F. Becattini – University of Florence - ISMD 2003

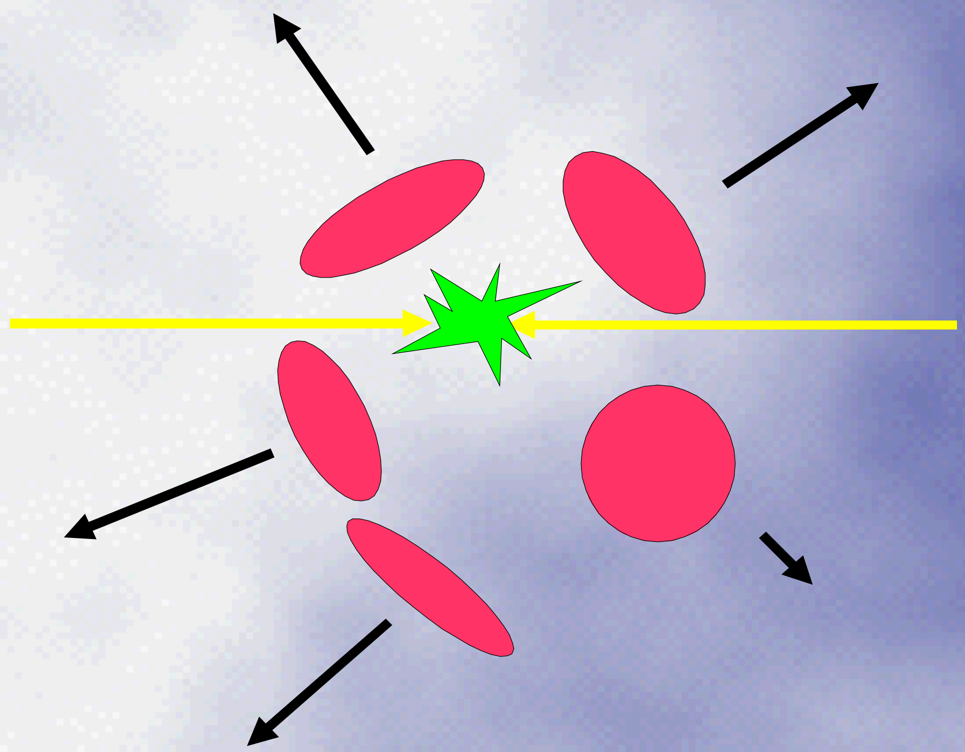
STATISTICAL HADRONIZATION and MICROCANONICAL ENSEMBLE

F. B., L. Ferroni, hep-ph 0307061 and a paper to appear

MOTIVATION: a Monte-Carlo code for the statistical model

OUTLINE

- Introduction
- Microcanonical hadron-resonance gas ensemble
- Monte-Carlo sampling algorithm
- Preliminary results and conclusions



Clusters: four-momentum P, charges $\mathbf{Q}$ and volume V

Statistical hadronization: any multi-hadronic state compatible with cluster quantum numbers is equally likely

→ MICROCANONICAL ENSEMBLE

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AIM : MONTE CARLO SAMPLING OF THE HADRON GAS MICROCANONICAL PHASE SPACE

sample the single hadronic channel first, then assign momenta

N_j : number of particles for the jth hadronic species

statistical weight of a single channel

$$\Omega_{\{N_1,\ \ldots,N_K\}} \equiv \sum_{states} \delta^4(P - P_{state})\big|_{\{N_1,\ \ldots,N_K\}}$$

phase space volume or microcanonical partition function

$$\Omega = \sum_{\{N_1,\ \ldots,N_K\}} \Omega_{\{N_1,\ \ldots,N_K\}} \equiv \sum_{states} \delta^4(P - P_{state})\,\delta_{Q,Q_{state}}$$

General expression for a cluster with volume V and four-momentum P as a *cluster expansion*

$$\Omega_{\{N_1,\ldots,N_K\}} = \lim_{\eta \to 0} \frac{1}{(2\pi)^4} \int_{-\infty - i\eta}^{+\infty - i\eta} dx^0 \int d^3\boldsymbol{x}\, \mathrm{e}^{iP\cdot x} \prod_{j=1}^{K} (\mp 1)^{N_j} \sum_{\{h_{n_j}\}} \prod_{n_j=1}^{N_j} (\mp 1)^{h_{n_j}} \frac{z_{j(h_{n_j})}^{h_{n_j}}(x)}{n_j^{h_{n_j}}\, h_{n_j}!}$$

with

$$z_{j(n)}(x) = \frac{(2J_j+1)V}{(2\pi)^3} \int d^3\boldsymbol{p}\, \exp[-i n x^0 \epsilon + i n \boldsymbol{x}\cdot\boldsymbol{p}] \qquad \sum_{n_j=1}^{N_j} n_j h_{n_j} = N_j$$

In the limit of Boltzmann statistics, with $\sum_{j=1}^{K} N_j \equiv N$

$$\Omega_{\{N_1,\ldots,N_K\}} = \left[\frac{V}{(2\pi)^3}\right]^N \prod_{i=1}^{N} \frac{(2J_i+1)}{N_i!} \int \prod_{i=1}^{N} d^3\boldsymbol{p}_i\, \delta^4\Big(P - \sum_i p_i\Big)$$

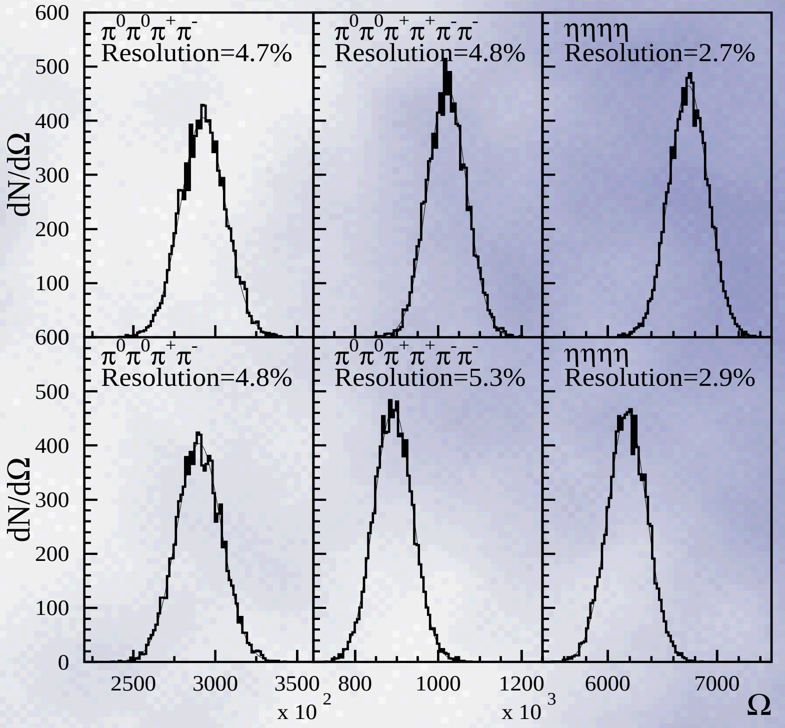
Calculable analytically only in the NR and UR regimes

- The problem of calculating $\Omega_{\{N_1, \ldots, N_K\}}$ for relativistic massive particles with suitable numerical methods tackled by several authors in the '50s
- None of several devised approximations satisfactory for all the allowed channels
- Cerulus and Hagedorn (Suppl. N. Cim. IX (1958) 646) succesfully implemented a Monte-Carlo method

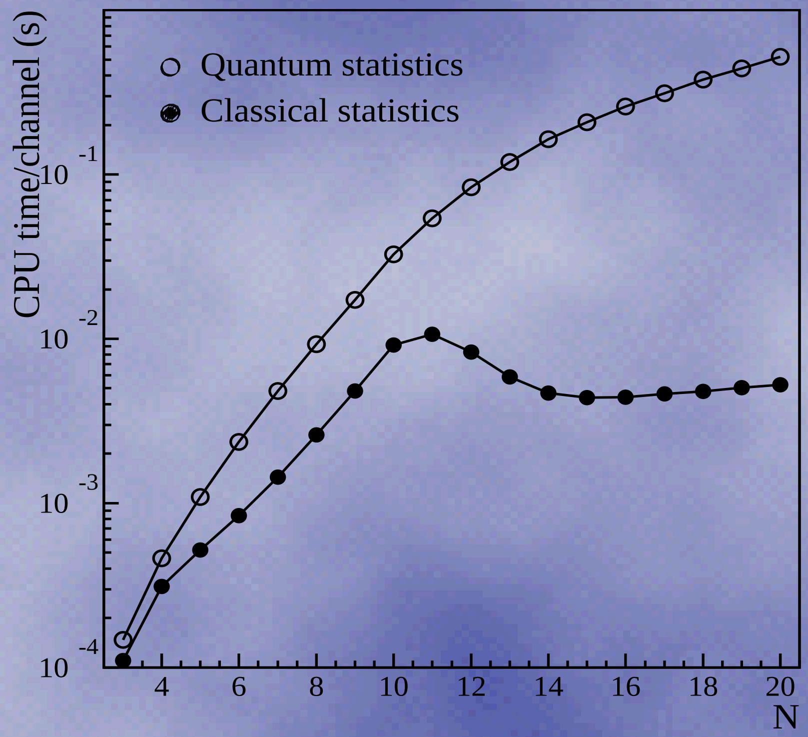
Ω distribution with 1000 MC steps

with QS

no QS



N π^0 channel - 2 GHz clock rate



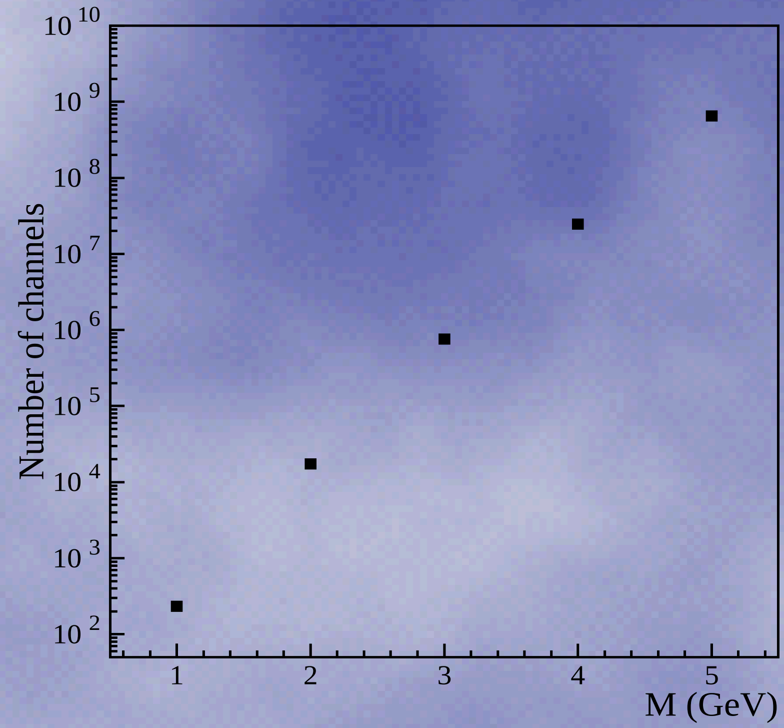
MAIN DIFFICULTY: SIZE

Number of light-flavoured hadronic species (PDG 2002): 271

$N_{max} = M \,/\, \langle m \rangle$

If $N_{max} \ll 271 \;\Rightarrow\; N_{channels} \approx \dfrac{271^{N_{max}}}{N_{max}\,!}$

Time needed to calculate all channels diverges exponentially with *M*

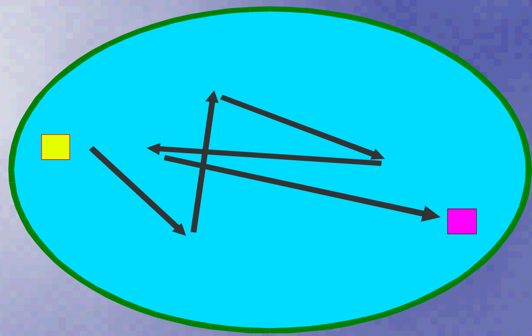


Need a non-deterministic sampling algorithm

METROPOLIS ALGORITHM

First proposed by Werner and Aichelin: Phys. Rev. C 52 (1995) 1584

Random walk in the channel
(or multihadronic configuration) space



Master equation

$$P_m(t+1) - P_m(t) = \sum_n P_n(t)\, w(n \to m) - P_m(t)\, w(m \to n)$$

if $\dfrac{w(n \to m)}{w(m \to n)} = \dfrac{\Omega_m}{\Omega_n}$ → at equilibrium $P_m \propto \Omega_m$

and a channel can be extracted

Design goal: minimize the number of steps needed to reach equilibrium

- Start from fast-sampling distribution as close as possible to the final one

- Clever choice of acceptance and proposal matrix so as to maximize the transition probability

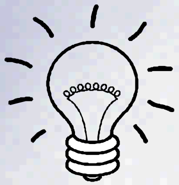
$$w(n \to m) = T(n \to m) A(n \to m)$$

OPTIMAL CHOICE
for A once T is known

$$A(n \to m) = min\left\{1, \frac{\Omega_m T(m \to n)}{\Omega_n T(n \to m)}\right\}$$

OPTIMAL CHOICE for T

$$T(n \to m) = \Omega_m \Longleftrightarrow A(n \to m) = 1$$



set T as the grand-canonical limit of the multihadron multiplicity distribution, the multi-Poisson

$$\Omega_m \to \prod_j \exp(-\langle n_j \rangle) \frac{\langle n_j \rangle^{N_j}}{N_j!} = T(n \to m)$$

$<n_j>$ = canonical averages

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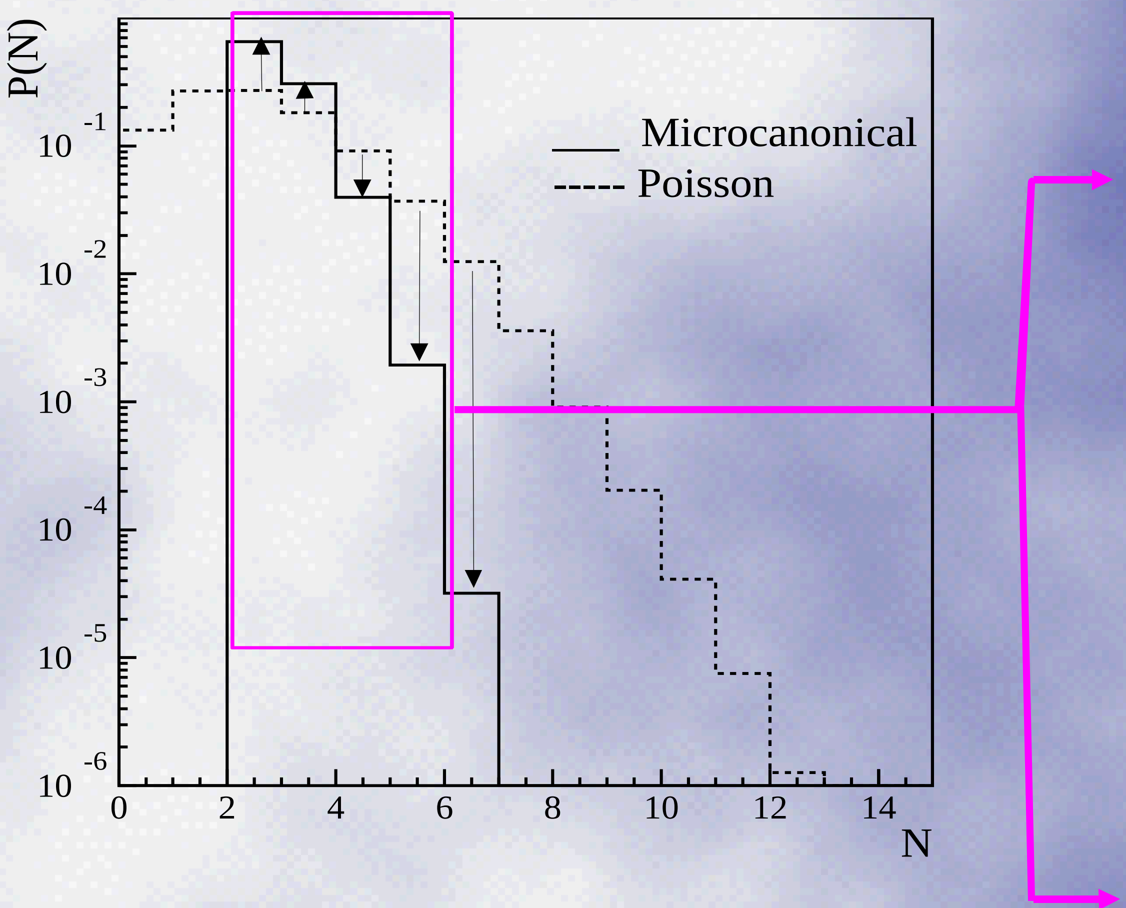
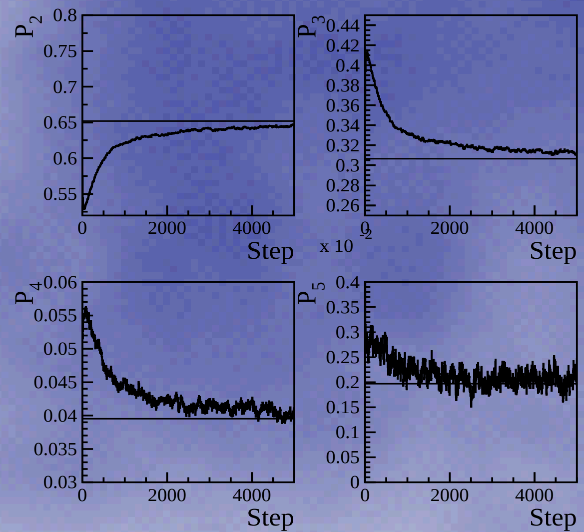
FAST CONVERGENCE TO EQUILIBRIUM !

M=2 GeV, free charges, M/V=0.4 GeV/fm^3

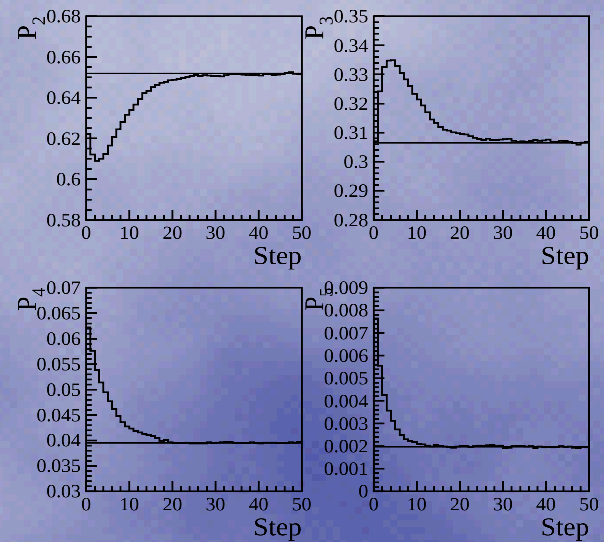


Total multiplicity distribution

'simple' updating rule

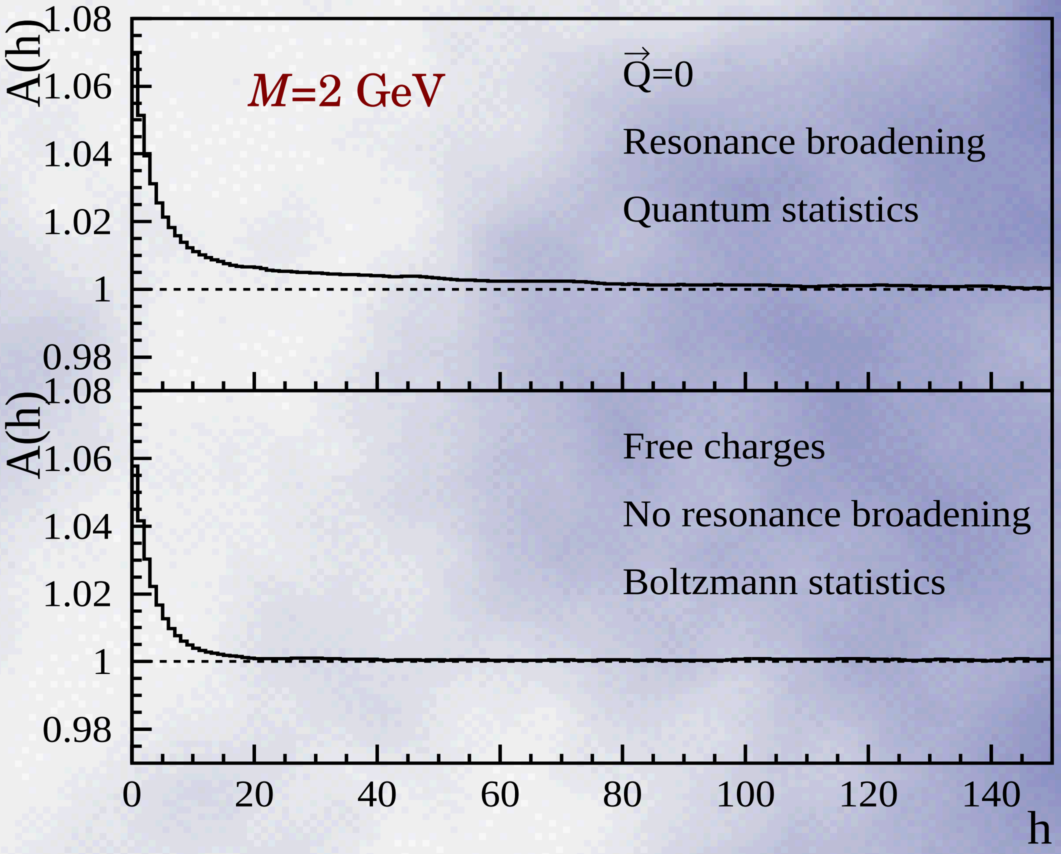


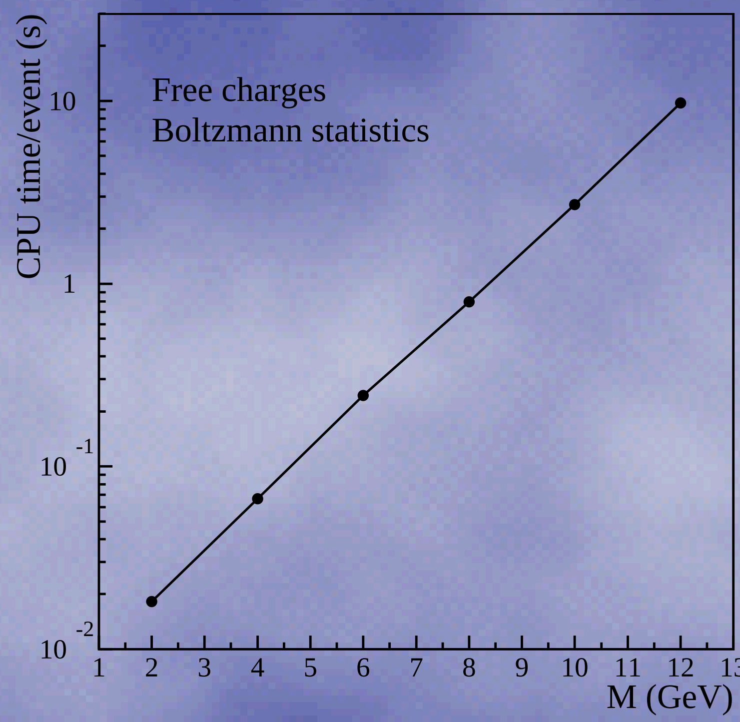
poissonian updating rule



Further advantages of the poissonian updating rule

- Efficient sampling
- Convergence faster for larger clusters (approach grand-canonical limit)
- Low auto-correlation lenght





COMPARISON BETWEEN MICRO AND CANONICAL AVERAGES

Statistical model calculations so far have been done within the canonical ensemble assuming the equivalence between the set of clusters and one global cluster

TEMPERATURE introduced through a saddle-point approximation for large M and V

$$\Omega = \sum_{states} \delta^4 (P - P_{state}) \delta_{\mathbf{Q},\mathbf{Q}_{state}} \sim \exp[M/T] Z_{can}(\mathbf{Q}) \qquad P = (M, \mathbf{0})$$

canonical average multiplicity

$$\langle n_j \rangle = \frac{(2J_j+1)V}{(2\pi)^3} \int d^3p \, \exp(-\sqrt{p^2+m_j^2}/T) \frac{Z_{can}(\mathbf{Q}-\mathbf{q}_j)}{Z_{can}(\mathbf{Q})}$$

HOW LARGE M AND V SHOULD BE ?

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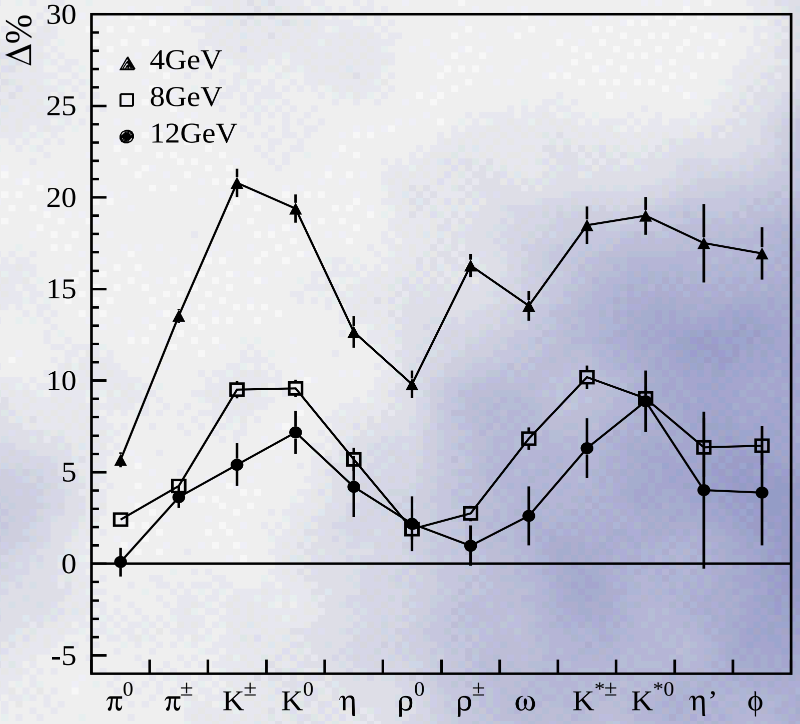
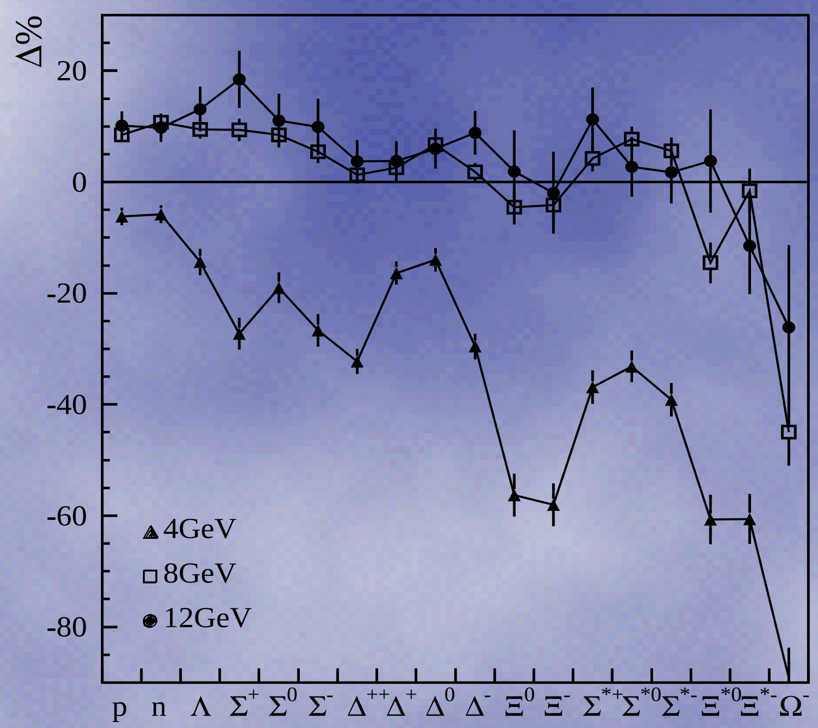
Study of $\dfrac{\langle n \rangle_{micro} - \langle n \rangle_{can}}{\langle n \rangle_{can}}$ with $\boldsymbol{Q} = 0$ and $\boldsymbol{M}/\boldsymbol{V} = 0.4$ GeV/fm^3





see also F. Liu et al.
hep-ph 0304174

Mass (GeV)	Temperature (MeV)
4	169.3
8	164.6
12	162.7

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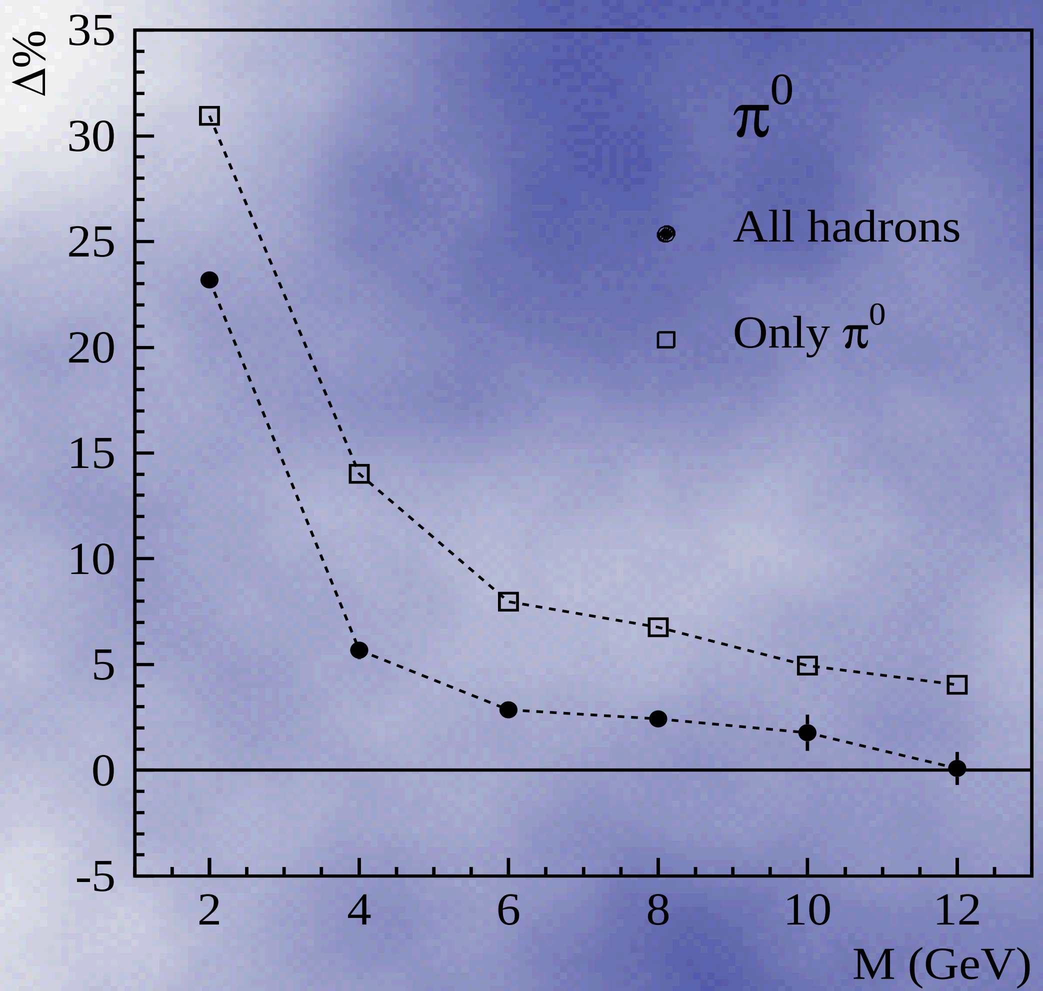
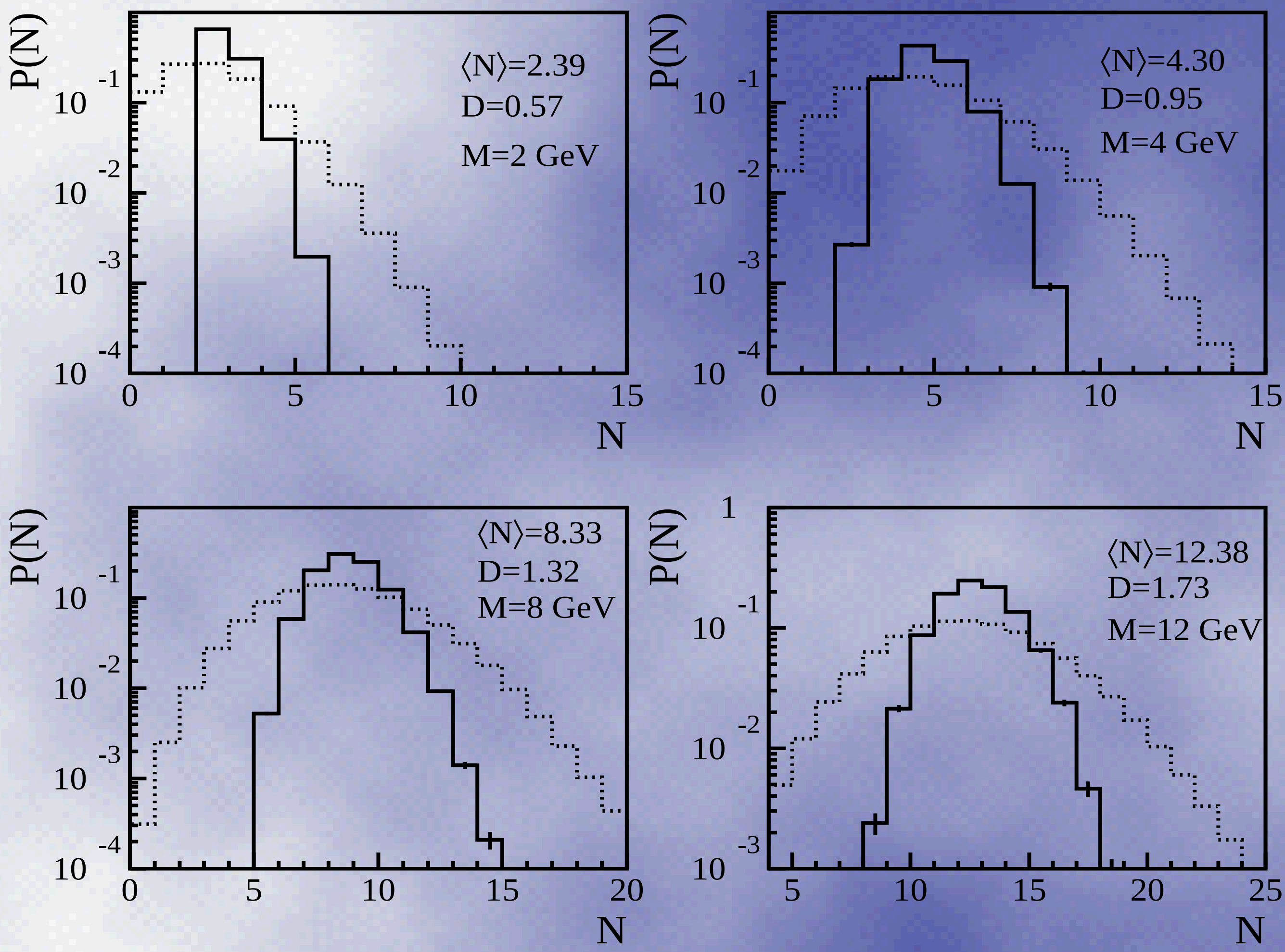
Increasing the number of degrees of freedom gets microcanonical and canonical averages closer



MICROCANONICAL VS CANONICAL MULTIPLICITY DISTRIBUTIONS

Free charges,

M/V=0.4 GeV/fm^3



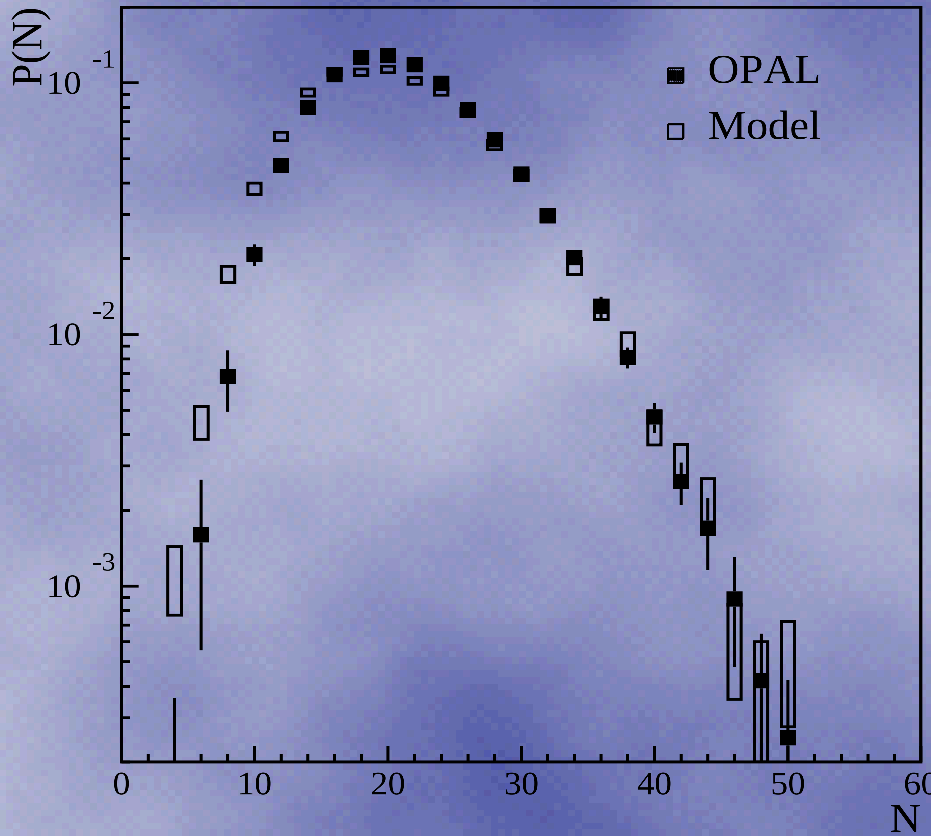
Preliminary simplified version of an e^+e^- event generator

- Run parton shower program
- Assign final (virtual) partons a volume $V = M / 0.35$ GeV/fm^3 (and $\gamma_S = 0.65$)
- Hadronize the clusters with Metropolis algorithm
- Check overall conservation of charge (accept or reject)

Charged track multiplicity distribution at

$\sqrt{s}$ = 91.2 GeV

10K events with parton shower in Jetset7.4



Average multiplicities

Mesoni	Misura	GSPS	LPS
π^0	9.61 ± 0.29	9.738 ± 0.031	9.783 ± 0.031
$\pi^\pm$	8.50 ± 0.10	8.644 ± 0.021	8.621 ± 0.020
$K^\pm$	1.127 ± 0.026	1.0955 ± 0.0074	1.0621 ± 0.0073
K^0_S	1.0376 ± 0.0096	1.062 ± 0.010	1.035 ± 0.010
η	1.059 ± 0.086	0.888 ± 0.0094	0.8858 ± 0.0094
ρ^0	1.40 ± 0.13	1.052 ± 0.010	1.050 ± 0.010
$\rho^\pm$	1.20 ± 0.22	0.9963 ± 0.0071	1.0053 ± 0.0071
ω	1.024 ± 0.059	0.9468 ± 0.0097	0.9646 ± 0.0098
K^{*0}	0.357 ± 0.022	0.3570 ± 0.0042	0.3448 ± 0.0041
$K^{*\pm}$	0.370 ± 0.012	0.3544 ± 0.0042	0.3412 ± 0.0041
η'	0.166 ± 0.047	0.0866 ± 0.0029	0.0890 ± 0.0030
ϕ	0.0977 ± 0.0058	0.1081 ± 0.0033	0.1141 ± 0.0034
$f_2(1270)$	0.188 ± 0.020	0.1006 ± 0.0032	0.1088 ± 0.0033
K^*_2	0.036 ± 0.011	0.02275 ± 0.0011	0.0225 ± 0.0011
$a_0(980)$	0.135 ± 0.054	0.0875 ± 0.0021	0.0823 ± 0.0020
f_0	0.1555 ± 0.0085	0.0674 ± 0.0026	0.0700 ± 0.0026
$f'_2(1525)$	0.0120 ± 0.0058	0.0087 ± 0.0009	0.0107 ± 0.0010

GSPS: Parton shower developed by R. Ugoccioni, A. Giovannini, S. Lupia

LPS: Parton shower in JETSET 7.4

Barioni	Misura	GSPS	LPS
p	0.519 ± 0.018	0.5963 ± 0.0077	0.58585 ± 0.0054
Λ	0.1943 ± 0.0038	0.2127 ± 0.0033	0.20015 ± 0.0032
Σ^+	0.0535 ± 0.0052	0.0512 ± 0.0023	0.0487 ± 0.0016
Σ^0	0.0389 ± 0.0041	0.05245 ± 0.0016	0.0479 ± 0.0015
Σ^-	0.0410 ± 0.0037	0.0451 ± 0.0015	0.0428 ± 0.0015
$\Sigma^\pm$	0.0868 ± 0.0087	0.0963 ± 0.0031	0.0915 ± 0.0030
Δ^{++}	0.044 ± 0.016	0.0906 ± 0.0021	0.0870 ± 0.0021
Ξ^-	0.01319 ± 0.00052	0.01305 ± 0.0008	0.01315 ± 0.00081
Σ^*	0.0118 ± 0.0011	0.02225 ± 0.0010	0.02165 ± 0.0010
$\Lambda(1520)$	0.0112 ± 0.0014	0.01205 ± 0.0008	0.0114 ± 0.0007
Ξ^*	0.00289 ± 0.00050	0.0047 ± 0.0005	0.0046 ± 0.0005
Ω^-	0.00062 ± 0.00012	0.0016 ± 0.0003	0.0005 ± 0.0002

Conclusions and Outlook

- Towards a Monte-Carlo code for the statistical model of hadronization. Next steps: inclusion of particle momenta and BEC (L. Ferroni, T. Gabbriellini, A. Keranen, collaboration with Nantes)
- Easily matcheable to parton shower cascade programs with pre-confinement cluster formation (HERWIG)
- Comparison between micro and canonical ensemble: for M >~ 8 GeV average multiplicities sufficiently close
- Many more tests possible