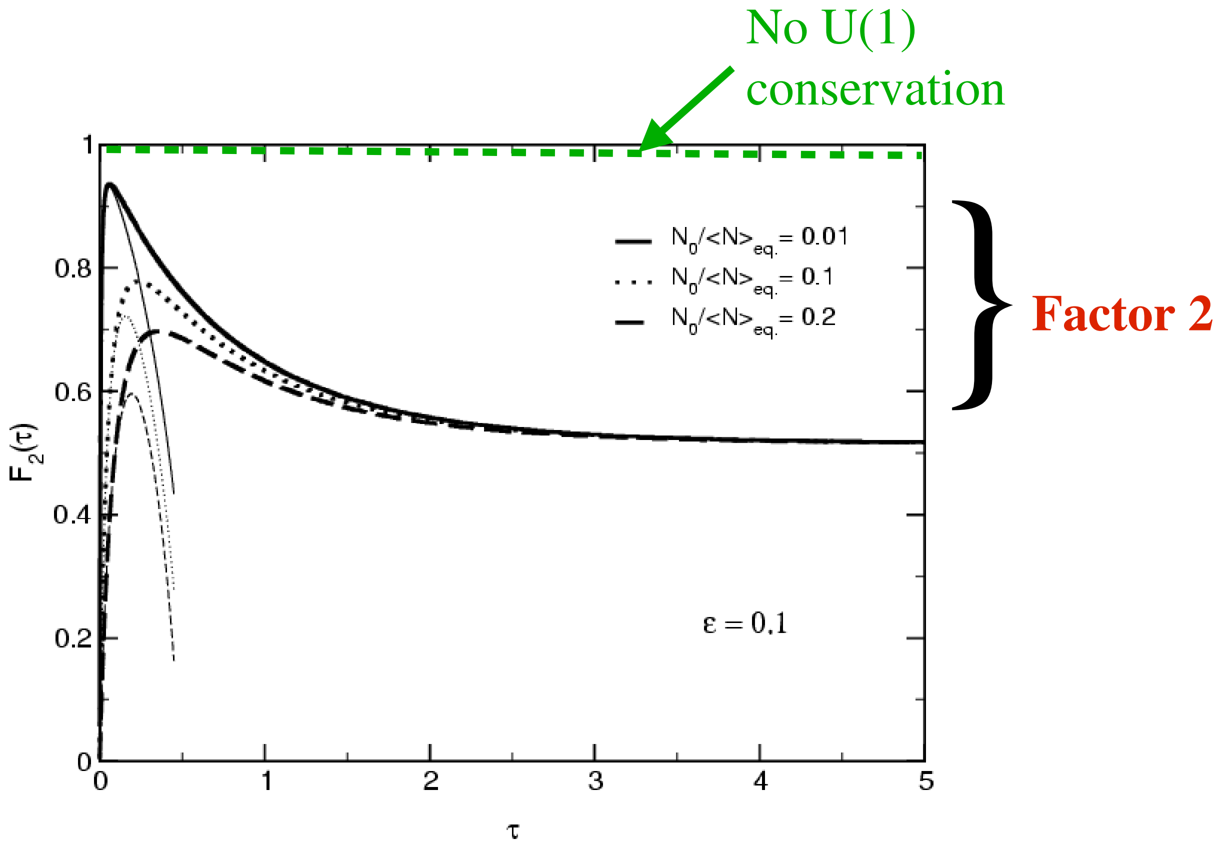
$\langle(\delta N)^2\rangle = \langle N \rangle$ grand canonical???

NO! many binomials make a Poisson!



Either 0 or 1 Omega from each sub-domain

Equilibrium at SIS-energies (2 GeV)



Summary

- Fluctuations measure response of system
- Event-by-event fluctuations simple measure 2 particle correlations
- Fluctuations:
  - Do we see the fractional charges?
  - Understand slight increase of trans. mom. fluctuations
- Equilibrium: Have we generated matter?
- Fluctuations are NEW approach.
- All we need are the 2-particle correlation functions